The truck scheduling problem at crossdocking terminals: exclusive versus mixed mode

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Outline

• What is cross-docking?
• Exclusive versus mixed mode
• Problem statement and notations
• Time-indexed formulation
• Minimizing number of double purpose gates
• Computational results
• Conclusions and future research
What is cross-docking?

- Warehouse management concept
- Items delivered by inbound trucks are immediately sorted out, reorganized, and loaded into outbound trucks
- Advantages:
  - faster deliveries
  - lower inventory costs
  - reduction of warehouse space requirement
- Storage and length of stay of a product in the warehouse are limited
- ⇒ Appropriate coordination of inbound and outbound trucks
The truck scheduling problem

• Succession of truck processing at dock doors
• Minimize storage usage during product transfer
• Internal organization of the warehouse: not explicitly taken into consideration
• We do not model the resources that may be needed to load or unload the trucks
Exclusive mode

Each dock is exclusively dedicated either to inbound or to outbound operations
Mixed mode

An intermixed sequence of inbound and outbound trucks to be processed per dock is allowed.

Problem statement and notations

- A set of incoming trucks $i \in I$ need to be unloaded
- A set of outgoing trucks $o \in O$ need to be loaded
- The processing time of truck $j \in I \cup O$ equals $p_j$
- Every truck has its release time $r_j$ (planned arrival time) and its deadline $\tilde{d}_j$ (latest allowed departure time)
- Precedence relations $(i, o) \in P \subset I \times O$: $w_{io}$ the number of pallets transshipped from $i$ to $o$
- $s_j$ is the starting time of the handling of truck $j$
- There are $n$ docks that can be used in mixed mode
Conceptual problem statement

\[
\min \sum_{(i,o) \in P} w_{io}(s_o - s_i)
\]

subject to

\[
\begin{align*}
    s_j & \geq r_j & \forall j \in I \cup O \\
    s_j + p_j & \leq \tilde{d}_j & \forall j \in I \cup O \\
    s_o - s_i & \geq 0 & \forall (i,o) \in P \\
    |A_t| & \leq n & \forall t \in T
\end{align*}
\]

\[A_t = \{ j \in I \cup O | s_j \leq t < s_j + p_j \} \]
the set containing all tasks being executed at time \( t \)

\[T \]
the set containing all time instants considered
Time-indexed (linear) formulation

For all inbound trucks $i \in I$ and all time periods $\tau \in \mathcal{T}_i$,

$$x_{i\tau} = \begin{cases} 1 & \text{if the unloading of inbound truck } i \text{ is started during time period } \tau, \\ 0 & \text{otherwise,} \end{cases}$$

with $\mathcal{T}_i = [r_i + 1, \tilde{d}_i - p_i + 1]$, the relevant time window for inbound truck $i$.

For all outbound trucks $o \in O$ and all time periods $t \in \mathcal{T}_o$,

$$y_{o\tau} = \begin{cases} 1 & \text{if the loading of outbound truck } o \text{ is started during time period } \tau, \\ 0 & \text{otherwise,} \end{cases}$$

with $\mathcal{T}_o = [r_o + 1, \tilde{d}_o - p_o + 1]$. 
Time-indexed formulation

\[
\min \sum_{(i,o) \in P} \sum_{\tau \in T} w_{io\tau} (y_{o\tau} - x_{i\tau})
\]

subject to

\[
\sum_{\tau \in T_i} x_{i\tau} = 1 \quad \forall i \in I \quad (1)
\]

\[
\sum_{\tau \in T_o} y_{o\tau} = 1 \quad \forall o \in O \quad (2)
\]

\[
\sum_{\tau \in T} \tau (x_{i\tau} - y_{o\tau}) \leq 0 \quad \forall (i, o) \in P \quad (3)
\]

\[
\sum_{i \in I} \sum_{u=\tau-p_i+1}^{\tau} x_{iu} + \sum_{o \in O} \sum_{u=\tau-p_o+1}^{\tau} y_{ou} \leq n \quad \forall \tau \in T \quad (4)
\]

\(x_{i\tau}, y_{o\tau} \in \{0, 1\}\)
Two different precedence constraints

\[
\sum_{\tau \in \mathcal{T}} \left( x_{i\tau} - y_{o\tau} \right) \leq 0 \quad \forall (i, o) \in P
\]

\[
\sum_{u=1}^{\tau} x_{iu} - \sum_{u=1}^{\tau} y_{ou} \leq 0 \quad \forall (i, o) \in P; \forall \tau \in \mathcal{T}
\]

- Aggregated versus disaggregated constraint
- Disaggregated is theoretically stronger
- The additional CPU time needed to solve the larger linear program does not always counterbalance the significant improvement of the bound
Generation of instances

- \( n \in \{10, 20, 30\} \)
- \( |I| \in \{3n, 4n, 5n\} \)
- \( |O| = \alpha \times |I| \) with \( \alpha = \{0.8, 1, 1.2\} \)
- \( p_j \in [\beta, 30] \) with \( \beta = \{10, 20, 30\} \)
- \( r_i \in [1, \frac{\delta \sum p_j}{n}] \) with \( \delta = \{0.3, 0.6, 0.9\} \)
- \( \tilde{d}_o \in [1.5 \times d_o, 5 \times d_o] \) with \( d_o = \max_{(i,o) \in P} \{r_i + p_o\} \)
- \( r_o \in [\max_{(i,o) \in P} \{r_i\}, \tilde{d}_o - p_o] \)
- \( \tilde{d}_i \in [1.5(r_i + p_i), \min_{(i,o) \in P} \{\tilde{d}_o - p_o\} + p_i, \max_{(i,o) \in P} \{\tilde{d}_o\}] \)
- \( w_{io} \in \left[\frac{0.8p_i}{\gamma}, \frac{1.2p_i}{\gamma}\right] \) with \( \gamma \in [1, \frac{p_i}{3}] \)
- \( |T| = \max_{o \in O} \{\tilde{d}_o\} \)
Computational results

Solving with Cplex ($T_{cpu} \leq 5$ minutes), using aggregated constraints

| $n$ | $|I|$ | exclusive mode | mixed mode |
|-----|------|----------------|------------|
|     |      | infeasible    | feasible   | optimal   | infeasible | feasible   | optimal   |
| 10  | 30   | 23.81%        | 65.08%     | 11.11%    | 0.00%      | 84.13%     | 15.87%    |
| 10  | 40   | 23.81%        | 74.60%     | 1.59%     | 0.00%      | 96.83%     | 3.17%     |
| 10  | 50   | 26.98%        | 63.49%     | 0.00%     | 0.00%      | 98.41%     | 0.00%     |
| 20  | 60   | 12.70%        | 82.54%     | 3.17%     | 0.00%      | 96.83%     | 3.17%     |
| 20  | 80   | 22.22%        | 73.02%     | 0.00%     | 0.00%      | 98.41%     | 0.00%     |
| 20  | 100  | 23.81%        | 61.90%     | 0.00%     | 0.00%      | 90.48%     | 0.00%     |
| 30  | 90   | 15.87%        | 77.78%     | 0.00%     | 0.00%      | 95.24%     | 4.76%     |
| 30  | 120  | 19.05%        | 65.08%     | 0.00%     | 0.00%      | 95.24%     | 0.00%     |
| 30  | 150  | 28.57%        | 53.97%     | 0.00%     | 0.00%      | 79.69%     | 0.00%     |
| total|      | 21.87%        | 68.61%     | 1.76%     | 0.00%      | 92.95%     | 3.00%     |

- average GAP with respect to LP solution is 13.28%
- average GAP with respect to a Lagrangian relaxation is 6.73%
- exclusive versus mixed mode: improvement of 8%
Minimizing number of double purpose gates

- switching completely to mixed mode might impact significantly the company organization
- it might not be needed that every gate has a double purpose
- determining the gain obtained when switching only a small number of docks
Second time-indexed formulation

- $\delta_i^* (\delta_o^*)$ is the optimal value of $\delta_i (\delta_o)$, the number of gates that we allow to unload (load) incoming (outgoing) trailers, on top of $n_i \ (n_o)$
- we solve the presented time-indexed formulation $\Rightarrow z^*$
- minimize $\delta = \delta_i + \delta_o$ and add the following constraints:

\[
\sum_{(i,o) \in P} \sum_{\tau \in T} w_{io\tau} (y_{o\tau} - x_{i\tau}) \leq z^* \tag{5}
\]
\[
\sum_{i \in I} \sum_{u=\tau-p_i+1}^{\tau} x_{iu} \leq n_i + \delta_i \quad \forall \tau \in T \tag{6}
\]
\[
\sum_{o \in O} \sum_{u=\tau-p_o+1}^{\tau} y_{ou} \leq n_o + \delta_o \quad \forall \tau \in T \tag{7}
\]
\[
n_i + n_o = n \tag{8}
\]
Computational results
Computational results
Conclusions and future research

Conclusions

• Truck scheduling problem at cross-docking terminals
• Time-indexed (integer programming) formulation
• Mixed mode versus exclusive mode
• Number of gates to be changed from exclusive to mixed mode

Future research

• Special case of the problem with $p_i = p \Rightarrow$ Generalisation of $Pm|r_i, \hat{d}_i, p_i = p| \sum w_i C_i$ (complexity open)
• Extension: as an alternative, pallets can also be stocked at the gate
Future research: staging

Questions?