A mass-flow MILP formulation for energy-efficient supplying in assembly lines

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Outline

1. Introduction
2. Problem definition
3. Energy analysis
4. Mathematical modeling
5. Experimentation
6. Conclusion
Introduction

ECO-INNOVERA Project - EASY (Energy Aware Feeding Systems)

- Skövde University / Volvo VCE
- Universidad de Navarra / VW Polo
- LAAS-CNRS, Toulouse

Motivations

- Develop more sustainable internal logistic practices
- Improve the energy efficiency of production systems
- Focus on the supplying system of assembly lines
Introduction

EASY’s objectives

- Energy oriented analysis of supplying systems
- Determination of the most significant energy parameters
- Design new effective optimization and simulation methods
- Help decision-makers to find a good balance between energy and economical efficiency - Energy awareness
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Problem description

Assembly line supplying

- **Assumptions**
  - Periodic supplying
  - A single vehicle / A single route

- **Decision variables**
  - Determine which workstations to serve at each period
  - Determine the number of components to be delivered

- **Constraints**:
  - Avoid component shortage / satisfy component demands along periods
  - Respect each workstation capacity / vehicle capacity
Performance objectives

Comparison
Energy minimization (vehicle) vs. Distance (Tours) minimization

Energy minimization
• Pollution Routing problem - Betkas and Laporte, Transportation Res. Part B, 2011
• Pollution Inventory Routing problem - Shamsi et al., Logistic Operations, 2014
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Energy analysis

A bit of physics

\[ E = \int P \times dt \]
\[ P = F \times v \]

- More relevant forces:
  - Traction force \((F_t) \Rightarrow F_t = m_T a(t)\)
  - Rolling resistance \((F_r) \Rightarrow F_r = m_T gC_r\)

- Energy equation:

\[ E = \int m_T \times (a(t) + gC_r) \times v(t)dt \]
Energy consumption profile

Assumption on the acceleration profile

Properties
- Any vehicle stop produces a pic of energy consumption.
- The consumed energy is proportional to the carried mass.
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Model basis

A transportation network

- A node \((i, t)\) in the network = a workstation \(i\) at period \(t\)
- A mass flow circulates from the depot along the network at each period \(t\) (plain arcs)
- A energy cost \(c_{i,j}\) is associated to each arc
- A component flow circulates from workstation \(i\) at period \(t\) to workstation \(i\) at period \(t + 1\) (dotted arcs)
Model basis

A 2-periods / 4-workstations network
A MILP formulation

Decision variables

- $M_{ij}^t$: mass going from $i$ to $j$ during period $t$
- $Z_i^t$: components delivered to workstation $i$ at period $t$
- $IL_i^t$: inventory level of workstation $i$ at period $t$
- $\phi_{ij}^t = 1$ whether the vehicle is going from $i$ to $j$ at period $t$

Constraints

- Mass, component, vehicle flows conservation
- Vehicle / stock capacity
- Demand satisfaction

Objective

- Energy minimization: $\sum_{i,j}^n \sum_{t}^{NT} c_{ij} M_{ij}^t$

NP-hard in the strong sense ...
Min \quad z = \sum_{i,j}^{NT} c_{ij} M_{ij}^t + \sum_{i,j}^{NT} c_{ij} m_v \phi_{ij}^t \quad (1)

\text{st:}

Z_i^t + IL_i^{t-1} - d_i^t = IL_i^t \quad \forall \quad (i, t) \quad (2)

Z_i^t - \frac{1}{m_i} (\sum_{j<i} M_{ji}^t - \sum_{j>i} M_{ij}^t) = 0 \quad \forall \quad (i, t) \quad (3)

Z_i^t + IL_i^{t-1} \leq c_i \quad \forall \quad (i, t) \quad (4)

\sum_{j<i} \phi_{ij}^t = \sum_{j>i} \phi_{ji}^t \quad \forall \quad (i, t) \quad (5)

M_{ij}^t \leq m_{max} \phi_{ij}^t \quad \forall \quad (i, j, t) \quad (6)

IL_i^t \geq 0 \quad \forall \quad (i, t) \quad (7)

M_{ij}^t \geq 0 \quad \forall \quad (i, j, t) \quad (8)

Z_i^t \in \mathbb{N} \quad \forall \quad (i, t) \quad (9)

\phi_{ij}^t \in \{0, 1\} \quad \forall \quad (i, t) \quad (10)
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Experimentation

Problem instances

- Instances adapted from those (IRP) of C. Archetti et al., Transportation Science (2007).
- Each workstation demand is periodic and assumed constant
- One single route among the customers is imposed
- New added parameters: energy cost, vehicle mass, component mass, maximum vehicle load
- 9 instances for each $NT = \{5, 10, 20\}$ and $N = \{5, 15, 30, 50\}$ values were created $\Rightarrow$ 108 instances

Optimization strategy

- First, distance (tours) minimization (GUROBI)
- Then energy minimization (GUROBI, Time Limit = 300sec)
- Comparison of the energy consumed by both solutions
Results

Time performance (sec)

For $NT = 20$ and $N = 50$, 2 instances were not solved to optimality ($GAP < 1\%$)
Results

Energy savings (%)

For every instance, the optimal number of tours remains unchanged when energy is minimized.
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Conclusion and perspectives

Conclusion

- Significant energy parameters: distances, masses, number of stops.
- An “effective” MILP model for energy minimization
- Energy optimization does not cause the increase of the number of tours

Perspectives

- Improvement of the problem instances
- Need of more advanced optimization techniques
- Generalization to the Inventory Routing Problem