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# Controller Design via Experimental Exploration with Robustness Guarantees

Tobias Holicki

# Introductory Comments

- This talk is highly inspired by the work [1].
- Related works are, e.g., [2], [3], [4], [5].
- The aim is to extend some of the aspects of [1] while focusing on a deterministic setup.

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- [1] [Marco et al.](#) “On the design of LQR kernels for efficient controller learning”. 2017
- [2] [Ferizbegovic et al.](#) “Learning Robust LQ-Controllers Using Application Oriented Exploration”. 2020
- [3] [Boczar, Matni, and Recht.](#) “Finite-Data Performance Guarantees for the Output-Feedback Control of an Unknown System”. 2018
- [4] [Kober, Bagnell, and Peters.](#) “Reinforcement learning in robotics: A survey”. 2013
- [5] [Berkenkamp and Schoellig.](#) “Safe and robust learning control with Gaussian processes”. 2015

# Contents

Motivation and Problem Setting

Selection of Test Controllers

Conclusions and Outlook

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## Setting and Goal

Let us consider the feedback interconnection

$$\begin{pmatrix} \dot{x}(t) \\ z(t) \\ e(t) \\ y(t) \end{pmatrix} = \left( \begin{array}{c|ccc} A & B_1 & B_2 & B_3 \\ \hline C_1 & D_{11} & D_{12} & D_{13} \\ C_2 & D_{21} & D_{22} & D_{23} \\ I & 0 & 0 & 0 \end{array} \right) \begin{pmatrix} x(t) \\ w(t) \\ d(t) \\ u(t) \end{pmatrix}, \quad w(t) = \Delta_0 z(t)$$

for some uncertain parameter  $\Delta_0$  contained in a known compact set  $\mathbf{\Delta}$ .

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**Goal:** We wish to find a state-feedback controller

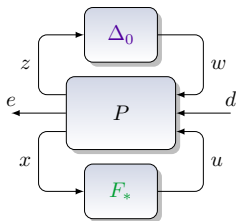
$$u(t) = F_* x(t)$$

which stabilizes  $\Delta_0 \star P$  and turns the closed-loop  $H_\infty$  norm is as small as possible.

I.e., we search for a minimizer of the function

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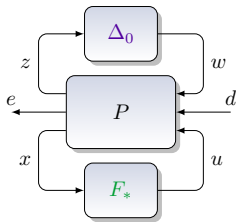
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**Issue:** Finding an (close-to-)optimal controller is difficult as  $\Delta_0$  is unknown.



# Standard Design Approaches

Via standard  $H_\infty$  design, we can compute for any fixed  $\Delta \in \mathbf{\Delta}$ :

$$\gamma_{\text{nom}}(\Delta) := \inf_{F \text{ stabilizes } \Delta \star P} \|\Delta \star P \star F\|_\infty.$$

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Via standard robust design (by exploiting knowledge of  $\mathbf{\Delta}$ ), we can compute upper bounds  $\gamma_{\text{sep}}$  on the **worst-case** closed-loop  $H_\infty$  norm:

$$\inf_{F \in \mathbb{F}} \sup_{\Delta \in \mathbf{\Delta}} \|\Delta \star P \star F\|_\infty \leq \gamma_{\text{sep}}.$$

Here, we abbreviate the set of robustly stabilizing controllers as

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Clearly, we have

$$\gamma_{\text{nom}}(\Delta_0) \leq \gamma_{\text{sep}}$$

and there might be a very large gap between both values.

## Setting and Goal (Continued)

**Additional Assumption:** E.g. by running and measuring multiple closed-loop experiments, the function

$J : F \mapsto \|\Delta_0 \star P \star F\|_\infty$  can be evaluated for finitely many controllers  $F_1, \dots, F_N$ .

**New Goal:** Based on this additional information, find a controller  $F$  such that  $J(F) = \|\Delta_0 \star P \star F\|_\infty$  is much closer to  $\gamma_{\text{nom}}(\Delta_0)$  than  $\gamma_{\text{sep}}$ .

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The above assumption suggests to perform a numerical minimization of a function that interpolates the data points

$$(F_1, J(F_1)), \dots, (F_N, J(F_N)).$$

This gives rise to the following essential questions.

- How can suitable test controllers  $F_1, \dots, F_N$  be selected systematically?
- How can the resulting data points be interpolated?

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# Robust Stability

**Issue:** Stability is a critical property as interconnecting a controller to the given system that is not stabilizing can lead to catastrophic results.

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**Remedy:** Following [1], we only search for robustly stabilizing controllers in  $\mathbb{F}$ .

It is not possible to include this safety requirement for free as we usually have

$$\gamma_{\text{nom}}(\Delta_0) = \inf_{F \text{ stabilizes } \Delta_0 * P} J(F) < \inf_{F \in \mathbb{F}} J(F).$$

- We show later on how to get closer to  $\gamma_{\text{nom}}(\Delta_0)$  by increasing the set of admissible controllers while still being able to guarantee safe operation.

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# Sampling and Gridding

**Issue:** It can be difficult to find controllers in  $\mathbb{F} \subset \mathbb{R}^{n_u \times n_y}$  based on gridding or sampling especially if

- the dimension of  $\mathbb{R}^{n_u \times n_y}$  is large,
- $\mathbb{F}$  is an unbounded set or
- $\mathbb{F}$  has measure zero in  $\mathbb{R}^{n_u \times n_y}$ .

**Remedy:** In contrast to [1], we propose a systematic approach to find such controllers based on gridding or sampling in the compact set  $\Delta$ .



# Motivation

As motivation, let us define the function (assuming it is well-defined)

$$\mathcal{F} : \mathbf{\Delta} \rightarrow \mathbb{F}, \quad \Delta \mapsto F \in \arg \min_{F \in \mathbb{F}} \|\Delta \star P \star F\|_{\infty}.$$

Then  $\mathcal{F}(\Delta)$  is a robustly stabilizing controller that yields the smallest  $H_{\infty}$  norm of  $\Delta \star P \star F$  among all robustly stabilizing controllers.

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By its definition we have

$$\|\Delta \star P \star \mathcal{F}(\Delta)\|_{\infty} \leq \|\Delta \star P \star F\|_{\infty} \quad \text{for all } F \in \mathbb{F} \quad \text{and all } \Delta \in \mathbf{\Delta}.$$

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For  $L := J \circ \mathcal{F} : \Delta \mapsto \|\Delta_0 \star P \star \mathcal{F}(\Delta)\|_{\infty}$  this implies

$$\inf_{F \in \mathbb{F}} J(F) = \inf_{F \in \mathbb{F}} \|\Delta_0 \star P \star F\|_{\infty} = L(\Delta_0) \leq L(\Delta) \quad \text{for all } \Delta \in \mathbf{\Delta}.$$

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**Why useful?** We can minimize  $L : \mathbf{\Delta} \rightarrow \mathbb{R}$  instead of  $J : \mathbb{F} \rightarrow \mathbb{R}$  based on I/O samples.

# Observations

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- The underlying problem is nonconvex and also nonsmooth in general.

**However**, as for robust controller design we can compute upper bounds on the optimal value and synthesize corresponding controllers!

# Robust Multi-Objective Design

**Lemma 1.** Let  $\Delta \in \mathbf{\Delta}$  be fixed. Then there is a controller  $F \in \mathbb{F}$  satisfying  $\|\Delta \star P \star F\|_\infty < \gamma$  if there exist a matrix  $M$  and symmetric  $Y, P$  satisfying

$$Y \succ 0,$$

$$P \in \mathbb{P}(\mathbf{\Delta}), \quad (\bullet)^T \left( \begin{array}{c|c} 0 & I \\ \hline I & 0 \\ \hline & P \end{array} \right) \left( \begin{array}{c} I \\ \hline \frac{-(AY + B_3 M)^T \quad -(C_1 Y + D_{13} M)^T}{0 \quad I} \\ \hline -B_1^T \quad -D_{11}^T \end{array} \right) \succ 0, \quad (\text{RS})$$

$$(\bullet)^T \left( \begin{array}{c|c} 0 & I \\ \hline I & 0 \\ \hline & P_\gamma^{-1} \end{array} \right) \left( \begin{array}{c} I \\ \hline \frac{-(A^\Delta Y + B_3^\Delta M)^T \quad -(C_2^\Delta Y + D_{23}^\Delta M)^T}{0 \quad I} \\ \hline -(B_2^\Delta)^T \quad -(D_{22}^\Delta)^T \end{array} \right) \succ 0. \quad (\text{NP}\Delta)$$

If the above LMIs are feasible, a suitable controller is  $F := MY^{-1}$ . Moreover,

$$\inf_{F \in \mathbb{F}} \|\Delta \star P \star F\|_\infty \leq \gamma_{\text{mo}}(\Delta)$$

for  $\gamma_{\text{mo}}(\Delta)$  being the infimal  $\gamma$  such that the above LMIs are feasible.



## Consequences and Remarks

Instead of using  $\mathcal{F}$  and for  $\varepsilon > 0$ , Lemma 1 suggests to employ the function

$\mathcal{F}_{\text{mo}} : \Delta \mapsto$  a corresp. close-to-optimal controller ( $\gamma = (1 + \varepsilon)\gamma_{\text{mo}}(\Delta)$ )

- $\mathcal{F}_{\text{mo}}(\Delta)$  is easily determined by solving a convex semi-definite program.
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Finally, we obtain suitable test controllers by choosing

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$$\gamma_{\text{nom}}(\Delta_0) \leq L(\Delta_0) \leq L_{\text{mo}}(\Delta_0) \leq (1 + \varepsilon)\gamma_{\text{mo}}(\Delta_0)$$

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- A minimizer of  $L$  is not necessarily a minimizer of  $L_{\text{mo}}$  and, conversely, a minimizer of  $L_{\text{mo}}$  is not necessarily a minimizer of  $L$ .
  - This is due to the conservatism in the convex design.

## Example

Let us consider a slight variation of an example from COMPI<sub>e</sub>ib [6] with

$$\Delta := \delta I, \quad \delta := [-1, 1], \quad \Delta_0 := \delta_0 I, \quad \delta_0 = 0.7.$$

We obtain

$$\gamma_{\text{nom}}(\delta_0) = 1.20, \quad \min_{\delta \in \delta} L_{\text{mo}}(\delta) = L_{\text{mo}}(0.66) = 1.39 \quad \text{and} \quad \gamma_{\text{sep}} = 2.02.$$

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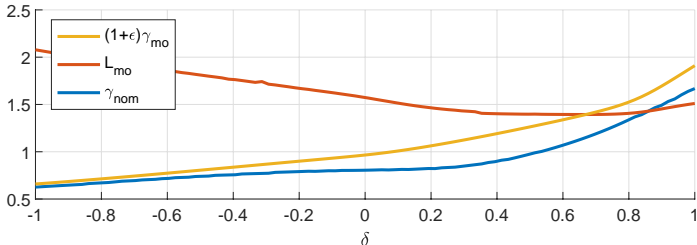
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- Minimizing  $L_{\text{mo}}$  leads as desired to better closed-loop  $H_\infty$  performance if compared to robust design.
- Safe operation is assured as robustly stabilizing controllers are designed.
- Here  $\mathbb{F}$  is a subset of  $\mathbb{R}^{4 \times 8}$  which has dimension 36 and turns sampling or gridding very tedious.
- The minimizer of  $L_{\text{mo}}$  is not necessarily equal to  $\delta_0$ .

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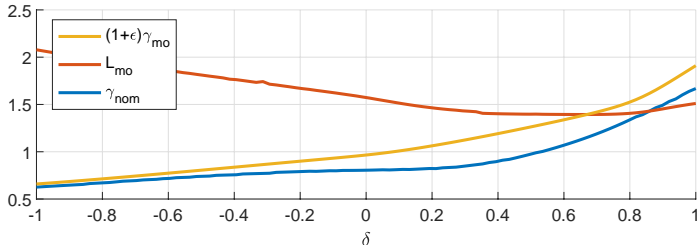


## Interesting Bonus Feature:

- We can assure that  $\delta_0$  is contained in  $[0.65, 0.9]$  as we have inequality

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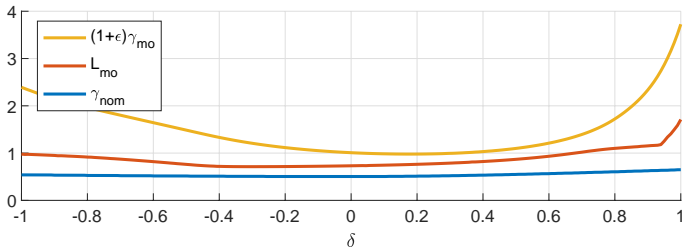
$$\gamma_{\text{nom}}(\delta_0) \leq L_{\text{mo}}(\delta_0) \leq (1 + \epsilon)\gamma_{\text{mo}}(\delta_0).$$

- This allows to repeat the procedure for  $\Delta$  replaced by  $\tilde{\Delta} := [0.65, 0.9]I$ .
- This yields even better controllers as easier robust problems are involved:

$$\gamma_{\text{nom}}(\delta_0) = 1.20, \quad \min_{\delta \in [0.65, 0.9]} L_{\text{mo}}(\delta) = 1.31 \quad \text{and} \quad \gamma_{\text{sep}} = 1.66.$$



## “Negative” Example



- Shrinking  $\Delta$  by a large amount is not always possible as the curves do not have to intersect at all.
- But it can as well be possible to iteratively apply the shrinking.

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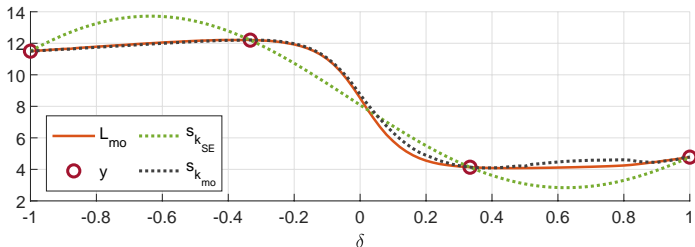
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- Evaluations of closed-loop experiments allow to design safe controllers with superior performance if compared to a standard robust design.
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## Outlooks:

- (Output-feedback) synthesis based on superior analysis results.
- How to handle time-varying uncertainties?
- Systematic approaches for higher dimensions.





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# Thank you!

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