University of Stuttgart
Germany

Revisiting the Dual Iteration

Tobias Holicki

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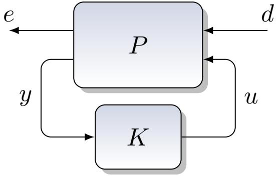
Static Output-Feedback H_∞ Design

Let us consider a system

$$\begin{pmatrix} \dot{x}(t) \\ \hline e(t) \\ y(t) \end{pmatrix} = \left(\begin{array}{c|cc} A & B & B_2 \\ \hline C & D & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right) \begin{pmatrix} x(t) \\ \hline d(t) \\ u(t) \end{pmatrix}.$$



Goal: Design a static output-feedback controller

$$u(t) = Ky(t)$$

such that the resulting closed-loop H_∞ norm is as small as possible.

Static Output-Feedback H_∞ Design

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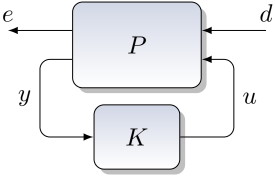

Goal: Design a static output-feedback controller

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such that the resulting closed-loop H_∞ norm is as small as possible.

Issue: Computing the optimal H_∞ norm and finding K is a hard nonconvex and also nonsmooth problem

Remedy: Heuristic approaches such as

- D-K iteration
- `hinfstruct` [1]
- Dual iteration [2]

[1] Apkarian and Noll. "Nonsmooth H_∞ Synthesis". 2006
[2] Iwasaki. "The dual iteration for fixed-order control". 1999

Elimination Lemma [3]

Lemma 1. Let $P \in \mathbb{S}^{p+q}$ with $\mathrm{in}(P) = (0, p, q)$ and U, V, W be given. Then there is a matrix Z satisfying

$$\begin{pmatrix} I_p \\ U^T Z V + W \end{pmatrix}^T P \begin{pmatrix} I_p \\ U^T Z V + W \end{pmatrix} \prec 0$$

if and only if

$$V_\perp^T \begin{pmatrix} I_p \\ W \end{pmatrix}^T P \begin{pmatrix} I_p \\ W \end{pmatrix} V_\perp \prec 0 \quad \text{and} \quad U_\perp^T \begin{pmatrix} -W^T \\ I_q \end{pmatrix}^T P^{-1} \begin{pmatrix} -W^T \\ I_q \end{pmatrix} U_\perp \succ 0.$$

Notation: $M_\perp$ is a basis matrix of the kernel of M.

Special Case: If $P = \begin{pmatrix} Q & I \\ I & 0 \end{pmatrix}$ and $W = 0$ the LMIs, respectively, read as

$$Q + U^T Z V + (U^T Z V)^T \prec 0, \quad V_\perp^T Q V_\perp \prec 0 \quad \text{and} \quad U_\perp^T Q U_\perp \prec 0.$$

[3] Helmersson. "IQC synthesis based on inertia constraints". 1999

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Static Output-Feedback

Theorem 2. Let $P_\gamma := \begin{pmatrix} I & 0 \\ 0 & -\gamma^2 I \end{pmatrix}$, $V := (C_2, D_{21})_\perp$ and $U = (B_2^T, D_{12}^T)_\perp$. Then there is a SOF controller K satisfying $\|P \star K\|_\infty < \gamma$ if and only if there exists a matrix X satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \\ 0 & I \end{pmatrix} V \prec 0, \quad (\bullet)^T \left(\begin{array}{cc|c} 0 & X^{-1} & \\ X^{-1} & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \begin{pmatrix} I & 0 \\ -A^T & -C^T \\ \hline 0 & I \\ -B^T & -D^T \end{pmatrix} U \succ 0$$

and

$$X \succ 0.$$

Moreover, we have

$$\inf_{K \text{ stabilizes } P} \|P \star K\|_\infty = \gamma_{\text{opt}}$$

for γ_{opt} being the infimal γ such that the above LMIs are feasible.

Full-Order Dynamic Output-Feedback

Theorem 3. Let $P_\gamma := \begin{pmatrix} I & 0 \\ 0 & -\gamma^2 I \end{pmatrix}$, $V := (C_2, D_{21})_\perp$ and $U = (B_2^T, D_{12}^T)_\perp$. Then there is a full-order controller K_{dof} satisfying $\|P \star K_{\mathrm{dof}}\|_\infty < \gamma$ if and only if there exist matrices X and Y satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \left(\begin{array}{cc} I & 0 \\ A & B \\ \hline C & D \\ 0 & I \end{array}\right) V \prec 0, \quad (\bullet)^T \left(\begin{array}{cc|c} 0 & Y & \\ Y & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \left(\begin{array}{cc} I & 0 \\ -A^T & -C^T \\ \hline 0 & I \\ -B^T & -D^T \end{array}\right) U \succ 0$$

and

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \succ 0.$$

Moreover, we have

$$\gamma_{\mathrm{dof}} \leq \gamma_{\mathrm{opt}}$$

for γ_{dof} being the infimal γ such that the above LMIs are feasible.

Full-Information Controller Design

For a full-information (FI) controller

$$u = F\tilde{y} = (F_1, F_2)\tilde{y} \quad \text{where} \quad \tilde{y} := \begin{pmatrix} x \\ d \end{pmatrix}$$

the resulting closed-loop system reads as

$$\begin{pmatrix} \dot{x}(t) \\ e(t) \end{pmatrix} = \begin{pmatrix} A(F) & B(F) \\ C(F) & D(F) \end{pmatrix} \begin{pmatrix} x(t) \\ d(t) \end{pmatrix} = \begin{pmatrix} A + B_2 F_1 & B + B_2 F_2 \\ C + D_{12} F_1 & D + D_{12} F_2 \end{pmatrix} \begin{pmatrix} x(t) \\ d(t) \end{pmatrix}. \quad (1)$$

Lemma 4. There is a FI controller F such that $\|(1)\|_\infty < \gamma$ if and only if there exists a matrix $Y \succ 0$ satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & Y & \\ Y & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \left(\begin{array}{cc} I & 0 \\ -A^T & -C^T \\ \hline 0 & I \\ -B^T & -D^T \end{array}\right) U \succ 0.$$

First Key Result

Once we have designed a suitable FI F, we can synthesize a SOF controller:

Theorem 5. There is a SOF controller K satisfying $\|P \star K\|_\infty < \gamma$ if there exists a matrix $X \succ 0$ satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \\ 0 & I \end{pmatrix} V \prec 0 \quad \text{and} \quad (\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A(F) & B(F) \\ \hline C(F) & D(F) \\ 0 & I \end{pmatrix} \prec 0.$$

Moreover, we have

$$\gamma_{\text{dof}} \leq \gamma_{\text{opt}} \leq \gamma_F$$

for γ_F being the infimal γ such that the above LMIs are feasible.

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Moreover, we have

$$\gamma_{\text{dof}} \leq \gamma_{\text{opt}} \leq \gamma_F$$

for γ_F being the infimal γ such that the above LMIs are feasible.

Applying elimination lemma to eliminate F from second LMI leads to:

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X^{-1} & \\ X^{-1} & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \left(\begin{array}{cc} I & 0 \\ -A^T & -C^T \\ \hline 0 & I \\ -B^T & -D^T \end{array}\right) U \succ 0.$$

Dual Design Problems

Full-actuation (FA) design:

$$\begin{pmatrix} \dot{x}(t) \\ e(t) \end{pmatrix} = \begin{pmatrix} A(E) & B(E) \\ C(E) & D(E) \end{pmatrix} \begin{pmatrix} x(t) \\ d(t) \end{pmatrix} = \begin{pmatrix} A + E_1 C_2 & B + E_1 D_{21} \\ C + E_2 C_2 & D + E_2 D_{21} \end{pmatrix} \begin{pmatrix} x(t) \\ d(t) \end{pmatrix}. \quad (2)$$

Lemma 6. There is a FA gain E s.th. $\|(2)\|_\infty < \gamma$ iff there is a matrix $X \succ 0$ with

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \\ 0 & I \end{pmatrix} V \prec 0.$$

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Theorem 7. There is a SOF controller K satisfying $\|P \star K\|_\infty < \gamma$ if there exists a matrix $Y \succ 0$ satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & Y & \\ Y & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \begin{pmatrix} I & 0 \\ -A(E)^T & -C(E)^T \\ \hline 0 & I \\ -B(E)^T & -D(E)^T \end{pmatrix} \succ 0 \ \ \& \ \ (\bullet)^T \left(\begin{array}{cc|c} 0 & Y & \\ Y & 0 & \\ \hline & & P_\gamma^{-1} \end{array}\right) \begin{pmatrix} I & 0 \\ -A^T & -C^T \\ \hline 0 & I \\ -B^T & -D^T \end{pmatrix} U \succ 0.$$

Moreover, we have

$$\gamma_{\mathrm{dof}} \leq \gamma_{\mathrm{opt}} \leq \gamma_E$$

for γ_E being the infimal γ such that the above LMIs are feasible.

Dual Iteration Algorithm

1. Compute the lower bound γ_{dof} and set $k = 1$.
2. Design an initial FI gain F.
3. Primal Step: Compute γ_F based on Theorem 5 and set $\gamma^k := \gamma_F$.
4. Design a corresponding FA gain E.
5. Dual Step: Compute γ_E based on Theorem 7 and set $\gamma^{k+1} := \gamma_E$.
6. If k is too large or γ^k does not decrease anymore stop and apply Theorem 7 to construct a close-to-optimal SOF controller K.

   Else set $k = k + 2$, design a corresponding FI gain F and go to Step 3.

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- **Key:** Exploiting the elimination lemma.
- Algebraically, the steps are very intuitive.
- Control theoretic interpretation?

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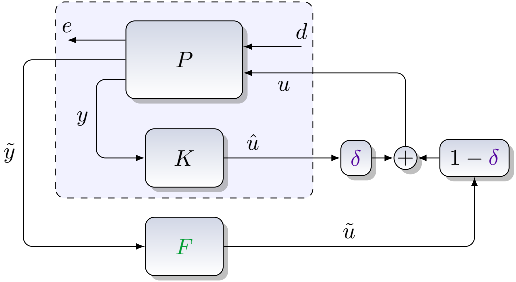
Modifying the Original System Interconnection

Suppose we have designed a FI controller $\tilde{u} = F\tilde{y}$. Then we can incorporate it into the original interconnection with a parameter $\delta \in [0, 1]$ as follows:



Note that

$$u = \delta \hat{u} + (1 - \delta)\tilde{u}.$$

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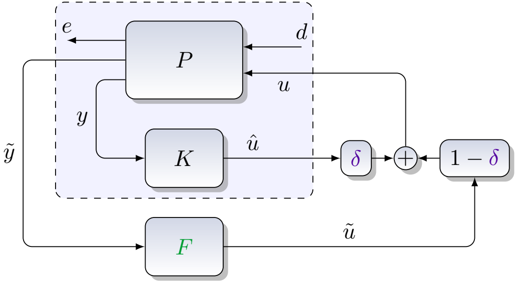

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➡ One can view δ as a homotopy parameter

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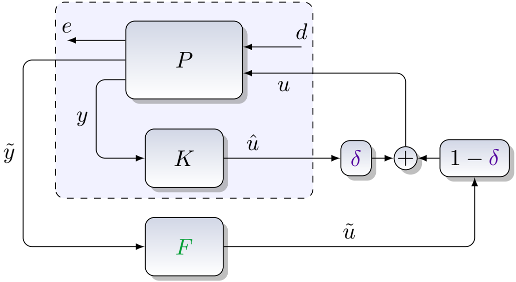

Key observation: Finding a robust SOF controller K can be turned into a convex problem!

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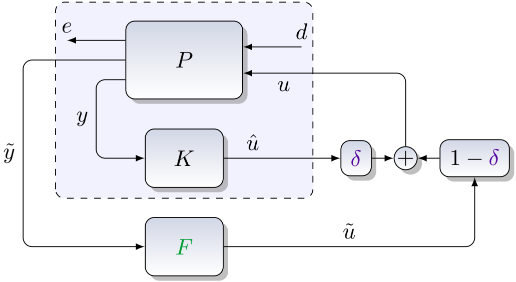

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Why? The corresponding generalized plant resembles the one appearing in robust estimation problems.

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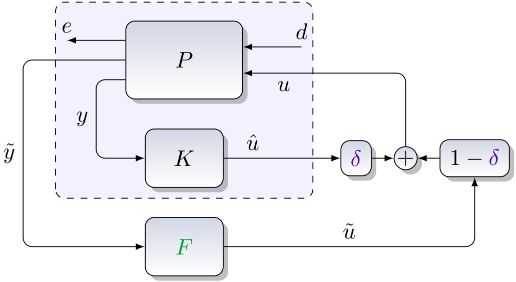

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Why? The corresponding generalized plant resembles the one appearing in robust estimation problems.

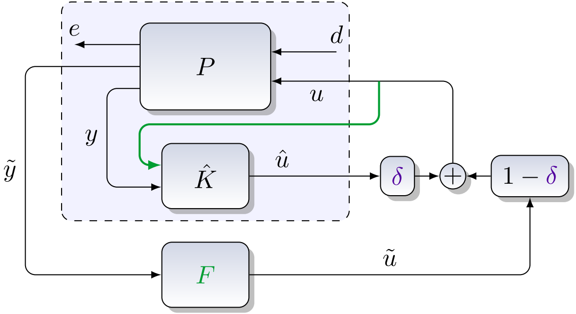
Issue: Computed gain bounds are conservative. We solve a convex but more difficult design problem since an uncertain parameter is involved.

Modifying the Original System Interconnection (2)

Remedy: We allow the controller to additionally include measurements of

$$u = \delta \hat{u} + (1 - \delta)\tilde{u}$$

- Since the controller $\hat{K}$ knows its output $\hat{u}$ this is essentially as good as measuring the new uncertain signal $\tilde{w} = \delta \tilde{z} = \delta(\hat{u} - \tilde{u})$.
- The important structure is preserved!



Main Results

Lemma 8. The OL system corresponding to the last diagram is given by

$$\begin{pmatrix} \dot{x}(t) \\ \hline e(t) \\ \hdashline \tilde{z}(t) \\ \hdashline \hat{y}(t) \end{pmatrix} = \left(\begin{array}{c|c:c:c} A + B_2F_1 & B + B_2F_2 & B_2 & 0 \\ \hline C + D_{12}F_1 & D + D_{12}F_2 & D_{12} & 0 \\ \hdashline -F_1 & -F_2 & 0 & I \\ \hdashline C_2 & D_{21} & 0 & 0 \\ F_1 & F_2 & I & 0 \end{array}\right) \begin{pmatrix} x(t) \\ \hline d(t) \\ \hdashline \tilde{w}(t) \\ \hdashline \hat{u}(t) \end{pmatrix}$$

with $\tilde{z} := \hat{u} - \tilde{u}$, $\tilde{w} := \delta\tilde{z}$ as well as $\hat{y} := \left(\begin{smallmatrix} y \\ u \end{smallmatrix}\right)$

- We can derive convex LMI criteria for designing $\hat{K} = (\hat{K}_1, \hat{K}_2)$, e.g., via elimination.
- We obtain a desired SOF controller via $K := (I - \hat{K}_2)^{-1}\hat{K}_1$.

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- Furthermore, as annihilator for $\left(\begin{smallmatrix} C_2 & D_{21} & 0 \\ F_1 & F_2 & I \end{smallmatrix}\right)$ we can choose $\left(\begin{smallmatrix} I & 0 \\ 0 & I \\ -F_1 & -F_2 \end{smallmatrix}\right) V$ with $V = (C_2, D_{21})_\perp$ as before.

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$$\begin{pmatrix} A+B_2F_1 & B+B_2F_2 & B_2 \\ C+D_{12}F_1 & D+D_{12}F_2 & D_{12} \\ -F_1 & -F_2 & 0 \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & I \\ -F_1 & -F_2 \end{pmatrix} V = \begin{pmatrix} A & B \\ C & D \\ -F_1 & -F_2 \end{pmatrix} V.$$

Main Results

Theorem 9. There is a SOF controller $\hat{K}$ such that the H_∞ norm of the last interconnection is smaller than γ for all $\delta \in [0,1]$ if there exists a symmetric matrix $X \succ 0$ satisfying

$$(\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A & B \\ \hline C & D \\ 0 & I \end{pmatrix} V \prec 0 \quad \text{and} \quad (\bullet)^T \left(\begin{array}{cc|c} 0 & X & \\ X & 0 & \\ \hline & & P_\gamma \end{array}\right) \begin{pmatrix} I & 0 \\ A(F) & B(F) \\ \hline C(F) & D(F) \\ 0 & I \end{pmatrix} \prec 0.$$

- These are exactly the same conditions as earlier!
- Thus we recovered the primal step in the dual iteration.

Remarks

- The second component of the dual iteration (the dual step) is related in a similar fashion to a problem that resembles robust feedforward design.

- It might be possible to obtain improved upper bounds, e.g., by
  - viewing δ as a scheduling parameter,
  - using dynamic IQCs for $\delta \in [0, 1]$ or
  - using parameter-dependent Lyapunov functions.

  However, we only obtained marginal improvements that do not justify the increased complexity.

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  However, we only obtained marginal improvements that do not justify the increased complexity.

- Main Point: $\hat{K}$ can also be designed based on parameter transformation.

  This allows to extend the scheme to situations were elimination is not possible, e.g.,
  - H_2 Performance
  - Closed-loop poles in LMI region
  - Multi-objective design
  - ...

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Comparison to a hinfstruct via Examples from [5]

Example	γ_{dof}	γ^1	γ^5	γ^9	γ_{his}
AC3	2.97	4.50	3.70	3.63	3.64
AC4	0.56	1.29	1.05	1.02	0.96
AC18	5.40	15.98	10.76	10.71	10.70
HE4	22.84	32.29	23.94	22.84	23.80
DIS1	4.16	4.85	4.34	4.34	4.19
WEC2	3.60	6.05	4.42	4.32	4.25
BDT1	0.27	0.30	0.27	0.27	0.27
EB2	1.77	2.03	2.02	2.02	2.02
NN14	9.44	23.51	17.48	17.48	17.48

Example	γ_{dof}	γ^1	γ^5	γ^9	γ_{his}
PAS	0.05	3.48	0.41	0.08	-
TF3	0.25	3.63	0.52	0.40	-
NN17	2.64	-	-	-	11.22

[5] Leibfritz. *COMPl$_e$ib: COnstraint Matrix-optimization Problem library - a collection of test examples for nonlinear semidefinite programs, control system design and related problems*. 2004

Comparison to a D-K iteration for Generalized H_2 Design

Name	γ_{dof}	γ^1	γ^3	γ^9	γ^9_{dk}	$\gamma^{21}_{\mathrm{dk}}$
AC6	1.91	2.05	1.99	1.98	2.07	2.07
AC9	1.39	1.74	1.41	1.40	8.34	5.20
AC11	1.56	1.96	1.83	1.79	1.85	1.85
HE1	0.08	0.14	0.11	0.09	0.09	0.09
HE5	0.82	12.54	1.56	1.15	3.76	3.67
REA2	0.90	0.93	0.91	0.90	0.92	0.92
AGS	4.45	4.75	4.67	4.67	4.68	4.68
WEC2	3.71	19.56	5.73	4.95	14.71	14.70
NN14	20.90	48.11	32.99	23.00	28.94	28.91

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Conclusions and Outlook

Conclusions:

- The dual iteration is an interesting heuristic scheme for solving nonconvex design problems.
- The individual steps can be viewed as convex robust design problems with homotopy parameter δ and with estimation / feedforward structure.

Conclusions and Outlook

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- The dual iteration is an interesting heuristic scheme for solving nonconvex design problems.
- The individual steps can be viewed as convex robust design problems with homotopy parameter δ and with estimation / feedforward structure.

Outlook:

- Robust design based on dynamic IQC analysis results
- Robust/static design for hybrid systems
- Consensus for heterogeneous multi-agent systems

University of Stuttgart
Germany

Thank you!

Tobias Holicki

tobias.holicki@imng.uni-stuttgart.de

University of Stuttgart
Department of Mathematics
Mathematical Systems Theory