Adaptive control for time-varying systems

Internship of Master 2 Robotic : Decision and Control

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Introduction

Internship's objectives

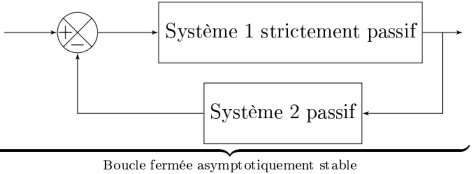
- Thesis from
  - Alexandru-Razvan Luzi[1]
  - Harmonie Leduc [2]
- Applied to a time-varying system
- Understand behaviors of the adaptive control

[1] Alexandru-Razvan Luzi. *Commande variante dans le temps pour le contrôle d'attitude de satellites*. PhD thesis, 2014.

[2] Harmonie Leduc. *Contrôle adaptatif robuste. Application au contrôle d'attitude de satellites*. PhD thesis, 2017.

Origins of adaptive control
Bibliographic review

- Appeared in the 50's [3]
- Based on systems' passivity



[3] Karl Aström. History of Adaptive Control. In John Baillieul and Tariq Samad, editors, *Encyclopedia of Systems and Control*, pages 1-9. Springer, London, 2014.

Introduction

Bibliographic review

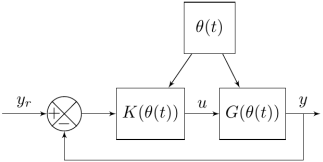


Figure: Diagram of gain scheduling

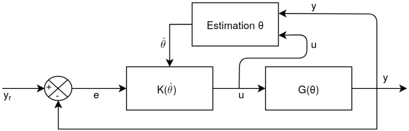


Figure: Diagram of indirect adaptive control

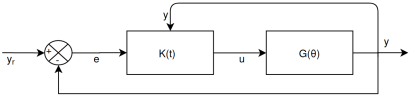


Figure: Diagram of direct adaptive control

Equations of direct adaptive control

$$u(t) = K(t)e(t) \tag{1}$$

$$\dot{K}(t) = Ge(t)e^T(t)\Gamma, \quad \Gamma > 0$$

Studied system
Presentation

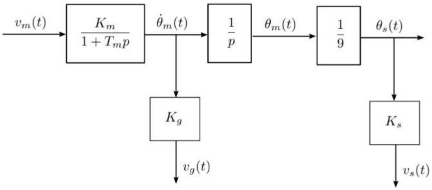


Figure: Block diagram of DC motor

Exogenous signals :

- Reference : $y_r = \begin{bmatrix} K_s \pi \\ 0 \end{bmatrix}$
- White noise measure : $b(t) : P(b(t)) = 10^{-3}$
- Disturbance : $p(t) = 0.5H(t)$

Studied system

Presentation

State space

$$x(t) = \begin{bmatrix} v_s(t) \\ v_g(t) \end{bmatrix}$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & \frac{K_s}{9K_g} \\ 0 & \frac{-1}{T_m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{K_g K_m}{T_m} \end{bmatrix} v_m(t) \\ y(t) = x(t) \end{cases} \quad (2)$$

Equilibrum points

$$\dot{x} = 0 \Leftrightarrow \begin{cases} \text{Null speed} \\ \text{Null engine command} \\ \text{Constant position} \end{cases} \quad (3)$$

First simulation

Correction by poles placement

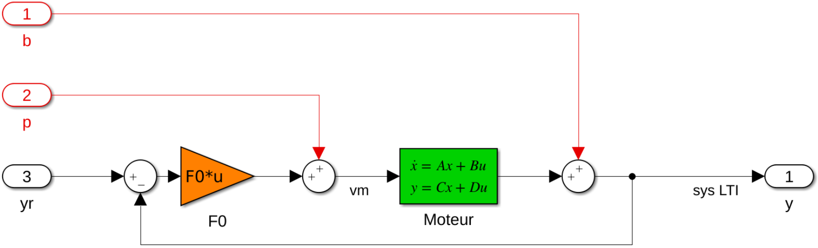


Figure: DC motor controlled by state feedback

$$rt_{5\%} = 2s = \frac{-3}{Re(eig_{A_{cl}})} = -1.5 \quad (4)$$

$$\begin{aligned} eig_{des} &= \{-1.5, \quad -1.6\} \\ \Rightarrow F_0 &= [0.0718 \quad -0.285] \end{aligned} \quad (5)$$

First simulation

Correction by poles placement

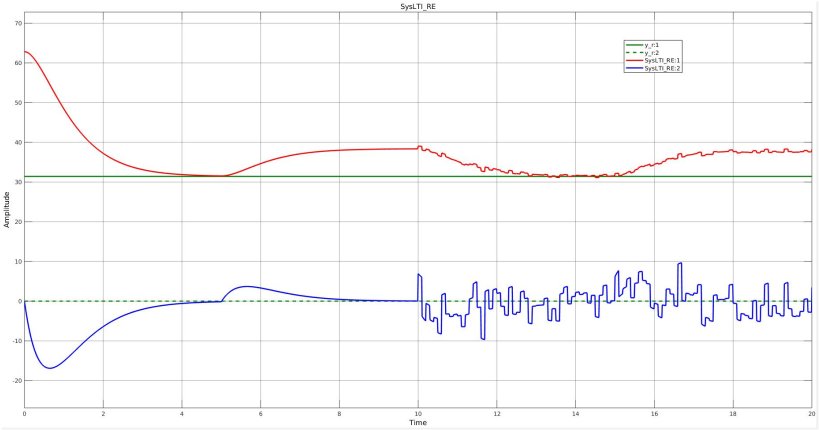


Figure: Motor controlled by state feedback (speed x10)

Modification of engine's model

Varying parameter θ

Modification of engine's model

$$x(t) = \begin{bmatrix} v_s(t) \\ v_g(t) \end{bmatrix}$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & \frac{K_s}{9K_g} \\ 0 & \frac{-1}{T_m} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{K_g K_{m_u}(t)}{T_m} \end{bmatrix} v_m(t) \\ y(t) = x(t) \end{cases} \tag{6}$$

$$\theta = K_{m_u}(t) = K_m(1 + \alpha sin(\omega t)) \tag{7}$$

$$I_{K_{m_u}} = [K_m(1-\alpha) \quad K_m(1+\alpha)] \Rightarrow |\alpha| < 1 \tag{8}$$

$$\alpha = 0.75 \quad \omega = 2\pi$$

Study of stability

Correction by poles placement

Definition : Quadratic stability [4]

The linear system

$$\dot{x}(t) = A(\theta(t))x(t), \quad \theta(t) \in \Theta$$

is quatrically stable if, and only if, there exists a matrix $P \in \mathbb{S}^+$ such that :

$$A(\theta)^T P + PA(\theta) < 0, \forall \theta \in \Theta$$

Quadratic stability implies robust stability, proved with the Lyapunov function $V(x) = x^T Px$

[4] B. R. Barmish. Necessary and sufficient conditions for quadratic stabilizability of an uncertain system. *Journal of Optimization Theory and Applications*, 46(4) :399-408, August 1985.

Study of stability

Correction by poles placement

Theorem : Quadratic stability of a polytope [5]

Consider the following polytope with $\bar{v}$ vertices :

$$A(\theta) = \sum_{v=1}^{\bar{v}} \theta_v A^{[v]}, \quad \theta_v \geq 0, \quad \sum_{v=1}^{\bar{v}} \theta_v = 1 \tag{9}$$

The system

$$\dot{x}(t) = A(\theta(t))x(t) \tag{10}$$

is quadratically stable if, and only if,

$$\forall v \in \{1 \dots \bar{v}\} : {A^{[v]}}^T P + PA^{[v]} < 0, P > 0 \tag{11}$$

[5] Junior de Souza Leite. *Sur la stabilité robuste des systèmes linéaires : une approche par des fonctions dépendantes de paramètres.* PhD thesis, 2005.

Study of stability
Correction by poles placements

Vertices of the polytope

$$B(t) = Co\{\underline{B}, \quad \overline{B}\} \tag{12}$$

$$\begin{cases} \dot{x}(t) = \underbrace{(A - \underline{B}F_0)}_{\underline{A_{cl}}} x(t) - \underline{B}F_0 y_r(t) \\ y(t) = x(t) \end{cases} \tag{13}$$

$$\begin{cases} \dot{x}(t) = \underbrace{(A - \overline{B}F_0)}_{\overline{A_{cl}}} x(t) - \overline{B}F_0 y_r(t) \\ y(t) = x(t) \end{cases} \tag{14}$$

Solve LMI

$$\begin{aligned} \underline{A_{cl}}^T P + P\underline{A_{cl}} &< 0 \\ \overline{A_{cl}}^T P + P\overline{A_{cl}} &< 0 \\ P &> I_2 \end{aligned} \tag{15}$$

$$P = \begin{bmatrix} 4.0559 & 9.5941 \\ 9.5941 & 45.1547 \end{bmatrix} \tag{16}$$

LPV motor controlled with poles placement

Correction by poles placement

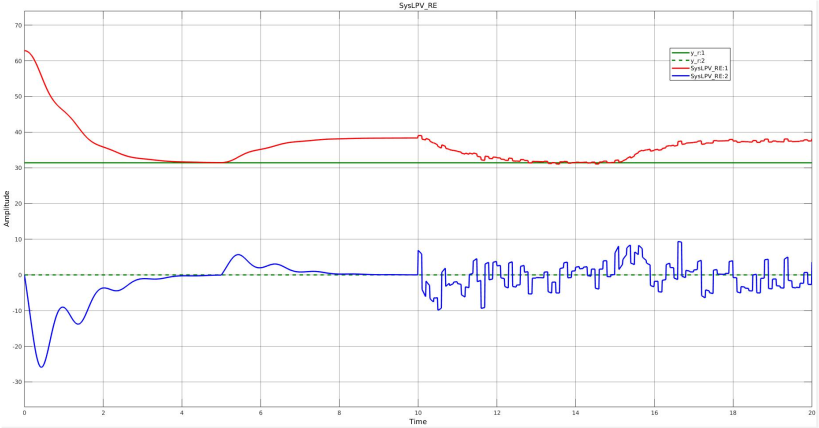


Figure: LPV engine controlled by state feedback (speed x10)

Implementation of the adaptive control

Equations of the adaptive command law

$$\begin{aligned} e(t) &= y_r(t) - y(t) \\ u(t) &= K(t)e(t) \\ \dot{K}(t) &= Ge(t)e(t)^T\Gamma, \quad \Gamma > 0 \end{aligned} \tag{17}$$

Implementation of the adaptive control

Notion of passivity

Theorem : Passivity of squared systems [6]

Consider the following linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{18}$$

This system is strictly passive if it exists $P \in \mathbb{S}^+$ and $Q > 0$ such as the following LMI is verified :

$$\begin{bmatrix} A^T P + PA + Q & PB - C^T \\ B^T P - C & 0 \end{bmatrix} \leq 0 \tag{19}$$

[6] Alexandru-Razvan Luzi. *Commande variante dans le temps pour le contrôle d'attitude de satellites*. PhD thesis, 2014.

Implementation of the adaptive control

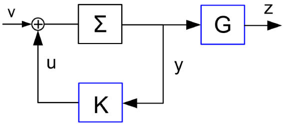
Notion of passivity

Definition : G-passivity [7]

Consider the linear system (18). This system is G-passive if, for a given matrix $G \in \mathbb{R}^{m \times p}$, there exists two matrices $P > 0$ and $Q > 0$ such that :

$$\begin{bmatrix} A^T P + PA + Q & PB - C^T G^T \\ B^T P - GC & 0 \end{bmatrix} \leq 0 \tag{20}$$

$$\Leftrightarrow \begin{cases} A^T P + PA + Q \leq 0 \\ PB = C^T G^T \end{cases} \tag{21}$$



[7] A L Fradkov. *Design of an adaptive system for stabilization of a linear dynamic plant*. page 9, 1974.

Implementation of the adaptive control

Notion of passivity

$$A = \begin{bmatrix} 0 & \frac{K_s}{9K_g} \\ 0 & \frac{-1}{T_m} \end{bmatrix} \quad (22)$$

$$\begin{cases} (A - BF_0)^T P + P(A - BF_0) + Q \leq 0 \\ PB = C^T G^T \end{cases} \quad (23)$$

$$G = B^T P \quad (24)$$

Study of stability

$$x_a = \begin{pmatrix} v_s \\ v_g \\ vec(K) \end{pmatrix} \quad (25)$$

$$V(x_a) = x^T P x + Tr[(K - F_0)\Gamma^{-1}(K - F_0)^T] \quad (26)$$

Implementation of the adaptive control
Simulation of the LTI system

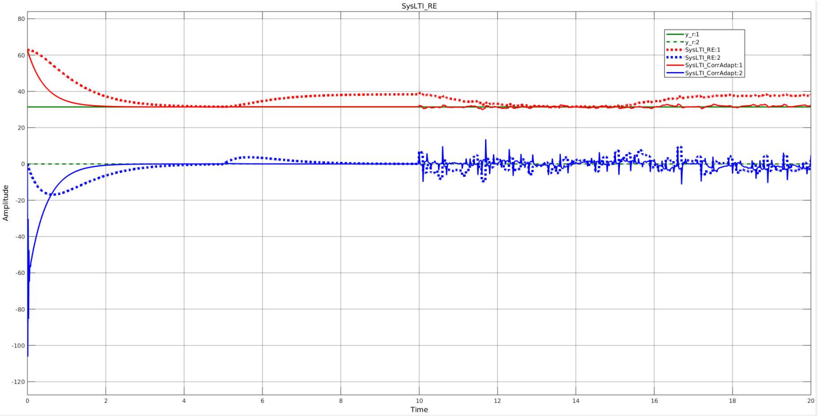


Figure: LTI system controlled by the adaptive controller, compared with the state feedback controller

Implementation of the adaptive control
Simulation of the LTI system

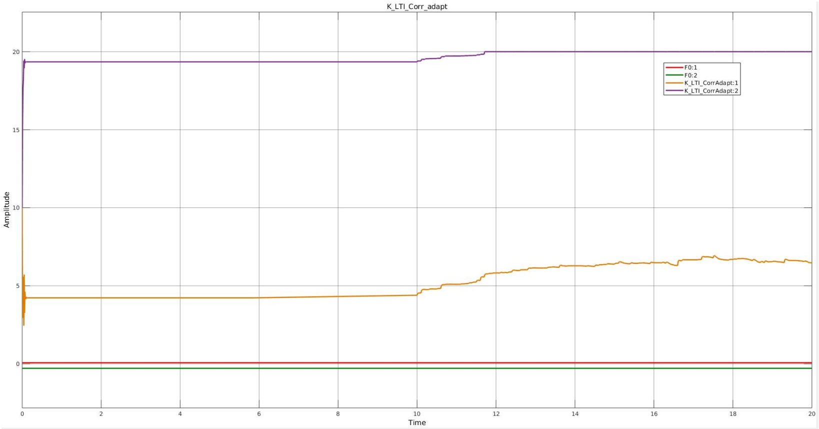


Figure: Evolution of $K(t)$

Implementation of the adaptive control

Time-varying parameter system

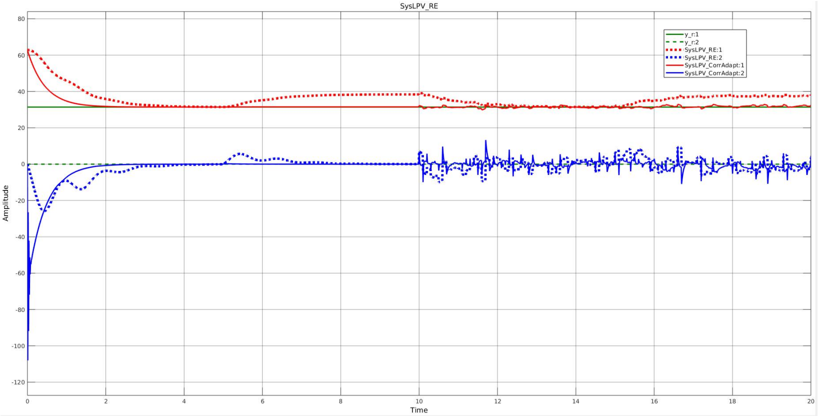


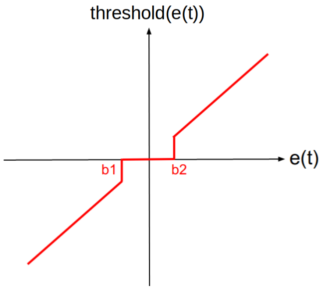
Figure: LPV system controlled by adaptive control law, compared with state feedback controller

Improvement of the adaptive controller

Rejection of the noise measurement

Adding a *threshold* function

$$\begin{aligned} e(t) &= y_r(t) - y(t) \\ u(t) &= K(t)e(t) \\ \dot{K}(t) &= G\,\textcolor{red}{Threshold(e(t))}e(t)^T\Gamma, \quad \Gamma > 0 \end{aligned} \tag{27}$$



Lyapunov function for the case 1

LTI system

Case 1 : $|e(t)| \geq \text{ threshold} \Rightarrow Threshold(e(t)) = e(t)$

$$\begin{cases} \dot{K}(t) = Ge(t)e^T(t)\Gamma \\ v_m(t) = K(t)e(t) \\ \dot{x}(t) = (A - BK(t))x(t) + BK(t)y_r(t) \end{cases} \tag{28}$$

$$x_a = \begin{pmatrix} v_s \\ v_s \\ vec(K) \end{pmatrix} \tag{29}$$

$$V(x_a) = x^T Px + Tr[(K - F_0)\Gamma^{-1}(K - F_0)^T], \quad \Gamma^{-1} > 0, \quad P > 0 \tag{30}$$

$$\begin{aligned} \dot{V}(x_a) &= 2x^T P\dot{x} + 2Tr[(K - F_0)\Gamma^{-1}(K \dot{-} F_0)^T] \\ &= 2x^T P(A - BF_0)x \end{aligned} \tag{31}$$

Lyapunov function for the case 2

LTI system

Case 2 : $|e(t)| < \text{ threshold} \Rightarrow Threshold(e(t)) = 0$

$$\begin{cases} \dot{K}(t) = 0 \\ v_m(t) = Ke(t) \\ \dot{x}(t) = (A - BK)x(t) + BKy_r(t) \end{cases} \tag{32}$$

$$V(x_a) = x^T Px + Tr[(K - F_0)\Gamma^{-1}(K - F_0)^T], \quad \Gamma^{-1} > 0, \quad P > 0 \tag{33}$$

$$\begin{aligned} \dot{V}(x_a) &= 2x^T P\dot{x} + 2Tr[(K - F_0)\Gamma^{-1}(K \dot{-} F_0)^T] \\ &= 2x^T P(A - BK)x \end{aligned} \tag{34}$$

Lyapunov function for the case 2
Case of the LTI system

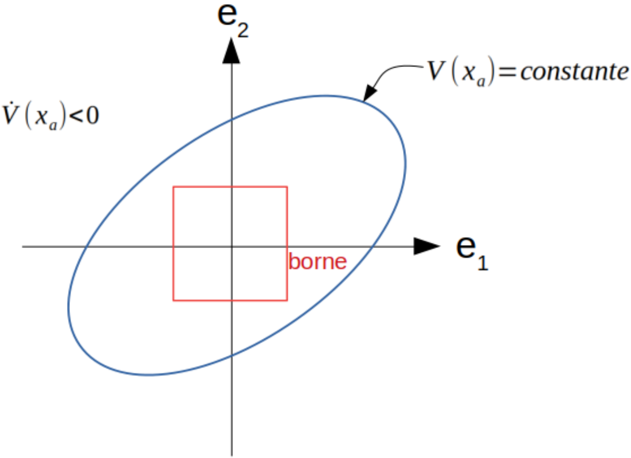


Figure: Lyapunov's equipotential

Improvement of the adaptive controller

Modification of the control law structure

Adding the σ-modification

$$\begin{cases} e(t) = y_r(t) - y(t) \\ v_m(t) = K(t)e(t) \\ \dot{K}(t) = (Ge(t)e^T(t) - \sigma(K(t) - F_0))\Gamma, \quad \Gamma > 0, \quad \sigma > 0 \end{cases} \tag{35}$$

Improvement of the adaptive controller

Equilibrium point of $\dot{K}(t)$

$$\dot{K}(t) = (Gee^T - \sigma(K_{eq} - F_0))\Gamma, \quad \Gamma > 0, \quad \sigma > 0 \qquad (36)$$
$$e(t) = y_r(t) - y(t) + p(t)$$

If $p(t) = 0$	If $p(t) \neq 0$
$\dot{K}(t) = 0$	$\dot{K}(t) = 0$
$\Leftrightarrow \sigma(K_{eq} - F_0)\Gamma = 0$	$\Leftrightarrow Gpp^T - \sigma(K_{eq} - F_0)\Gamma = 0$
$\Leftrightarrow K_{eq} = F_0$	$\Leftrightarrow K_{eq} = \sigma^{-1} Gpp^T - F_0$

(37)

σ-modification

Use on the LTI system

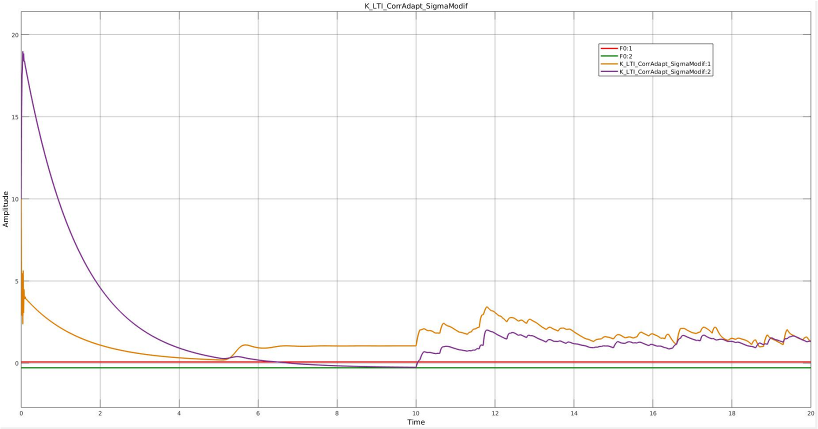


Figure: Evolution of $K(t)$ from the adaptive controller with σ-modification

σ-modification
Use on the LTI system

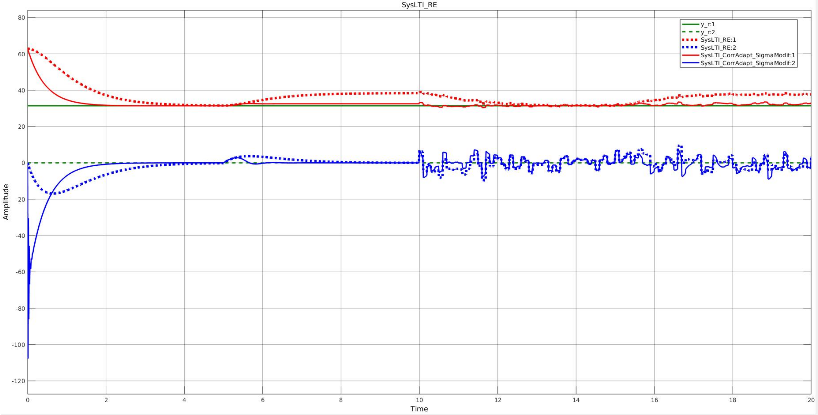


Figure: Simulation of the LTI system controlled with adaptive control law with σ-modification, with speed x10

σ-modification

Use on the LPV system

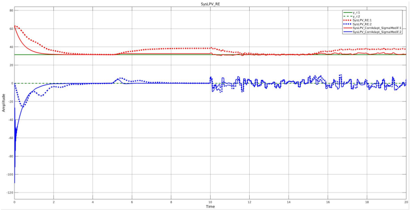


Figure: Simulation of the LPV system controlled with adaptive control law with σ-modification, with speed x10

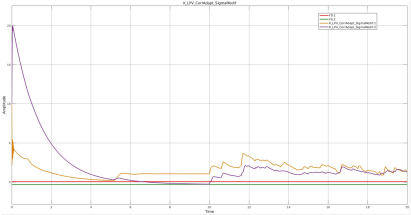


Figure: $K(t)$ from the adaptive controller with σ-modification

Impact of the parameters of the control law

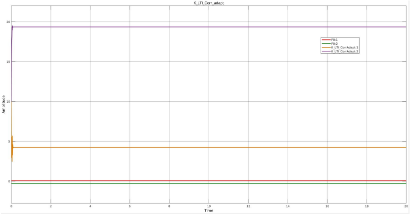


Figure: $K(t)$ with $\Gamma = 0.1$

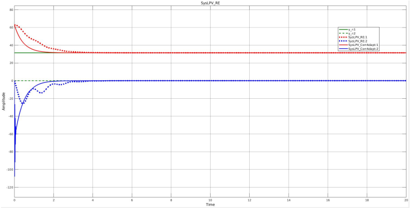


Figure: $y(t)_{LPV_{CorrAdapt}}$ with $\Gamma = 0.1$

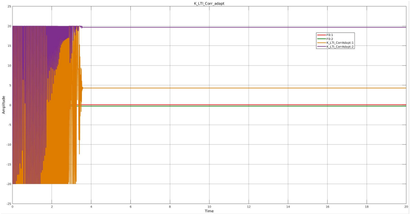


Figure: $K(t)$ with $\Gamma = 100$

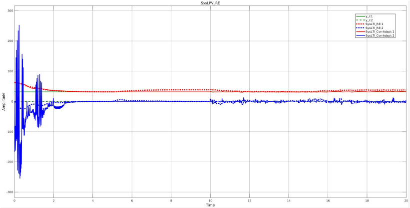


Figure: $y(t)_{LPV_{CorrAdapt}}$ with $\Gamma = 100$

Impact of the parameters of the control law

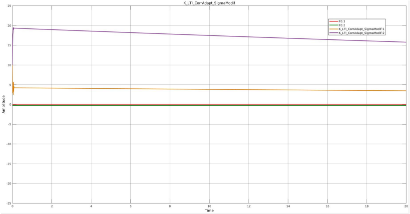


Figure: $K(t)$ with $\sigma = 0.1$

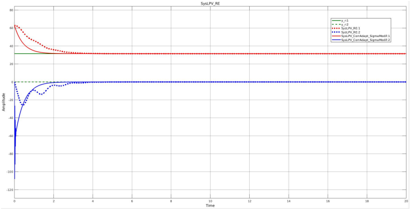


Figure: $y(t)_{LPV_{CorrAdapt+\sigma\, modif}}$ with $\sigma = 0.1$

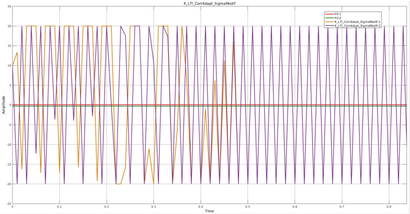


Figure: $K(t)$ with $\sigma = 10^3$

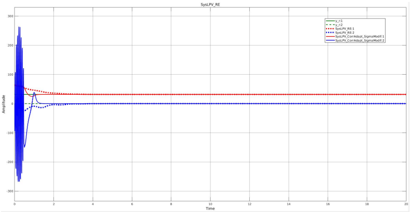


Figure: $y(t)_{LPV_{CorrAdapt+\sigma\, modif}}$ with $\sigma = 10^3$

Conclusion

- First static correction
- Addition of a time-varying parameter
- Implementation of the adaptive control law
- Improvement of the adaptive control law
- Study of the variation of parameters

Outlooks

- Different frequencies for sine wave in $K_{m_u}(t)$
- Add integral effect to cancel static error
- $V(x_a, \theta) = x^T P(\theta) x + Tr[(K - F_0(\theta))\Gamma^{-1}(K - F_0(\theta))^T]$

Two firsts thesis

- Alexandru-Razvan Luzi (2014) : *Time-varying control for satellite attitude control*
    - Replace the current commutation control law with adpative controllers
    - Avoid saturation of the controller
- Harmonie Leduc (2017) : *Robust adaptive control. Application to satellite attitude control*
    - Reinforce previous results with robustness proofs
    - Only for constant bounded parameters

About the thesis

Adaptive control robust to time-varying parameters for satellite attitude control
Supervisor : Dimitri Peaucelle

Objective : Design theoretical and practical frame of adaptive control laws for satellite attitude control, robust to continuous or discontinuous temporal variations of parameters

Thanks for your attention