Output feedback stabilization of non-uniformly observable control systems

Seminar at LAAS

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Sommaire

Introduction

Dynamic output feedback stabilization

Consider a nonlinear control system with measured output:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{1}$$

where x is the state, y is the output and u is the control.

Semi-global dynamic output feedback stabilization problem:
For each compact set $K \subset \mathbb{R}^n$, find a dynamic output feedback

$$\begin{cases} \dot{\hat{x}} = \hat{f}(\hat{x}, u, y) \\ u = \lambda(\hat{x}, y) \end{cases} \tag{2}$$

and a compact set $\hat{K}$ such that $(,0)$ is an asymptotically stable equilibrium with basin of attraction containing $K \times \hat{K}$ of (1)-(2).

Control value at equilibrium: $u \equiv \lambda(0, h())$.

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State feedback stabilization problem: Find a feedback λ such that the origin is a globally asymptotically stable equilibrium point of the vector field $x \mapsto f(x, \lambda(x))$.

Idea: Design an observer system

$$\dot{\hat{x}} = \hat{f}(\hat{x}, u, y) \tag{3}$$

such that $\hat{x} - x \to 0$ for all initial conditions in $K \times \hat{K}$ and use the control $u = \lambda(\hat{x})$ with λ globally stabilizing. The closed-loop system is given by

$$\begin{cases} \dot{x} = f(x, \lambda(\hat{x})) \\ y = h(x) \\ \dot{\hat{x}} = \hat{f}(\hat{x}, \lambda(\hat{x}), y). \end{cases} \tag{4}$$

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Summary of the strategy:

1. Find a globally stabilizing state feedback,
2. Design an observer,
3. Show that dynamic output feedback stabilization is achieved.

Definition 1 (Observability and Uniform observability).
A control system with measured output is said to be observable in time $T > 0$ and for a given input u if for all pair of initial conditions (x_1, x_2),

$$\left\{\forall t \in [0, T], y(t; x_1) = y(t; x_2)\right\} \Longrightarrow x_1 = x_2. \tag{5}$$

If it is observable for any time $T > 0$ and **for any input** u, then it is said to be uniformly observable in small time.

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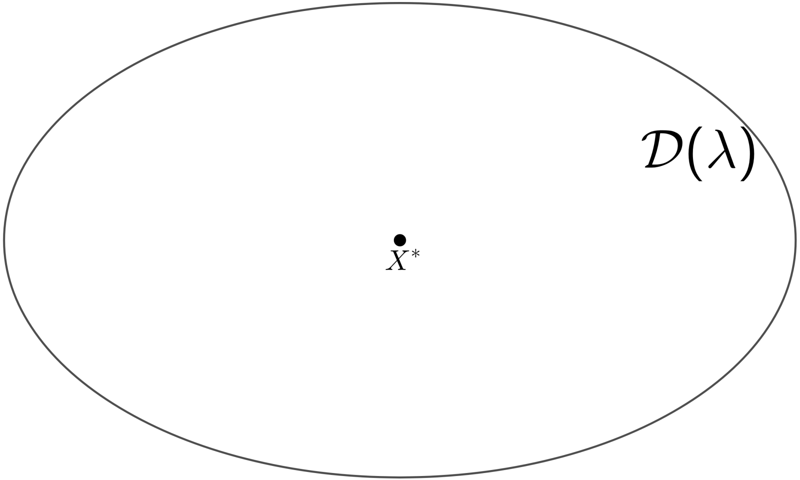
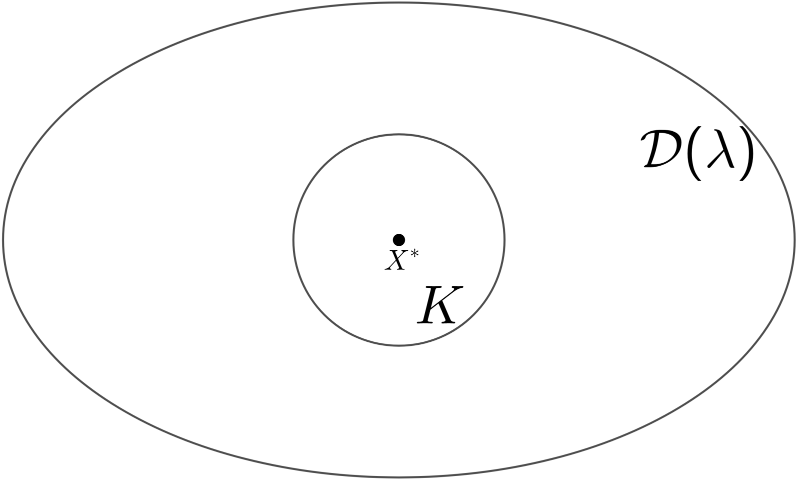


Figure 1: Dynamic output feedback stabilization diagram

Dynamic output feedback stabilization



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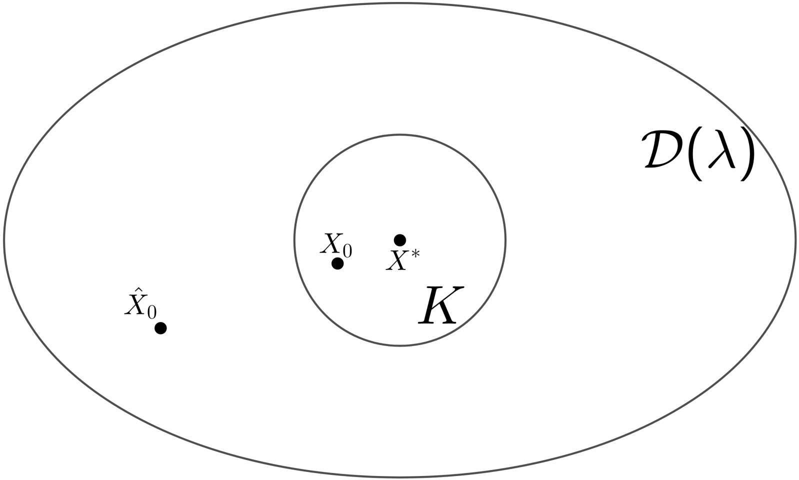

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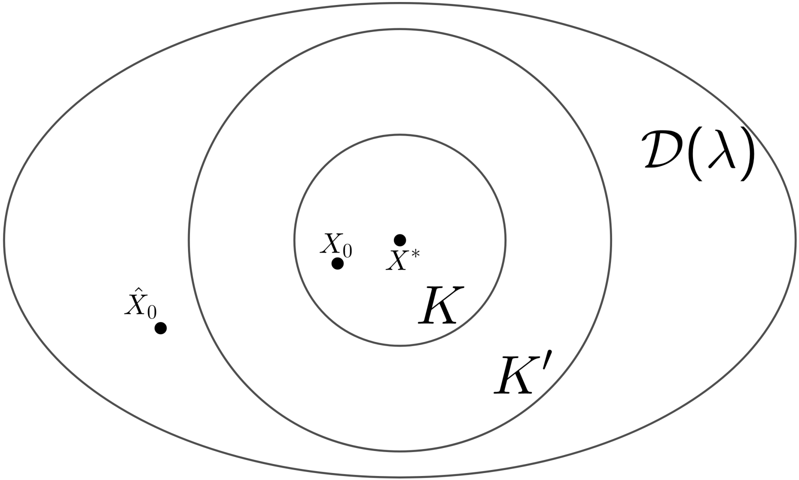

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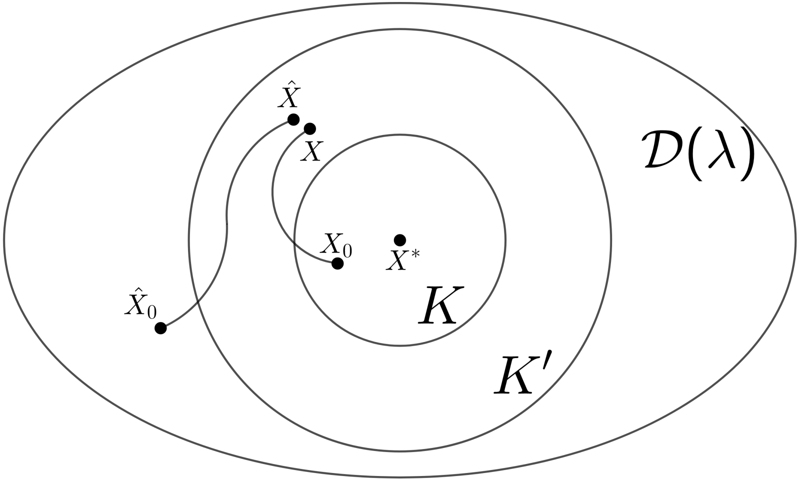

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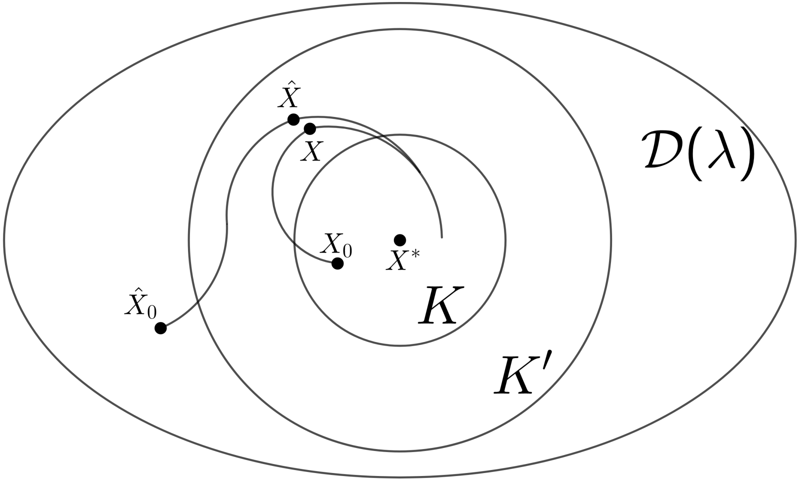

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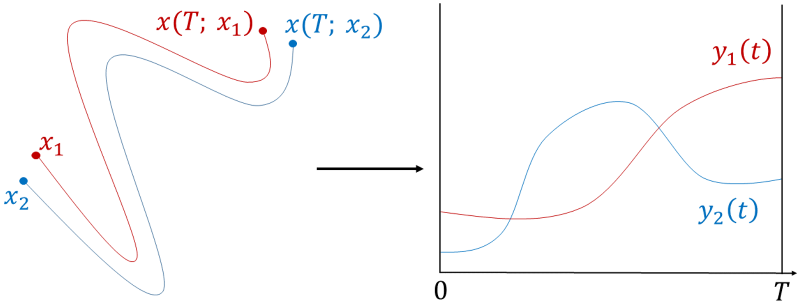

Figure 1: Observability

Uniform observability

Theorem 1 (A. Teel and L. Praly 1994).
If a system is

- *globally state feedback stabilizable*
- *and uniformly observable in small time,*

then it is also semi-globally stabilizable by dynamic output feedback.

Problem: It is not generic for a dynamical system to be uniformly observable (Gauthier and Kupka 2001).

What if there is no uniform observability?

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Non-uniformly observable systems

We distinguish 2 cases:

1. The system is not uniformly observable, but the target point corresponds to an input that makes the system observable.
2. The control is singular at the target point.

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   Example:
$$\dot{x} = \begin{pmatrix} 0 & 1+u \\ -1 & 0 \end{pmatrix} x, \quad y = \begin{pmatrix} 1 & 0 \end{pmatrix} x.$$
   Observability matrix:
$$\begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1+u \end{pmatrix}.$$
   The pair (C, A) is observable for $u \equiv \lambda(0) = 0$, but not for $u = -1$.
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Feedback perturbation

Objective: Get observability of the chosen control u.

General idea: Feedback modification

$$u = \lambda(\hat{x}) \longrightarrow u = \tilde{\lambda}(\hat{x}).$$

- Time-varying feedback
  - Excite the system to estimate the state, then control to stabilize, and start again...
  - Coron 1994: local stabilization
  - Shim and Teel 2003: practical stabilization
- Smooth autonomous perturbation
  - Lagache, Serres, and Gauthier 2017: additive perturbation

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Strategy of Lagache, Serres, and Gauthier 2017:

1. Show that there exists a (smooth) small perturbation δ of λ such that the control $(\lambda + \delta) \circ \hat{x}$ makes the system observable.
2. Show that with this control, the observer converges to the state (and remains in a fixed compact set).
3. Show that we achieve stabilization.

Remarks:

- Example of quantum control;
- **Unobservable** target;
- **Practical** stabilization and **exact** stabilization.

Towards a generalization?

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Observable target

Context

Systems under consideration: SISO bilinear systems with linear output

$$\begin{cases} \dot{x} = (A + uB)x + bu \\ y = Cx \end{cases} \tag{5}$$

where $x \in \mathbb{R}^n$, $u, y \in \mathbb{R}$, $A, B \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{1\times n}$ and $b \in \mathbb{R}^n$.

Observer system:

$$\dot{\hat{x}} = (A + uB)\hat{x} + bu - PC^*C(\hat{x} - x) \tag{6}$$

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- Assume that the target is observable: (C, A) is observable.
- Assume that the system is state feedback stabilizable.
- We are able to perform Step 1.
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Recall that the system

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is observable in time $T > 0$ for a given input u if for all pair of initial conditions (x_1, x_2),

$$\left\{ \forall t \in [0, T], y(t; x_1) = y(t; x_2) \right\} \Longrightarrow x_1 = x_2. \tag{8}$$

Let $\omega(t) = x(t; x_1) - x(t; x_2)$ for all $t \in [0, T]$. Then

$$\begin{cases} \dot{\omega} = (A + uB)\omega \\ C\omega = y(t; x_1) - y(t; x_2). \end{cases} \tag{9}$$

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Avoiding observability singularities

Let $\lambda : \mathbb{R}^n \mapsto \mathbb{R}$ be a state stabilizing feedback with basin of attraction $\mathcal{D}(\lambda)$. Let $K = K_1 \times K_2 \times K_3 \subset \mathcal{D}(\lambda) \times \mathbb{R}^n \times \mathrm{S}_{++}(n)$ be a compact set.

Theorem 2.

If the pairs (C, A) and (C, B) are observable and $0 \notin K_1$, then there exist $\eta > 0$, $k > 0$ and a dense open subset $\mathcal{O} \subset \mathcal{N}(k, K_1, \eta)$ such that for all initial condition in $K \times \mathbb{S}^{n-1}$ the solution of $\dot{\omega} = (A + (\lambda + \delta)(\hat{x})B)\omega$ with $\delta \in \mathcal{O}$ satisfies

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For $k \in \mathbb{N}$, $K \subset \mathbb{R}^n$ compact and $\eta > 0$:

$$\mathcal{N}(k, K, \eta) = \{\delta \in C^\infty(\mathbb{R}^n, \mathbb{R}) \mid \|\delta\|_{k,K} < \eta\}$$

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- The statement of the theorem is stronger than observability of the system for any time $T > 0$.
- The proof is based on transversality theory.
- A crucial point in the proof is to show that there exists a positive integer k such that the solution of the observer system satisfies $\hat{x}^{(k)}(0) \neq 0$. We have shown that this is true for Kalman and Luenberger observers.

$$\begin{pmatrix} C\omega \\ C\dot{\omega} \\ C\ddot{\omega} \\ \vdots \\ C\omega^{(\ell-1)} \end{pmatrix} = \begin{pmatrix} C \\ C(A+uB) \\ C(A+uB)^2 + \dot{u}CB \\ \vdots \\ C(A+uB)^{\ell-1} + CP_{n-1}(\dot{u}, u^{(2)}, \dots, u^{(\ell-2)}) \end{pmatrix} \omega \quad (12)$$

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Dissipative systems

We have not been able to show the asymptotic stabilization.

Dissipative systems:

$$\dot{x} = A(u)x + bu, \qquad y = Cx \tag{13}$$

with

$$x^* A(u) x \leqslant 0, \qquad \forall x \in \mathbb{R}^n,\ \forall u \in \mathbb{R}. \tag{14}$$

Theorem 3.

Assume that λ is a globally stabilizing state feedback, $(C, A(0))$ is observable, and $A(u)$ is dissipative for all $u \in \mathbb{R}$. Then for any compact set $K \subset \mathbb{R}^n$, there exists $\alpha > 0$ such that $(0,0)$ is an asymptotically stable equilibrium point with basin of attraction containing $K \times K$ of

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Sketch of the proof:

1. Local asymptotic stability of the target.
   Proof: Linearization of the system at 0.
2. Bounded trajectories converge to the target.
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3. All trajectories are bounded.
   Proof: Choose α large enough, so x does not exit the stability domain.

Remarks:

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Unobservable target

An example

Let $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Consider the problem of stabilization at 0 of the following system:

$$\begin{cases} \dot{x} = Jx + bu, \\ y = x_1^2 + x_2^2. \end{cases} \tag{16}$$

Natural choice of **state feedback**:

$$\lambda(x_1, x_2) = -2x_2. \tag{17}$$

Observability at 0:

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Immersion into a dissipative system

Idea: Immersion into a bilinear dissipative system with linear output

$$\tau(x_1, x_2) = \left(\frac{x_1^2 + x_2^2}{2}, x_1, x_2\right). \tag{19}$$

Let $z = \tau(x)$. Then

$$\begin{cases} \dot{x} = Jx + bu \\ y = |x|^2 \\ x(0) \in \mathbb{R}^2 \end{cases} \xrightarrow[z=\tau(x)]{} \begin{cases} \dot{z} = A(u)z + Bu \\ y = Cz \\ z(0) \in \{z \in \mathbb{R}^3 \mid 2z_1 = z_2^2 + z_3^2\} =: \mathcal{P} \end{cases} \tag{20}$$

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with $K(u) = \begin{pmatrix} \alpha \\ 0 \\ u \end{pmatrix}$ and $\alpha > 0$. Then,

$$A(u) - K(u)C = \underbrace{\begin{pmatrix} 0 & 0 & u \\ 0 & 0 & -1 \\ -u & 1 & 0 \end{pmatrix}}_{\text{dissipative}} - \underbrace{\alpha C^* C.}_{\text{Luenberger correction term}} \tag{22}$$

Immersion into a dissipative system

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$$\begin{cases} \dot{\varepsilon} = (A(u) - K(u)C)\,\varepsilon \\ \dot{\hat{z}} = A(u)\hat{z} + Bu - K(u)C\varepsilon. \end{cases} \tag{21}$$

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1. Add a perturbation to the feedback:

$$\lambda_\delta(z) = -2z_3 + \delta z_1. \tag{23}$$

2. If $(\varepsilon_0, \hat{z}_0) \neq (0,0)$, then $u = \lambda_\delta(\hat{z})$ makes the system observable in time T for any $T > 0$.
   Proof: Compute the first derivatives of $C\omega$ where $\dot{\omega} = A(u)\omega$ and $\omega_0 \neq 0$.
3. If the trajectories is bounded, then $\varepsilon \to 0$.
   Proof: Otherwise, $R > |\varepsilon| > r > 0$ since $|\varepsilon|$ is **non increasing**.
   Then, the input is **persistent**, thus $\varepsilon \to 0$ (see Celle et al. 1989).
   Then $\hat{z} \to 0$. Proof: Choose δ small enough.
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Proposition 4.

Let U be a compact subset of $\mathbb{R}^6$. There exist $\delta_0 > 0$, $\alpha_0 > 0$ such that for all $\delta \in (\delta_0, +\infty)$ and all $\alpha \in (0, \alpha_0)$, $(0,0)$ is an asymptotically stable equilibrium point of the system

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Immersion into an **infinite-dimensional** dissipative bilinear system:

$$\tau(x) : \theta \in \mathbb{S}^1 \mapsto \exp\left(i\mu(x_1 \cos(\theta) + x_2 \sin(\theta))\right). \qquad (25)$$

Let $z = \tau(x)$. Then

$$\begin{cases} \dot{x} = Jx + bu \\ y = |x| \\ x(0) \in \mathbb{R}^2 \end{cases} \quad \xrightarrow[z=\tau(x)]{} \quad \begin{cases} \dot{z} = A(u)z \\ J_0(\mu y) = Cz \\ z_0 \in Im\,\tau \subset L^2(\mathbb{S}^1) \end{cases} \qquad (26)$$

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$$A(u) = -\frac{\partial}{\partial\theta} + iu\sin(\theta), \qquad C = \langle \cdot, 1 \rangle_{L^2(\mathbb{S}^1)}$$

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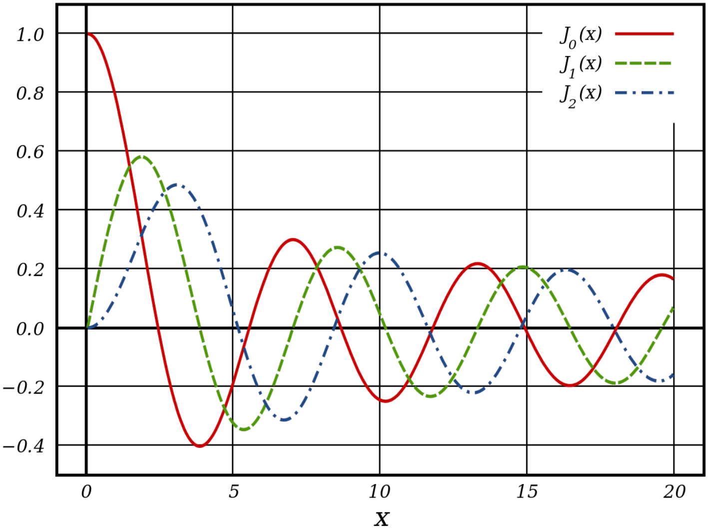

Immersion into an infinite-dimensional dissipative system

Infinite-dimensional observer system:

$$\begin{cases} \dot{\hat{z}} = A(u)\hat{z} - C^* C\varepsilon \\ \dot{\varepsilon} = (A(u) - C^* C)\varepsilon. \end{cases} \tag{27}$$

with $A(u)$ skew-adjoint for all u.

This system has been investigated in Celle et al. 1989.

Main differences with the finite dimensional case:

- Find a pseudo-inverse of τ;
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Concluding remark: The next step is a generalization of the result, first to other examples, then to a class of systems.

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Thank you for your attention

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