Free-matrices min-projection control for high frequency DC-DC converters

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Research on power converters control

Why should we get an interest in converters ?

- Their omnipresence
- Relevant issues
  - Efficiency
  - Reliability

... , how Automatic Control can improve those properties?

Plan

Switched Affine Systems

$$\dot{z}(t) = \mathbf{A}_\sigma z(t) + \mathbf{b}_\sigma, \qquad (1)$$

- $z(t) \in \mathbb{R}^n$: state vector
- $\sigma \in \mathbb{K} := \{1, ..., N\}$: control input

$\mathbf{A}_\sigma \in \{\mathbf{A}_1, ..., \mathbf{A}_N\}$ and $\mathbf{b}_\sigma \in \{\mathbf{b}_1, ..., \mathbf{b}_N\}$ are matrices of suited dimension

Assumption 1

Existence of a couple $(z_e, \lambda) \in \mathbb{R}^n \times \Lambda_\mathbb{K}$

$$\Omega_e = \{z_e \in \mathbb{R}^n,\ \mathbf{A}_\lambda z_e + \mathbf{b}_\lambda = 0, \quad \lambda \in \Lambda_\mathbb{K}\},$$

Notation

$$\Lambda_\mathbb{K} := \left\{\lambda = [\lambda_1, ..., \lambda_N] \in \mathbb{R}^N : \lambda_i \geq 0,\ \textstyle\sum_{i=1}^N \lambda_i = 1\right\},$$

$$\mathbf{A}_\lambda := \textstyle\sum_{i\in\mathbb{K}} \lambda_i \mathbf{A}_i, \qquad \mathbf{b}_\lambda := \textstyle\sum_{i\in\mathbb{K}} \lambda_i \mathbf{b}_i.$$

Switched Affine Systems

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Error equation

$$\dot{x}(t) = \mathbf{A}_\sigma x(t) + \mathcal{B}_\sigma, \qquad (2)$$

- $x(t) := z(t) - z_e$
- $\mathcal{B}_\sigma = \mathbf{A}_\sigma z_e + \mathbf{b}_\sigma$

Periodic sampled-data control

- Control laws that consider min-projection strategy : (Deaecto and Geromel, 2017), (Hauroigne et al., 2011) and (Hetel and Fridman, 2013)
  - → No suitable for high frequency systems
  - → Depends on the Lyapunov matrix or systems matrices
- Control law that consider high frequency model : (Ventosa-Cutillas et al., 2018)
  - → Invariance of the set is not proven

The discretization with δ-operator

Assumption 2

The sampling period is very small, $T << 1$

δ-operator (Middleton and Goodwin, 1986)

For any function ξ from $\mathbb{R}^+$ to $\mathbb{R}^n$, the vector $\delta\xi_k$, at any sampling instant $t_k \in \mathbb{R}^+$, is defined as follows:

$$\delta\xi_k := \frac{1}{T}(\xi_{k+1} - \xi_k), \qquad \forall k \geq 0,$$

Notation

$\delta\xi_k = \delta\xi(t_k)$ and $\xi_k = \xi(t_k)$

The discretization with δ-operator

Hence, we have

$$\delta x_k = A_\sigma x_k + B_\sigma, \tag{3}$$

where matrices A_σ and B_σ, with $\sigma \in \mathbb{K}$, are given by

$$A_\sigma = \frac{1}{T}(e^{\mathbf{A}_\sigma T} - I), \qquad B_\sigma = \frac{1}{T}\int_0^T e^{\mathbf{A}_\sigma (T-s)} ds \mathcal{B}_\sigma$$

Remark 1

When $T \to 0$

$A_\sigma \to \mathbf{A}_\sigma$ and $B_\sigma \to \mathcal{B}_\sigma$

Attractor set and stability

Lyapunov function

Attractors expressed as a level set of a Lyapunov function

$$V(x, x_c) = (x - x_c)^T P(x - x_c), \tag{4}$$

Attractor

$$\mathcal{A}(x_c) := \{x \in \mathbb{R}^n \mid V(x, x_c) < 1\} \tag{5}$$

Stability

The set $\mathcal{A}(x_c)$ is Uniformly Globally Asymptotically Stable if

1. if $x_k \notin \mathcal{A}(x_c)$, then $T\delta V(x_k, x_c) = V(x_{k+1}, x_c) - V(x_k, x_c) < 0$
2. $x = 0 \in \mathcal{A}(x_c)$
3. whenever $x_k \in \mathcal{A}(x_c)$, then $x_{k+1} \in \mathcal{A}(x_c)$

Switching control law

- σ depends on x_k

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$$\sigma \in \underset{i \in \mathbb{K}}{\operatorname{argmin}} \begin{bmatrix} x_k \\ 1 \end{bmatrix}^T N_i \begin{bmatrix} x_k \\ 1 \end{bmatrix} \tag{6}$$

$N_\lambda := \sum_{i \in \mathbb{K}} \lambda_i N_i$

Switching control law

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- Suitable for high frequency

Main Result

Theorem 1

For a given $z_e \in \Omega_e$ and a given parameter $\mu \in (0,1)$, assume that matrices $P \in \mathbb{R}^{n\times n} \succ 0$, $h \in \mathbb{R}^n$, and $N_i = N_i^T \in \mathbb{R}^{(n+1)\times(n+1)}$ are the solutions to the optimization problem

$$\begin{aligned} \min_{\mu,P,h,N_i} \quad & -\log(\det(P)) \\ s.t. \quad & P \succ 0, \\ & LMI_1(A_i, B_i, P, h, N_i, \lambda) \prec 0, \quad \forall i \in \mathbb{K}, \\ & LMI_2(B_\lambda, P, h, \lambda) \succ 0, \end{aligned}$$

Then, the control law ensures that $\mathcal{A}(-P^{-1}h)$ is UGAS for system and of a minimum volume.

Proof

Assumption 3

$P, h, N_i, i \in \mathbb{K}:$ a solution to the convex problem, for a given $\mu \in (0,1)$

At any t_k, the control law ensures :

$$\begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top (N_\lambda - N_\sigma) \begin{bmatrix} x_k \\ 1 \end{bmatrix} \geq 0$$

$\forall x \in \mathbb{R}^n$, s.t. $V(x, -P^{-1}h) > 1$, we have :

$$\begin{bmatrix} x \\ 1 \end{bmatrix}^\top \begin{bmatrix} P & h \\ h^\top & h^\top P^{-1} h - 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} > 0$$

$$\delta V = \begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top \Psi_\sigma \begin{bmatrix} x_k \\ 1 \end{bmatrix}$$

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$$\delta V = \begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top \Psi_\sigma \begin{bmatrix} x_k \\ 1 \end{bmatrix} < 0$$

Using S-procedures, into the existence of a positive scalar $\mu > 0$, then using the Schur complement, we obtain LMI_1

Proof

1. $\rightarrow$ Computing convex combination, weighted by λ, we obtain LMI_2
2. $\rightarrow$ With

$$
\begin{aligned}
V(x_{k+1}, -P^{-1}h) &= V(x_k, -P^{-1}h) + T\delta V \\
&= V(x_k, -P^{-1}h) + T \begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top \Psi_\sigma \begin{bmatrix} x_k \\ 1 \end{bmatrix} \\
&\leq (1-\mu)V(x_k, -P^{-1}h) + \mu
\end{aligned}
$$

Example Boost converter

$$\dot{z}(t) = \mathbf{A}_\sigma z(t) + \mathbf{b}_\sigma$$

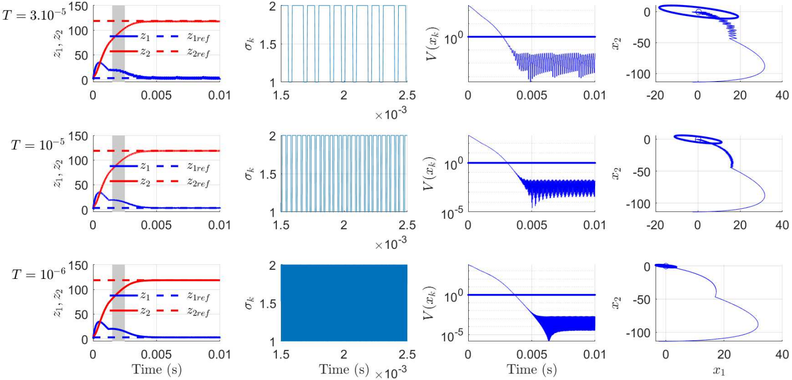
$$\mathbf{A}_\sigma = \begin{bmatrix} -\frac{R}{L} & \frac{(1-\sigma)}{L} \\ \frac{(\sigma-1)}{C_o} & -\frac{1}{R_o C_o} \end{bmatrix}, \quad \mathbf{b}_\sigma = \begin{bmatrix} \frac{V_{in}}{L} \\ 0 \end{bmatrix}, \quad z_e = \begin{bmatrix} 3 \\ 120 \end{bmatrix} \Rightarrow \lambda = \begin{bmatrix} 0.22 \\ 0.78 \end{bmatrix}$$

T	μ	$(det(P))^{\frac{1}{2}}$	x_c
3.10^{-5}	0.113	393.42	$\begin{bmatrix} 1.669 & -4.099 \end{bmatrix}^\top$
10^{-5}	0.013	54.08	$\begin{bmatrix} 0.1506 & -0.3214 \end{bmatrix}^\top$
10^{-6}	0.001	5.57	$\begin{bmatrix} 0.0146 & -0.03012 \end{bmatrix}^\top$

Values

$V_{in} = 100V$, $R = 2\Omega$, $L = 500\mu H$, $C_o = 470\mu F$ and $R_o = 50\Omega$.

Example Boost converter



Conclusions & Perspectives

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- The introduction of a free-matrices min-projection control law.
- Control design for discrete time switched affine systems using the delta-operator for high-frequency.

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- Control design for discrete time switched affine systems using the delta-operator for high-frequency.

Perspectives

- Robustness to parameter uncertainties or variation in the sampling period.

Thank you !

Main result

Theorem 1

For a given $z_e \in \Omega_e$ and a given parameter $\mu \in (0,1)$, assume that matrices $P \in \mathbb{R}^{n\times n} \succ 0$, $h \in \mathbb{R}^n$, and $N_i = N_i^T \in \mathbb{R}^{(n+1)\times(n+1)}$ are the solutions to the optimization problem

$$\min_{\mu,P,h,N_i} \quad -\log(\det(P)) \tag{7}$$

$$s.t. \qquad P \succ 0, \tag{8}$$

$$\begin{bmatrix} \Psi_i + N_\lambda - N_i - \frac{\mu}{T}\begin{bmatrix} 0 & 0 \\ * & 1 \end{bmatrix} & \frac{\mu}{T}\begin{bmatrix} P \\ h^\top \end{bmatrix} \\ * & -\frac{\mu}{T}P \end{bmatrix} \prec 0, \quad \forall i \in \mathbb{K}, \tag{9}$$

$$\text{He}\left(h^\top B_\lambda\right) + T\sum_{i\in\mathbb{K}} \lambda_i B_i^\top P B_i \succ 0, \tag{10}$$

Main result

Theorem 1 (cont.)

where

$$\Psi_i := \mathrm{He}\left(\begin{bmatrix} A_i^\top \\ B_i^\top \end{bmatrix} \begin{bmatrix} P & h \end{bmatrix}\right) + T \begin{bmatrix} A_i^\top \\ B_i^\top \end{bmatrix} P \begin{bmatrix} A_i & B_i \end{bmatrix}, \quad (11)$$

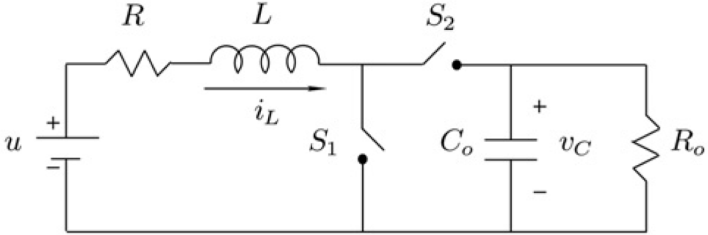
$$N_\lambda := \sum \lambda_i N_i. \quad (12)$$

Then, the switching control law given by

$$\sigma \in \underset{i \in \mathbb{K}}{\mathrm{argmin}} \begin{bmatrix} x_k \\ 1 \end{bmatrix}^T N_i \begin{bmatrix} x_k \\ 1 \end{bmatrix} \quad (13)$$

ensures that $\mathcal{A}(-P^{-1}h)$ is UGAS for system and of a minimum size.

Example Boost Converter



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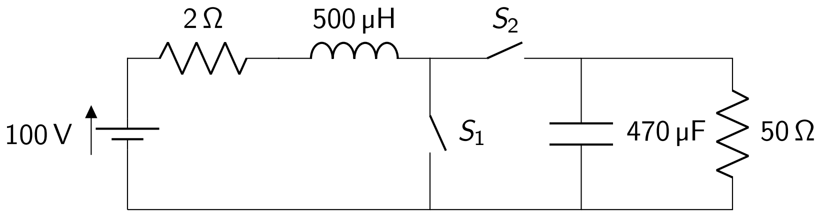

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