Magnetic force modelling and nonlinear switched control of an electromagnetic actuator

58TH IEEE Conference on Decision and Control

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- Convergence of the complete system

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- Intuitive global strategy
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System presentation :

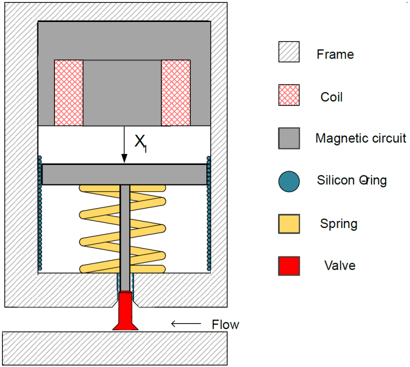

FIGURE – Schematic of the EMA

State space representation :

$$x = \begin{pmatrix} x_1 & \text{Moving part position} \\ x_2 & \text{Moving part speed} \\ x_3 & \text{Magnetic coil current} \end{pmatrix} \quad (1)$$

Problem statement

State space system :

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \dfrac{1}{m}\left[-F_{mag}(x_1, x_3) - \lambda x_2 - K(x_1 - x_0)\right], \\ \dot{x}_3 = \dfrac{1}{L(x_1, x_3)}\left[u - Rx_3 + x_2 x_3 \dfrac{\partial L}{\partial x_1}\right]. \end{cases} \tag{2}$$

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Paper goal :

- Develop a new magnetic force model.
- The actuator position x_1 has to track a reference signal y_r

Magnetic force measurment

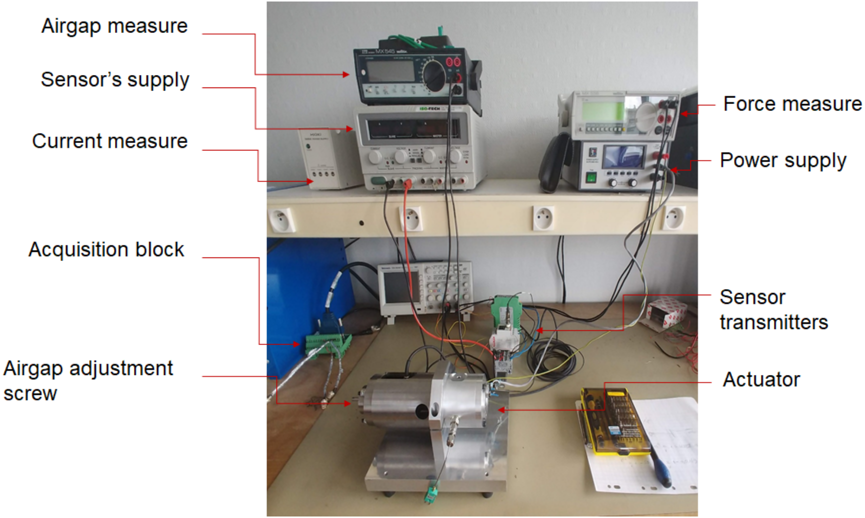

FIGURE – Testbench of force measurment

Magnetic force measurment

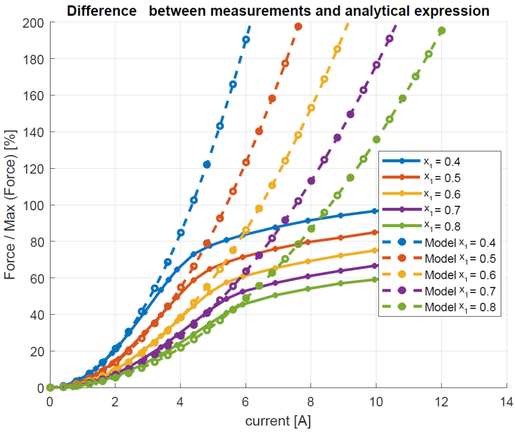

FIGURE – Comparison between analytical model and measurements

Magnetic force modelling

A method from [Yan, 2000]and [Wang, 2002]was adapted to take into account the magnetic saturation in the force modelling :

$$F_{mag}(x_1, x_3) = \begin{cases} F_{mag}^{lin} & \text{if } x_3 \leq i_s(x_1), \\ F_{mag}^{sat} & \text{if } x_3 > i_s(x_1). \end{cases} \tag{3}$$

with

$$\begin{cases} F_{mag}^{lin} & = & \frac{1}{2} x_3^2 \frac{dL}{dx_1} \\ F_{mag}^{sat} & = & p_1(x_1) e^{p_2(x_1) x_3} + p_3(x_1) e^{p_4(x_1) x_3} + cor(x_1). \end{cases} \tag{4}$$

Remark : The functions $p_i(x_1)$ and $cor(x_1)$ are given by a parameter identification from optimization tools

Magnetic force modelling

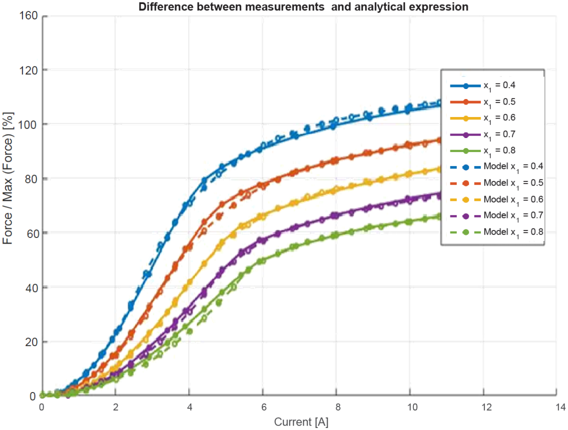

FIGURE – Comparaison between switched analytical model and measurements

Mechanical subsystem stabilization by Backstepping

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Aim : Find the desired current x_{3d} that stabilises the subsystem.

Problem : Complicated to express x_{3d} due to the expression of $F_{mag}(x_1, x_3)$.

Solution : More convenient to find the desired magnetic force F_d to stabilize this susbsytem.

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Theorem

Consider α_1, α_2 two positives scalars, the virtual control law $F_d = m\left(az_1 + (b + \alpha_2)z_2 - d\right)$ with $a = 1 - \alpha_1^2 + \frac{\lambda}{m}\alpha_1 - \frac{K}{m}$, $b = \alpha_1 - \frac{\lambda}{m}$ and $d = \frac{K}{m}(y_r - x_0)$. makes the subsystem (5) converge to $(y_r, 0)$.

Proof

Step 1 : define errors variables such that x_1 follow the reference signal y_r :

$$\begin{cases} z_1 = x_1 - y_r, \\ z_2 = x_2 + \alpha_1 z_1, \end{cases}$$

Step 2 : use a Lyapunov function to prove stability

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2$$

The derivative of V_1, if $F_{mag} = F_d$ is equal to :

$$\dot{V}_1 = -\alpha_1 z_1^2 - \alpha_2 z_2^2 \leq 0 \quad \forall z_1, z_2$$

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Remark

Notice that, by construction F_{mag} is always positive, while the expression of the desired force F_d may be not always positive.

Definition : validity region

Lemma

The estimation of the validity region is defined by the largest level line of $V(z_1, z_2)$ where there is a single intersection point between V and $F_d = 0$. This set is defined as : $\exists C \in \mathbb{R}^+$ such that

$$\mathbb{D} = \{(z_1, z_2) \in \mathbb{R}^2 | V(z_1, z_2) \leq C\}.$$

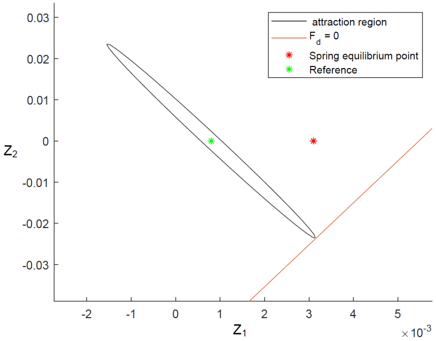

FIGURE – Approximation of the validity region for $\alpha_1 = \alpha_2 = 100$

Local convergence of the complete system

The full system (6) is now considered :

$$\left\{\begin{array}{rcl}\dot{z}_1 & = & -\alpha_1 z_1 + z_2, \\ \\ \dot{z}_2 & = & \dfrac{1}{m}\left[-F_{mag}(z_1, x_3) - \lambda(z_2 - \alpha_1 z_1) - K(z_1 + y_r - x_0)\right] \\ & + & \alpha_1 z_2 - \alpha_1^2 z_1. \\ \dot{x}_3 & = & \dfrac{1}{L(z_1, x_3)}\left[u - Rx_3 + (z_2 - \alpha_1 z_1)x_3 \dfrac{\partial L}{\partial z_1}\right].\end{array}\right. \tag{6}$$

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Theorem

Assume the initial condition $x_{ini} \in \mathbb{D}$, then the control

$$u = \frac{1}{g_F(z)}\left[-\alpha_3\left(F_{mag} - F_d\right) + \frac{z_2}{m} + \dot{F}_d - f_F(z)\right],$$

with $\alpha_3 > 0$, $z = (z_1, z_2, x_3)$ and $\dot{F}_{mag} = f_F(z) + g_F(z)u$ where makes the system (6) stable and makes the position converges to y_r.

Expansion of the validity region

In order to optimize the size of the domain $\mathbb{D}$, let consider a more general Lyapunov function candidate.

$$V = Z^T P Z, \tag{7}$$

with $Z = (z_1, z_2)$ and P a definite positive matrix.

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If $F_d = -\frac{K}{m}(y_r - x_0) - \alpha_1 m z_1 - \alpha_2 m z_2$, $\exists P > 0$, $\exists Q > 0$ *such that* $\dot{V} = A^T P + PA < -Q$ *and* $\dot{V} < -\alpha V$ *with* $\alpha > 0$ *and* $V = Z^T P Z$.

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Tool : Use a LMI procedure to find a P matrix :

objective function

$$Min\ tr(P)$$

under constraints

$$A^T P + PA < -\alpha P$$

Expansion of the validity region

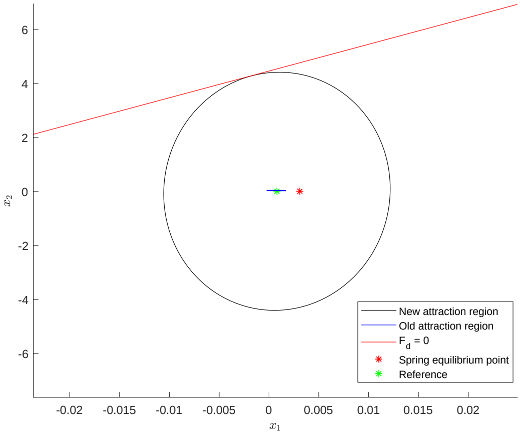


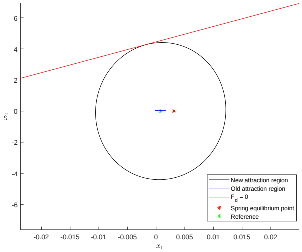
FIGURE – validity region optimisation

The spring equilibrium point $X0$ is now included in $\mathbb{D}$

Intuitive global strategy

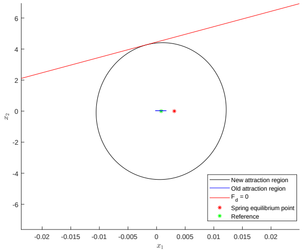
- *case 1* : $x(t) \in \mathbb{D}$
  The control u of theorem 1 and $F_d = m\left(-\alpha_1 z_1 - \alpha_2 z_2 - d\right)$ are choosen and the system converges to the desired equilibrium point y_r

- *case 2* : $x(t) \in \bar{\mathbb{D}}$
  The control $u = 0$ is enforced and there is a time where the trajectorie $x(t)$ hits $\mathbb{D}$ because $X0$ is attractive and it returns to the *case 1*.



Hybrid global strategy

- *case 1* : $x_{ini} \in \mathbb{D}$ The control u of Theorem 1 is choosen and the system converges to the desired equilibrium point $[y_r, 0]^T$
- *case 2* : $x_{ini} \in \{F_d < 0\}$ The control $u = 0$ is choosen and as the spring equilibrium point $X0 \in \mathbb{D}$, there exists $t_1 > t_0$ where $x(t_1) \in \mathbb{D}$.
- *case 3* : $x_{ini} \in \{F_d > 0\} \cap \bar{\mathbb{D}}$ The control u of Theorem 1 is choosen and the trajectories $x(t)$ have two options : $x(t)$ enter in $\mathbb{D}$ or in $\{F_d = 0\}$.



Hybrid modelling of the closed-loop system

Using a token M to take into account the fact that the trajectory $x(t)$ has ever been in region $\{F_d < 0\}$.

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So let consider the flow set

$$\mathbb{F}_f := \{\{M = 1\} \times \{F_d \geq 0\} \text{ or } \{M = 0\} \times \{z | V(z) \leq C\}\} . \qquad (8)$$

Let consider the jump set

$$\mathbb{D}_f := \{\{M = 1\} \times \{F_d \leq 0\} \text{ or } \{M = 0\} \times \{z | V(z) > C\}\} . \qquad (9)$$

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The closed loop system can be rewritten as

$$\begin{cases} \left.\begin{array}{ll} \dot{z} & = f(z(t), u_M(z)) \\ \dot{M} & = 0 \end{array}\right\} \text{ if } (z, M) \in \mathbb{F}_f \\ \\ \left.\begin{array}{ll} z^+ & = z \\ M^+ & = M - 1 \end{array}\right\} \text{ if } (z, M) \in \mathbb{D}_f \end{cases} \tag{10}$$

where

$$u = \begin{cases} u_1(z) & = & \frac{1}{g_F(z)}\left[-\alpha_3 z_3 + \frac{z_2}{m} + \dot{F}_d - f_F(z)\right] \\ u_0(z) & = & 0 \end{cases} \tag{11}$$

Convergence of the complete system

with $M(t_0) = 1$. Inspired by [Goebel et Al., 2009] , the system (10) satisfies the conditions which ensures the well-posedness of the closed-loop system.

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Theorem

Assume the closed-loop system (10), and consider the compact set $\mathbb{A} = \{x = x_{eq}\ ,\ M \in \{0,1\}\}$ *then* $\mathbb{A}$ *is globaly asymptotically stable.*

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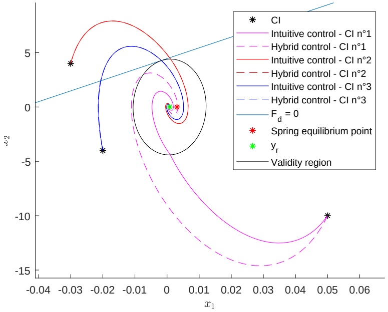

FIGURE – Dynamics of the controlled subsystem

Position tracking simulation :

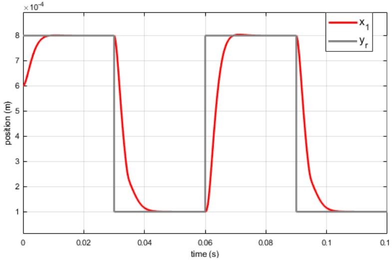

FIGURE – Position tracking simulation

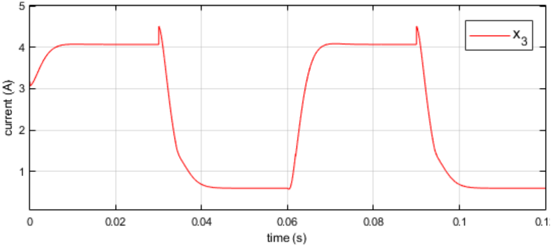

FIGURE – Current simulation

Conclusion

- The tracking has been achieved in simulation and more recently in the testbed
- Possible improvement : A more generic Lyapunov function with a new form of F_d may enlarge the set $\mathbb{D}$.
- A future paper will sum up the global work : A theoritical to an experimental work.