Evolutionary multi-objective optimization of colour pixels based on dielectric nanoantennas

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The rational design of photonic nanostructures consists of anticipating their optical response from systematic variations of simple models. This strategy, however, has limited success when multiple objectives are simultaneously targeted, because it requires demanding computational schemes. To this end, evolutionary algorithms can drive the morphology of a nano-object towards an optimum through several cycles of selection, mutation and cross-over, mimicking the process of natural selection. Here, we present a numerical technique that can allow the design of photonic nanostructures with optical properties optimized along several arbitrary objectives. In particular, we combine evolutionary multi-objective algorithms with frequency-domain electrodynamical simulations to optimize the design of colour pixels based on silicon nanostructures that resonate at two user-defined, polarization-dependent wavelengths. The scattering spectra of optimized pixels fabricated by electron-beam lithography show excellent agreement with the targeted objectives. The method is self-adaptive to arbitrary constraints and therefore particularly apt for the design of complex structures within predefined technological limits.

Over the last decade, the field of nanophotonics (or nanooptics) has been developing rapidly, mainly driven by plasmonics, because noble metal nanoparticles allow plasmon resonances to be spectrally tuned1 as well as the tailoring of a number of optical properties including directional scattering2, polarization conversion3, optical chirality4 or nonlinear effects5. Recently, high-index dielectric nanoantennas have gained increasing interest thanks to their ability to provide exceptionally strong electric6–7 and magnetic8–10 resonances, tunable from the ultraviolet to the near-infrared11–13. In analogy to plasmonics, it is possible to design functionalities like transmissive metasurfaces14, enhanced nonlinear effects15,16 or directional scattering17.

When designing photonic nanostructures, a particular geometry is usually selected from qualitative considerations, and its properties are subsequently studied systematically. Regarding applications, a more convenient approach is to define the requested properties and design a nanostructure that optimally exhibits the desired features. For the latter approach, a structure model has to be developed, which, based on a certain set of parameters, can describe a large variety of particle geometries. However, this leads to huge parameter spaces that usually cannot be explored systematically. Also, trial-and-error is not an efficient search strategy. Evolutionary optimization strategies are more promising techniques, which, by mimicking natural selection, are able to find the fittest parameter sets for a complex non-analytical problem18.

In the field of nanophotonics, evolutionary algorithms have been applied to the maximization of field enhancement19–22, scattering from plasmonic particles23,24 and to the design of hybrid plasmonic/dielectric antennas25,26. Such methods have also been used successfully on more technological applications, such as electron-beam field-emission sources27, waveguide couplers28 or core–shell nanoparticles for photothermal medical treatment29.

These studies were limited to the maximization of one target property at a specific wavelength and polarization. Such single-objective scenarios are the simplest forms of optimization problems, and a structure that concurrently matches multiple objectives will, in general, be more difficult to design. In a recent study, genetic multi-objective optimization was used on plasmonic waveguides. A figure of merit describing the waveguide and its robustness against geometrical variations were maximized simultaneously30. Evolutionary multi-objective optimization (EMO) strategies31 could lead to considerable improvements in the design of wavelength-dependent (multi-)directional scattering32, multiresonant antennas33 or polarization-dependent tailored optical behaviour34. Nanoantennas possessing multiple resonances, for instance at fundamental and harmonic frequencies, may also be optimized by EMO to enhance nonlinear effects or fluorescence spectroscopy35–37.

In this Article, we present a combination of EMO and the ‘Green dyadic method’ (GDM) for self-consistent full-field electrodynamical simulations38. We applied the EMO–GDM technique to design dielectric (silicon) nanoantennas that concurrently maximize scattering at different wavelengths, dependent on the polarization of the incident light. Finally, from the outcome of the EMO, we fabricated Si nanostructures on a silicon-on-insulator (SOI) substrate and measured their optical response by confocal dark-field scattering microscopy, yielding excellent agreement with the optimization predictions. Possible applications of such nanoscaters are holographic colour filters11 or colour rendering and printing close to the diffraction limit. The latter has been demonstrated either using plasmonic12–19 or dielectric nanostructures11. Polarization-dependent, dual-colour pixels have been reported recently using plasmonic nanoparticle arrays. Although plasmonic nanoantennas provide widely tunable single-mode responses from simple geometries (pillars in ref. 1, cuboids in ref. 39 and crosses in ref. 40), dielectric nanostructures often support high-order and degenerate modes in a narrow spectral range. Therefore, an EMO scheme is of particular interest for the design of multiresonant dielectric nanostructures.

Evolutionary optimization of scattering efficiency

Evolutionary optimization numerical techniques mimic the selection process that drives the evolution of species in nature. Each
individual of an initial population is first evaluated along one (single-objective optimization) or several (multi-objective optimization) fitness functions in a so-called fitness function. Only the best individuals are selected and the next generation is obtained from a reproduction procedure, as illustrated in Fig. 1b. The new generation undergoes the same evaluation—selection—reproduction process and, after several cycles, individuals with optimized properties are obtained.

In the present study, we propose to maximize, simultaneously, the scattering efficiencies $Q_{\text{scat}}$ of silicon nanoantennas at a first wavelength $\lambda_X$ for an incident polarization along $X$ and at a second wavelength $\lambda_Y$ for an incident polarization along $Y$. $Q_{\text{scat}}$ is defined as the ratio between the scattering cross-section $\sigma_{\text{scat}}$ and the geometrical cross-section of a nanostructure. We therefore consider a set (or ‘population’) of nanostructures that are compared using a fitness function consisting of the scattering efficiencies $Q_{\text{scat}}(\lambda_X)$ and $Q_{\text{scat}}(\lambda_Y)$. The initial generation consists of a collection of $N$ antennas with randomly initialized designs. At each optimization step, the scattering efficiencies of all the antennas are compared and the geometries yielding the maximum $Q_{\text{scat}}(\lambda_X)$ and $Q_{\text{scat}}(\lambda_Y)$ are selected to generate the set of $N$ antennas used for the next iteration. For more information about the determination of fitness and selection in multi-objective optimization see ref. 41. This process of evaluation, selection and reproduction is repeated until the maximum number of iterations is reached. At the end of the optimization process, a set of optimal solutions, known as the Pareto front, is obtained (Fig. 1c). These optimal (or non-dominated) solutions cannot be further optimized in one of the objectives (increasing $Q_{\text{scat}}(\lambda_X)$ for instance), without worsening the other target value (decreasing $Q_{\text{scat}}(\lambda_Y)$ for instance). Convergence towards the Pareto front during the optimization process is illustrated in Supplementary Section D. For electrodynamical simulations we use a volume integral technique in the frequency domain, the Green dyadic method42 (see Methods).

The ‘population’ of antenna morphologies to be considered in the computation must be diverse enough to explore, after several generations, a significant fraction of possible solutions. However, this requires a model with a large number of parameters, significantly slowing down convergence. Furthermore, the optimized geometries must remain within the limits of fabrication capabilities, and hence have neither too many features, nor features that are too small. For these reasons we use a very simple model, based on four individual silicon elements with variable dimensions and positions, placed on a SiO$_2$ substrate (refractive index $n \approx 1.5$) within a limited area of $600 \times 600$ nm$^2$. A sketch of the model is presented in Fig. 2a.

Both, the $x$- and $y$-dimension of each block are allowed to vary between 60 and 160 nm in steps of 20 nm, corresponding to the precision of a state-of-the-art electron-beam lithographic system. The height $H$ is fixed to 100 nm, equal to the silicon overlayer thickness of the SOI substrate. Overlapping antennas are allowed; such antennas are fused together, resulting in a maximum possible size of $320 \times 320$ nm$^2$. The constrained area ensures that the plane-wave excitation in our simulations is a good approximation for the
In a first step, we test the EMO–GDM technique on a simple problem. A single target wavelength of $\lambda_{\text{max}} = 630$ nm is selected at which $Q_{\text{scat}}$ is maximized for $X$ and $Y$ polarization, simultaneously. The structures of the final population and the corresponding Pareto front after evolution over 200 generations are shown in Fig. 3a,b. The geometries of the initial population are compared to those on the Pareto front in Supplementary Section C.

The geometries found by evolutionary optimization are then transformed into a lithographic mask, which we use to produce the silicon nanostructures on an SOI substrate (see Methods). Figure 3a shows a comparison of the design with scanning electron microscopy (SEM) images of the sample. Simulated (Fig. 3c,d) and experimental (Fig. 3e,f) spectra are in very good agreement. We note that the higher-order resonance around $\lambda = 450$ nm for structures at the edge of the Pareto front is enhanced in the experiment compared to the simulations. This is due to a cavity effect in the SiO$_2$ layer of the SOI substrate, which is not taken into account in the GDM simulations (Supplementary Section I).

The outermost individuals on the Pareto front (particles 1 and 40) correspond to equivalent results of a single-objective optimization using one target wavelength and polarization. We observe, in these cases, that all four sub-antennas are combined during the evolution to form a single rod-like antenna along the target polarization direction. In agreement with the literature, this yields an optimum scattering efficiency with respect to the considered polarization ($1: Y, 40: X$), at the expense of very low scattering for the perpendicular polarization$^{12}$. To obtain comparably high scattering efficiencies for both polarizations (particle 20 and neighbours), the evolution produces cross-like antennas.

**Evolutionary optimization of double resonant nanostructures**

In a next step we study the maximization of $Q_{\text{scat}}$ at two different wavelengths $\lambda_X = 550$ nm and $\lambda_Y = 450$ nm for mutually crossed polarizations. The randomly initialized population of 20 individuals at the beginning of the evolution (red), the Pareto front (green), selected structure designs, as well as corresponding spectra are shown in Fig. 4. The individuals at the Pareto front borders, labelled ‘1’ and ‘3’, correspond to single-objective optimizations for $\lambda_Y$ and $\lambda_X$, respectively. Inspecting the three selected structures in more detail leads to the following observations.

Obviously, twin structures like 1 and 2 seem to be preferred, because they result in an increase in the overall scattering efficiency. Indeed, structures 1 and 2 both consist of two dimer antennas that, if taken individually, have about 30% (respectively 10%) lower $Q_{\text{scat}}$ than the twin structure. Furthermore, the peak positions in the scattering spectra are slightly shifted and match the target wavelengths only in the combined antenna.

We point out that the rather symmetric relative positioning of the two dimers is crucial to achieve optimum scattering efficiency. The configuration found by the evolutionary optimization is very close to the ideal positions. A marginally stronger scattering can be obtained for both structures 1 and 2 when the dimers are placed on the same horizontal axis, but the possible gain is as low as $\sim$3% and 1%, respectively.

Finally, particle 3 in Fig. 4 consists of a single dimer structure only, which we attribute to the constrained maximum antenna size in our model. The maximization of the scattering at the longer target wavelength ($\lambda_X = 550$ nm) requires a larger amount of material than the shorter wavelength $\lambda_Y$. The scattering efficiency can be further improved by allowing the algorithm to use larger or more constituents.

**Proof of principle for $\lambda_X = \lambda_Y$**

In a second step, we test the EMO–GDM technique on a simple problem. A single target wavelength of $\lambda_{\text{max}} = 550$ nm is selected at which $Q_{\text{scat}}$ is maximized for $X$ and $Y$ polarization, simultaneously. The structures of the final population and the corresponding Pareto front after evolution over 200 generations are shown in Fig. 3a,b. The geometries of the initial population are compared to those on the Pareto front in Supplementary Section C.

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In the Supplementary Information, a detailed analysis of structures 1 and 2 is provided (Supplementary Section E and Supplementary Figs 6 and 7), as well as a demonstration that the different geometry of structure 3 can be explained by the limited amount of silicon allowed in the computation (Supplementary Section F).

To further illustrate the EMO-GDM technique, we performed several multi-objective optimizations for different combinations of target wavelengths. The wavelength $\lambda_X = 550$ nm is fixed, while the other ($\lambda_Y$) is varied from 450 to 650 nm in steps of 10 nm. Each simulation consists of an initial population of 20 random individuals, which is evolved for 200 generations. At the end of the evolution, the optimized structure with the closest scattering efficiencies for X and Y polarization,

$$|Q_{\text{scat}}(\lambda_X) - Q_{\text{scat}}(\lambda_Y)| = \min$$

is chosen from each simulation (like structure 2 in Fig. 4).

Figure 5 presents the resulting structures (Fig. 5a) and their GDM-simulated spectra for $X$- and $Y$-polarized incident light (Fig. 5b). The different $\lambda_Y$ are indicated by a colour coding from blue ($\lambda_Y = 450$ nm) to red ($\lambda_Y = 650$ nm). As explained in the previous subsection, for increasing wavelengths, the four sub-antennas tend to combine in only two structures (instead of more constituents for the shortest wavelengths), which is due to the limited amount of allowed material. For the same reason, at wavelengths above 600 nm, all sub-antennas are merged into one single structure, and for the longest wavelengths the available material is not sufficient to yield a satisfactory maximization (for an analysis of the role of the amount of material being constrained, see Supplementary Sections F and G).

For an experimental verification, we fabricated Si structures corresponding to the optimized colour-tuned nanoantennas. SEM images (Fig. 5c) and polarization-filtered dark-field spectra (Fig. 5d: top, filter along $X$; bottom, filter along $Y$) are shown in Fig. 5. Polarization-filtered dark-field images (Fig. 5d) of colour-switching pictograms, composed of the optimized structures, demonstrate the polarization dependence of the scattered wavelengths. For the largest structures, a high-order resonance appears in the blue part of the spectrum. This resonance, already visible in the simulations and experiments of Fig. 3, is indicated in Fig. 5b,d with dashed lines to distinguish it from the low-order resonance of the smaller nanostructures, which appear in the same spectral region. The simulated and experimental data in Fig. 5 have been normalized at the targeted wavelength. In the experiment, the relative contribution of the high-order resonance to scattering is reinforced by the Si/SiO2/Si cavity of the underlying SOI substrate. Supplementary Section I provides the complete set of experimental and computed data to allow for a quantitative comparison, and discusses the influence of the underlying cavity for the largest nanostructures.

From a closer look at the individual structures, we observe that the 'symmetric' optimization with $\lambda_X = \lambda_Y = 550$ nm results in a non-symmetric particle. We would intuitively expect a symmetric antenna to be ideally suited for equally strong scattering under both $X$ and $Y$ polarization. The evolutionary optimization, being a non-analytic routine, should at least result in some quasi-symmetric structures; however, this is not the case here. As before, this can be explained by the finite amount of material available in our structure model. Because the T-shaped part of the antenna already consists of three of the four sub-antennas, the fourth sub-antenna is added as a square block of maximum allowed dimensions, and it is impossible for the algorithm to generate a symmetric structure within the given constraints. As shown in the Supplementary Information, a simulation with $\lambda_X = \lambda_Y = 450$ nm,
as well as an optimization with relaxed constraints on the antenna size, results in quasi-symmetric structures, as intuitively expected (Supplementary Section I).

Again, for $\lambda_x = \lambda_y = 550$ nm, interference between both parts of the antenna results in an optimum scattering efficiency at the target wavelength and therefore exact positioning of the constituents is crucial. A change in the spacing between the T-shaped and squared sub-structures by $\Delta x = 100$ nm will already result in a decrease of more than 5% in scattering efficiency for at least one polarization. An analysis of the $\lambda_x = \lambda_y = 550$ nm antenna is provided in the Supplementary Information (Supplementary Section E and Supplementary Fig. 8).

Polarization-encoded micro images

To illustrate the previous results we produced small images, only a few micrometres large, composed of EMO-optimized antennas. The absolute scattering cross-section $\sigma_{\text{scat}}$ was used as the optimization target. An additional spacing of 250 nm was used between the individual particles, resulting in pixel sizes of $850 \times 850$ nm$^2$ ($\sim 30,000$ d.p.i.), close to the diffraction limit.

Polarization-filtered dark-field images are presented in Fig. 6. Depending on the orientation of the polarization filter (left, X; right, Y), one single arrow is visible, pointing in the corresponding direction, while the second arrow vanishes in a blue background. Furthermore, the logos of CNRS and the CEMES laboratory are nested into one image, encoded in perpendicular polarizations. A scheme of the lithographic mask (red) and an SEM image (grey) of a zoom into the logos, indicated by small yellow squares, is shown at the bottom. We attribute the slightly reminiscent signatures of the hidden motifs to intensity variations due to the arrangement of the antennas in grating-like 2D arrays (Supplementary Section J).

**Figure 4** | Pareto front example of an optimization run with $\lambda_x = 550$ nm and $\lambda_y = 450$ nm. a. Spectra of selected antennas (indicated by numbers), where either a single wavelength is optimized (1 and 3) or both resonance wavelengths are scattered approximately equally (2). X- and Y-polarized illumination are plotted in blue and red, respectively. b. Random initial population (red) and Pareto front after optimization (green) with target wavelengths $\lambda_x = 550$ nm and $\lambda_y = 450$ nm. Selected structures are sketched in the insets, showing areas of $600 \times 600$ nm$^2$.

**Figure 5** | Experimental demonstration of several dual-resonant silicon structures based on EMO. a,b. EMO design of multi-resonant dielectric particles (a) and simulated scattering spectra for $\lambda_x = 550$ nm (indicated by a black dashed line) and various $\lambda_y$ (b). c,d. SEM images (c) and polarization-filtered scattering spectra (d) of the corresponding nanofabricated sample, normalized to the peak closest to the target wavelength. Insets in d show polarization-filtered dark-field microscopy images of the full set of structures ($20 \times 4$ µm$^2$) and of the LAAS laboratory logo ($35 \times 23$ µm$^2$). The lines framing the blue letters ($\lambda_y = 450$ nm) are optimized for $\lambda_y = 650$ nm (upper line) and $\lambda_y = 570$ nm (lower line). Areas in a and c are $600 \times 600$ nm$^2$.© 2016 Macmillan Publishers Limited, part of Springer Nature. All rights reserved.
Figure 6 | Polarization-filtered dark-field images of micrometre-scale pictures designed by EMO. Micrometre-scale pictures composed of 24 × 24 (arrows) and 100 × 100 (logos) EMO-GDM-designed particles. A linear polarization filter is added before the camera, oriented along X (top, left) and along Y (top, right). The dimensions of the arrow images are 15 × 15 μm² and those of the logos are 60 × 60 μm². Bottom image: zoom into the logo picture. SEM image is in grey (scale bar, 500 nm) and sketch of the lithographic mask is in red, highlighted by small yellow squares in the dark-field images. The yellow arrow and blue emission indicate incident and scattered light, respectively.

Conclusions

We have presented a technique of EMO coupled to full-field dynamical simulations for the automatic design of photonic nanostructures. We have demonstrated that our approach is able to design double-resonant silicon nanoantennas even within a very simple structure model. We found that all accessible parameters were nearly perfectly optimized by the evolutionary algorithm. Furthermore, for maximum compatibility with fabrication methods, technological limitations were included as boundary conditions in the model. Thanks to these additional requirements, the measured spectra of samples produced on an SOI substrate showed excellent agreement with the predictions of the optimizations.

A great advantage of the EMO–GDM technique is its flexibility and the ability to self-adapt to arbitrary limitations. Additional constraints can easily be implemented because no analytical treatment of the input model needs to be performed. Inadequate structures, inconsistent with the constraint functions, are discarded automatically during the evolution, and only technologically convenient designs are generated. The method can also be extended to the rigorous design of metasurfaces, where interference between the unit cells needs to be considered. Periodic boundary conditions can be included in the GDM by means of an appropriate Green’s dyad\(^{35,44}\). In this way, the distance between substructures on the metasurface may also be included as a free parameter in the optimization. We believe that multi-objective optimization of photonic nanostructures has tremendous potential for many kinds of possible applications in near- and far-field nano-optics, for example in the design of multiresonant, broadband light harvesting or nonlinear nanostructures.

Methods

Methods and any associated references are available in the online version of the paper.

Received 27 May 2016; accepted 16 September 2016; published online 24 October 2016

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Acknowledgements
The authors thank P. Salles and G.-M. Caruso for technical assistance. This work was partly supported under a ‘Campus Gaston Dupouy’ grant by the French government, Région Midi-Pyrénées, and the European Union (ERDF), by the computing facility centre CALMIP of the University of Toulouse (grant no. P12167) and by the LAAS-CNRS micro and nanotechnologies platform, a member of the French RENATECH network.

Author contributions
P.R.W., V.P. and A.A. designed the research. C.G., A.A. and P.W. implemented the codes and performed the simulations. A.L. and G.L. fabricated the samples by EBL. P.R.W and V.P. performed the dark-field scattering experiments. All authors contributed to the data analysis, figure preparation and manuscript writing.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to P.R.W., A.A. and V.P.

Competing financial interests
The authors declare no competing financial interests.
Methods

EMO–GDM method. We used the Python interface of the parallel EMO toolkit paGMO/pyGMO (ref. 45) and in particular its implementation of the ‘SMS–EMOA’ algorithm46. A comprehensive introduction to EMO is provided in ref. 31.

All interfacing between the EMO and the electrodynamical full-field solver is implemented in Python. The fitness of each nanoparticle is calculated using the GDM, which is implemented in fortran to yield high computational speed.

The target nanoparticle is discretized in \( N \) cubic meshpoints of side length \( b \), for each of which a dipolar response is assumed. This approach eventually leads to a system of \( 3N \) coupled equations that relates an incident electric field \( E_0 \) to the field \( E \) due to the particle’s response:

\[
E_n = M \cdot E
\]

The field in the structure can then be obtained by an inversion of the matrix \( M \), which is composed of \( 3 \times 3 \) submatrices

\[
M_{ij} = \delta_{ij} - \alpha_{ij}(\omega) G(r_i, r_j, \omega)
\]

Here, \( I \) is the Cartesian unitary tensor, \( \delta_{ij} \) the Kronecker delta function and \( (\text{centimetre}-\text{gram}-\text{second units}) \)

\[
\alpha_{ij}(\omega) = \frac{r_i(\omega) - \epsilon_{\infty}(\omega)}{4\pi} \nu_{ij}
\]

\( \nu_{ij} \) is the volume of each cubic cell, in our case \( \nu_{ij} = b^3 \). For the permittivity \( \epsilon_r \) we use the dispersion of silicon from ref. 46 and assume a constant environment of \( \epsilon_{\infty} = 1 \). \( G \) is the Green’s dyad, which couples the dipolar elements \( i \) and \( j \) and is composed of a vacuum and a surface term

\[
G(r_i, r_j, \omega) = G_v(r_i, r_j, \omega) + G_{\text{surf}}(r_i, r_j, \omega)
\]

which can be found in the literature47. To account for the divergence of Green’s function at \( \nu_{ij} = 0 \), a normalization scheme

\[
G_v(r_i, r_j, \omega) = IC(\omega)
\]

is introduced, which for a cubic mesh is given by

\[
C(\omega) = -\frac{4\pi}{3} \frac{1}{\epsilon_{\infty}(\omega) \nu_{ij}}
\]

and has to be adapted, together with the cell volume, if a different meshing is used (like, for example, a hexagonal compact grid)48. Note that we neglected a weak radiative term in equation (7); this is discussed in ref. 47 and references therein.

Finally, the matrix inversion is carried out using standard lower-upper decomposition and the scattering efficiencies can be calculated from the near-field \( E \) inside the particle49.

A great advantage of GDM is that the presence of a substrate (in our case \( n = 1.5 \)) can be taken into account by means of an appropriate Green’s dyadic function (equation (2)), which can be calculated in the non-retarded approximation at almost no supplementary computational cost. In comparison to finite-difference time-domain (FDTD) simulations, a frequency-domain method has further advantages with regard to our purpose of designing a doubly resonant nanostructure. So-called perfectly matched layers are not needed and only the nanoparticle itself is subject to the volume discretization. Generally, this results in a quicker convergence.

The amount of silicon per antenna is not constant in our model and, as the duration of a simulation depends on the structure size, the optimizations generally tend to be slower for longer resonance wavelengths because of the resulting larger particles. Nevertheless, evolutions of populations with 20 individuals over 200 generations take no longer than \( 10–15 \) h on one single core of a 2.8 GHz Intel Xeon E5-1603 CPU. We note that the results are always reproducible, yielding very similar structures and scattering efficiencies from multiple runs (Supplementary Section D).

Nanofabrication technique by top-down approach. Samples were fabricated by a top-down approach that couples electron-beam lithography (EBL) with anisotropic plasma etching. This was used to pattern the designed nanostructures45,50 on an SOI wafer as the substrate (Si, 95 nm; buried oxide layer, 145 nm). EBL was carried out with a Raith 150 writer at an energy of 30 keV on a thin (60 nm) negative-tone resist layer, namely hydrogen silsesquioxane (HSQ). After exposure, the HSQ was developed by immersion in 25% tetramethylammonium hydroxide (TMAH) for 1 min. HSQ patterns were subsequently transferred to the silicon top layer by reactive ion etching in a \( \text{SF}_6/\text{Cl}_2 \) plasma-based chemistry down to the buried oxide layer.

In the EMO runs, the minimum feature size was set to 60 nm to avoid removing small features of the structures during liftoff. The structures were discretized and placed on a grid by steps of 20 nm to match the precision of the EBL. SEM images of individual structures are shown and are compared to the mask layout in Figs 3, 5 and 6.

Confocal dark-field microscopy. Confocal optical dark-field microscopy was performed on a conventional spectrometer (Horiba XploRA). A spectrally broad white lamp was focused on the sample by a ×50, NA 0.45 dark-field objective, backscattered, polarization-filtered and dispersed by a 300 grooves per mm grating onto an Andor iDus 401 charge-coupled device. The intensity distribution of the lamp as well as the spectral response of the optical components was taken into account by subtracting the background measured on bare SOI and normalizing the measured spectra to a white reference sample.

References


