

PhD Thesis Proposal

Advanced Control Synthesis for Switched Affine Systems under Unmeasured Disturbances

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1 Titre et Description

Title: Advanced Control Synthesis for Switched Affine Systems under Unmeasured Disturbances: A Non-Smooth Optimization and Hybrid Systems Approach.

Description: This PhD thesis focuses on the synthesis of a novel control architecture for Switched Affine Systems (SAS) subject to exogenous disturbances. Building upon the established expertise of the supervisors in hybrid/switched systems and state-space representations of power converters [9, 19, 8], this thesis addresses the fundamental challenge of stabilizing SAS when the equilibrium point is uncertain due to the inevitable effect of unmeasured perturbations.

The project hinges upon a novel nonlinear control structure based on four ingredients: a switched observer for simultaneous state and disturbance reconstruction, a nonlinear observer for the estimation of the unknown equilibrium to be stabilized, integrated with an optimization mechanism allowing for efficient online gradient-based selection of an optimal operating point and a switched stabilizer. The core theoretical innovation is threefold: (i) managing the inherent non-smoothness of cost functions (e.g., L_1 -norm penalties) through sub-gradients and differential inclusions (ii) exploring the use of modern nonquadratic Lyapunov certificates of asymptotic stability of the error dynamics leveraging recent advances in nonlinear integral action [1] (iii) using modern hybrid dynamical systems tools and reduction theory [16] to establish desirable stability and optimality properties of the interconnected scheme. This work aims to transcend standard quadratic constraints by to ensure global asymptotic stability in complex switching environments.

2 Contexte scientifique, situation par rapport à l'état de l'art

Switched Affine Systems (SAS) are hybrid systems described by a family of affine dynamical subsystems and a switching rule that determines the active subsystem. Switched Affine Systems constitute a modeling framework uniquely suited to capture the discontinuous nature of high-frequency electronic transistor switching. Designing a stabilizing switching rule for such systems is a challenging task, due to their discontinuous nature and the nontrivial description of their equilibrium states [4]. While traditional stability analysis has largely relied on Common Quadratic Lyapunov Functions (CQLF) [4, 7, 14, 5], these methods generally assume a known equilibrium and strong quadratic dissipativity properties that are not enjoyed by certain plants. Among various fields where SAS systems are encountered, power converters enjoy a growing interest, driven by the development of electric applications such as hybrid and electric vehicles and renewable energy [17, 3]. It often happens that the above-mentioned equilibrium point may change when the system is affected by disturbances or perturbations. In particular, power converters may be perturbed when the supply voltage is inaccurate, or if the load is varying. When perturbations are measured, this information can be used to directly determine the new desired equilibrium point (as seen in our previous work [10]).

In the presence of perturbations, the literature is sparse, which leaves many open challenges. Current methodologies either focus on robust attenuation [18, 20] or observer-based rejection

[13, 3, 6, 2, 11, 12]. However, these works are associated with significant shortcomings stemming from parametric dependence on the disturbance and on the modeling approximation errors induced by simplifying assumptions on the SAS dynamics. In [3], the estimated perturbation is used to reconstruct the equilibrium point, assuming explicit knowledge of the expression of the equilibrium point as a function of some state components and of the perturbation. In [6], a gridding approach is proposed to compute the desired equilibrium point offline, for a discrete set of perturbation values, completed with an interpolation mechanism, which is inherently approximate. Both contributions do not consider the direct influence of the perturbation and the switching of the dynamic matrices on the state of the SAS. Moreover, they do not consider the fact that there may exist multiple equilibria associated with a given perturbation value.

L. Zaccarian and Y. Ariba recently proposed [9] a modular architecture for SAS that exploits hybrid reduction arguments [16] to prove stability of the error dynamics for a control scheme integrating switched feedback control, nonlinear observation, and gradient-based optimization. This project intends to relax the strong underlying assumptions of the initial scheme proposed in [9], which relies on common quadratic Lyapunov functions, smooth cost function selections and generally strong assumptions on the SAS dynamics. Examples taken from the power converters literature, overviewed in [10] and [9], in addition to the hybrid dynamical systems techniques in [19] will serve as a guideline in the development of the work.

3 Objectifs, avec mise en évidence : des verrous, de l'aspect novateur, prospectif, des théories/méthodes/outils envisagés, des apports

The ultimate *goal (Objectif)* of the PhD thesis is to develop a generalized, robust control framework for SAS that ensures stability while optimizing the performance as measured by a cost function (e.g. accounting for the energy efficiency in power converters). The *challenges (verrous)* can be summarized in (i) Non-Smoothness and Sparsity: Integrating L_1 -type cost functions in our scheme, to promote control sparsity, requires bridging gradient-based optimization with sub-gradient methods and differential inclusions; (ii) Relaxation of Lyapunov Constraints: Moving beyond conservative common quadratic Lyapunov functions assumptions by utilizing recent nonlinear Lyapunov redesigns. (iii) Interconnected Stability: Proving the convergence of the integrated architecture (Observers + Optimizer + Stabilizer) with rigorous hybrid dynamical systems tools.

The *Innovation and Prospective Aspects (aspect novateur, prospectif)* associated with the thesis are related to a paradigm shift from fixed-frequency switching to generalized Lyapunov-based switching logics in SAS, governed by modern hybrid Lyapunov techniques: an area rarely explored in the existing literature, which may have important experimental outcomes in power electronics. A further important outcome is the easy applicability of the proposed techniques to experimental scenarios, due to the computationally attractive gradient-based methods to deal with optimality, which do not require online optimization. Experimental validation on simple prototypes is also foreseen, therefore contributing to the recent trend of the MAC group (and of the DO department) to invest more on experimental illustrations of the relevance of nontrivial mathematical control theory (such as nonlinear control laws exploiting nonsmooth optimization, hybrid Lyapunov theory, among others).

On the greater societal scale, the thesis will develop theoretical results geared towards improving the efficiency and optimizing the control of electrical actuators and power converters [17]. As reported by the GdR SEEDS¹ [17], a national research association from the CNRS, Automatic control should play a central role to design control strategies that ensure the required properties for the interconnected systems, for regular operations as well as for downgraded emergency

¹Groupement de Recherche Systèmes d'Énergie Électrique dans leurs Dimensions Sociétales.

modes. The GdR MACS² also emphasizes that improving the resilience of complex systems in an environment with (unmeasured) perturbations is one of the major topics to be addressed [15]. The developments foreseen in this thesis are the natural combination of the two past PhD theses [9, 19] completed in 2024 and supervised, respectively, by the two *encadrants* of this proposal.

4 Plan de travail

The thesis will be structured in four main phases, allowing for the conception and design of an innovative control strategy and combining both theoretical and practical aspects.

1. **State-of-the-art, equilibrium characterization and control architecture:** The student will begin with a comprehensive literature review on switched affine systems, converter control and related topics. He/she will become familiar with previous work, in particular the aforementioned past PhD theses. A first primary work will be to characterize the evolution of the equilibrium point when perturbations affect the SAS. The general control architecture consisting of 3 components: the state/perturbation observer, the equilibrium selection optimizer, and the switching control law will have to be reviewed and accurately classified.
2. **Stability analysis, observers convergence and optimization mechanism:** Regarding the control system design, the student will firstly address the redesign of the state/perturbation observer and the equilibrium observer. Given the complexity of the control system architecture, a key step will be then to formally prove the stability of the overall interconnected system using gradient-based optimization in the closed-loop, under relaxed assumptions.
3. **Test bench and experimental validation:** We aim at validating all theoretical results through simulations (MATLAB/Simulink and PSIM or PLECS) as well as experimental tests. The student will therefore become familiar with these tools: simulation software and rapid-control prototyping toolchain. Regarding the physical system, specific type of converter will be selected. Several options could be considered in order to test different aspects of the control system. The design and the implementation of the test bench may be carried out in collaboration with various partners, such as the University of Palermo (Italy), the University of Tarragona (Spain), the University of Seville (Spain), or the IRT Saint Exupéry (Toulouse). The supervisors have past collaborations with each one of these institutions.
4. **Refinement of control laws and hybrid system mechanisms:** Focusing on the control law, we aim at revisiting argmin-based laws, and exploring recent nonquadratic Lyapunov functions structures. Moreover, we also intend to embed hybrid mechanisms in the control algorithm to allow for implementations with non-periodic switching, thus departing from the conventional fixed-frequency PWM strategy towards a more general Lyapunov-based switching logic.

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²Groupement de Recherche Modélisation, Analyse et Conduite des Systèmes Dynamiques.

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