Module I3.5GE

Signaux - Systèmes et Automatique Linéaire

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version 4.0

Informations pratiques

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Forum : Moodle. Favorise le partage, l'auto-formation, la capitalisation d'informations...

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Organisation du cours

- Pré-requis : cours de l'an dernier, mathématiques depuis le CP !
- 8h en amphi et 2h de TD : cours + exercices. Présentation sur transparents.
- 4h de TP.
  - utilisation de MATLAB® et Simulink®
  - évaluation sur QCM.
- 1 examen final (1h).

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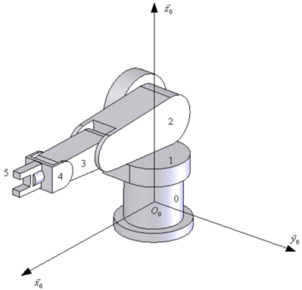
Sur Moodle

- Documents : transparent cours + support rédigé + exercices + sujets TP.
- Forum.

Contents

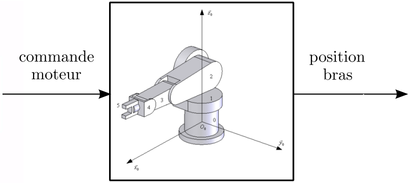
Introductory example

We want to control the angular position of a robotic arm.



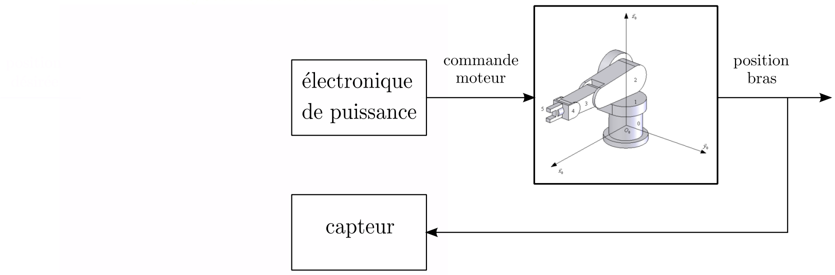
What do we do ? What do we need ?

Standard structure of closed-loop control



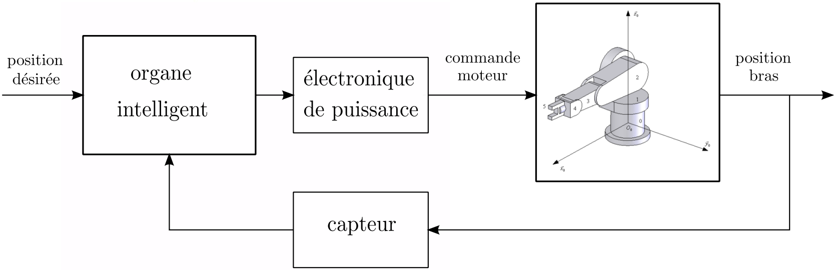
- A motor drives the robotic arm.

Standard structure of closed-loop control



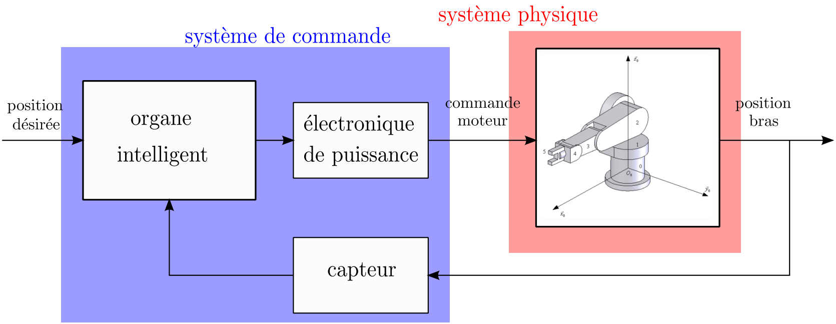
- A motor drives the robotic arm.
- A sensor measures the arm position and a power system controls the motor.

Standard structure of closed-loop control



- A motor drives the robotic arm.
- A sensor measures the arm position and a power system controls the motor.
- An “intelligent” system provides a control strategy.

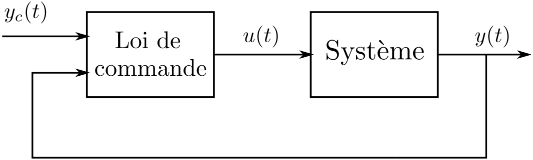
Standard structure of closed-loop control



- A motor drives the robotic arm.
- A sensor measures the arm position and a power system controls the motor.
- An “intelligent” system provides a control strategy.
- The control system may be implemented on an electronic board, a computer, a microcontroller...

What is automatic control ?

Automatic control is an interdisciplinary branch of engineering and mathematics that deals with the behavior of dynamical systems. It is the application of control theory for regulation of processes/physical systems without direct human intervention.



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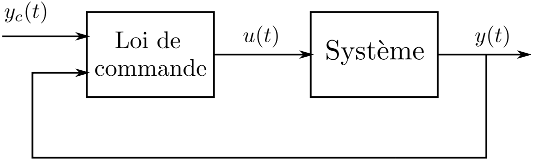
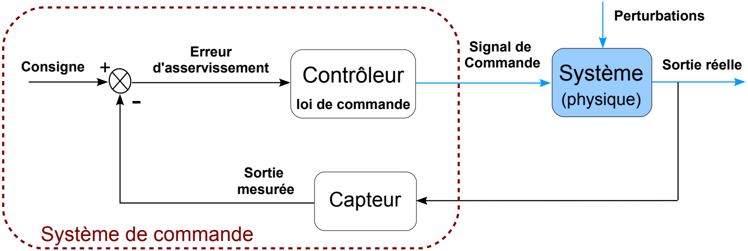


⋆ **Goal :** To design a control law to make the system output track a reference signal.

⇒ feedback control

Closed-loop control structure

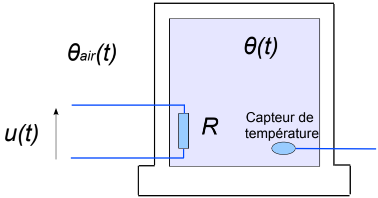


- Goal : make the ouput converge to a desired value (setpoint).
- The controller generates the control signal based on the difference between the setpoint and the measured output.
- Some disturbances may affect the system behavior.

Thermodynamic application

Temperature regulation

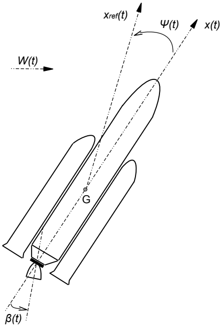
- **Input :** voltage $u(t)$.
- **Output :** temperature in the oven $\theta(t)$.
- **Disturbance :** heat exchange with the environment $\theta_{air}(t)$.



Aeronautic application

Launcher stabilization

- **Input :** angle $\beta(t)$.
- **Output :** angle $\Psi(t)$ with respect to the reference trajectory.
- **Disturbance :** wind, varying mass of fuel.

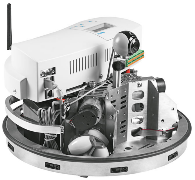


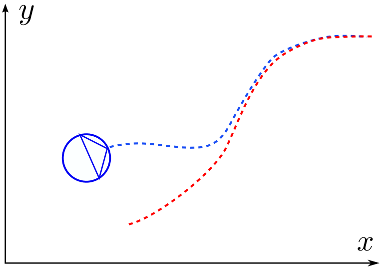
Robotic application

Path tracking for mobile robot

- **Input :** speed of motor wheels.
- **Output :** position and speed of the robot.
- **Disturbance :** ground surface, obstacles.

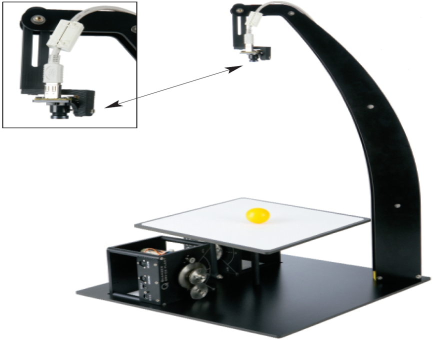
Mobile robot Robotino®

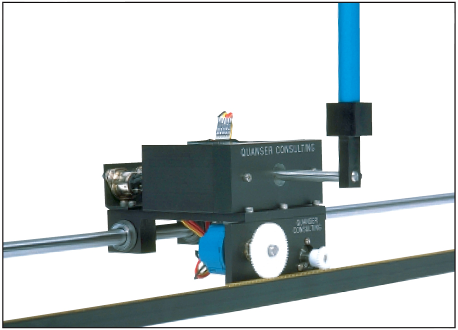




Academic applications

Stabilization of a ball on a surface and an inverted pendulum





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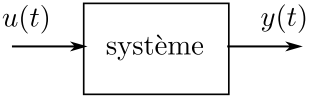
Contents

Modelisation

The study of a dynamical system requires a *model*.

$\Rightarrow$ mathematical relationship : $u(t) \;\rightarrow\; y(t)$?



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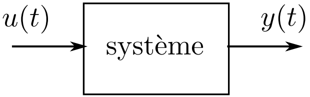

In the case of **linear time-invariant systems**, models are expressed as linear differential equation with constant parameters :

$$a_n y^{(n)} + \ldots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + \ldots + b_1 \dot{u} + b_0 u$$

Laplace transform

The Laplace transform of a function $f(t)$ is a function of the complex variable s :

$$\hat{f}(s) = \int_0^{\infty} e^{-st} f(t) dt$$

with notations : $\hat{f}(s) = \mathcal{L}\Big\{f(t)\Big\}$ and $f(t) = \mathcal{L}^{-1}\Big\{\hat{f}(s)\Big\}$.

Propriétés :

- Linearity : $\mathcal{L}\Big\{af(t) + bg(t)\Big\} = a\hat{f}(s) + b\hat{g}(s)$, a and b are constant.
- Derivation : $\mathcal{L}\Big\{\frac{d}{dt}f(t)\Big\} = s\hat{f}(s) - f(0)$.
- Integration : $\mathcal{L}\Big\{\int_0^t f(\theta)d\theta\Big\} = \frac{1}{s}\hat{f}(s)$.
- Delay : $\mathcal{L}\Big\{f(t-\tau)\Big\} = e^{-s\tau}\hat{f}(s)$, $\tau > 0$ and $f(t) = 0\ \forall t < 0$.
- Final value : $\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\hat{f}(s)$.

Laplace transform table

Time functions $f(t)$ are defined only for all $t \geq 0$.

Function	**time domain** $f(t)$	**Laplace transform** $\hat{f}(s)$
Dirac impulse	$\delta(t)$	1
step	1	$\frac{1}{s}$
ramp	t	$\frac{1}{s^2}$
power n-th	$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
decreasing exponential	e^{-at}	$\frac{1}{s+a}$

Laplace transform table

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Function	**time domain** $f(t)$	**Laplace transform** $\hat{f}(s)$
sine	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
cosine	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
sine with exponential decrease	$e^{-at} \sin(bt)$	$\frac{b}{(s+a)^2 + b^2}$
cosine with exponential decrease	$e^{-at} \cos(bt)$	$\frac{s+a}{(s+a)^2 + b^2}$
power n-th with exponential decrease	$\frac{t^n}{n!} e^{-at}$	$\frac{1}{(s+a)^{n+1}}$

Transfer function

The Laplace transform provides the transfer function of a system.

$$5y^{(3)}(t) + 2\ddot{y}(t) + \dot{y}(t) + 4y(t) \ = \ \dot{u}(t) + 2u(t)$$

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$$\Big\downarrow \ \mathcal{L}$$

$$5s^3\hat{y}(s) + 2s^2\hat{y}(s) + s\hat{y}(s) + 4\hat{y}(s) = s\hat{u}(s) + 2\hat{u}(s)$$

$$\Big\downarrow$$

$$G(s) = \frac{\hat{y}(s)}{\hat{u}(s)} = \frac{s+2}{5s^3 + 2s^2 + s + 4}$$

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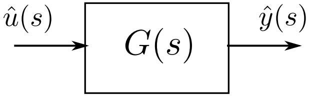
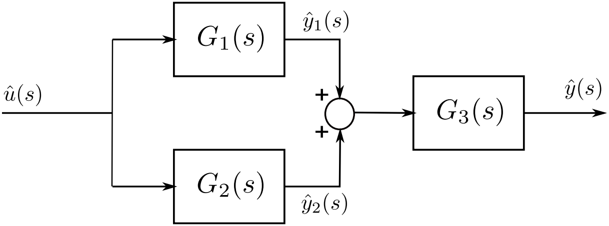
New input-output relationship

$$\boxed{\hat{y}(s) = G(s)\hat{u}(s)}$$



Block diagram

Transfer functions are convenient to study complex systems



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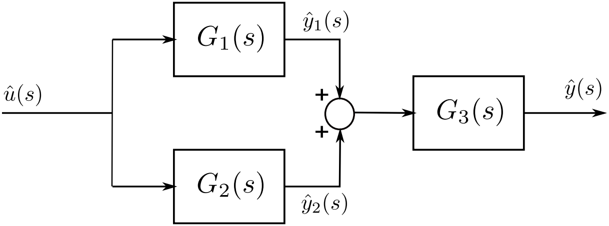


we have :

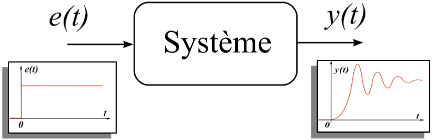
$$\begin{cases} \hat{y}_1(s) & = & G_1(s)\hat{u}(s) \\ \hat{y}_2(s) & = & G_2(s)\hat{u}(s) \\ \hat{y}(s) & = & G_3(s)\Big(\hat{y}_1(s) + \hat{y}_2(s)\Big) \end{cases}$$

equivalent transfer :

$$\hat{y}(s) = \underbrace{G_3(s)\Big(G_1(s) + G_2(s)\Big)}_{F(s)}\hat{u}(s)$$

Time response

Compute the explicit expression of the output $y(t)$ for a given input $e(t)$.



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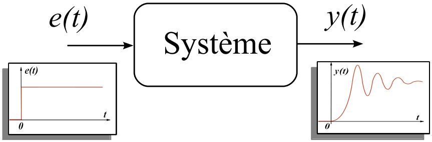


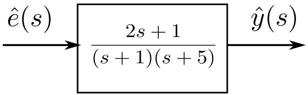
Solving method with transfer functions :

1. Express the output $\hat{y}(s) = G(s)\hat{e}(s)$.
2. Partial fraction expansion of $\hat{y}(s)$.
3. Check for the matching function in the table

$$\hat{y}(s) \xrightarrow{\mathcal{L}^{-1}} y(t)$$

   to get the time domain expression.

Example : Let us calculate the time response of the system to a step input



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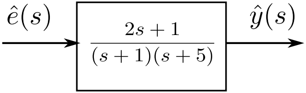

let us express the output

$$\hat{y}(s) = \frac{2s+1}{(s+1)(s+5)}\hat{e}(s) = \frac{2s+1}{(s+1)(s+5)}\frac{1}{s}$$

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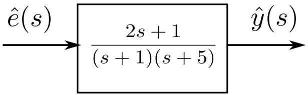

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the output can be rewritten as

$$\hat{y}(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+5}$$

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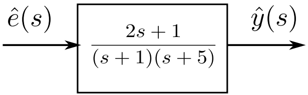

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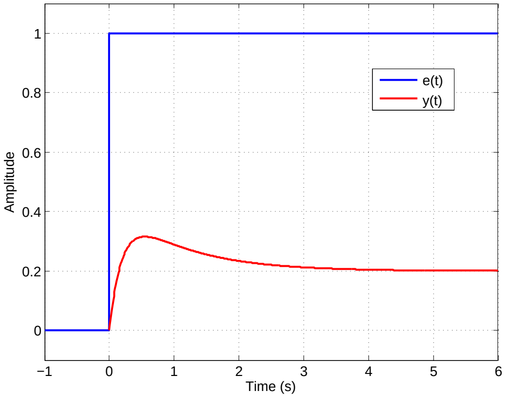
with :

$$\left\{ \begin{array}{lcl} A & = & s\hat{y}(s)\big|_{s=0} = \frac{1}{5} \\ B & = & (s+1)\hat{y}(s)\big|_{s=-1} = \frac{1}{4} \\ C & = & (s+5)\hat{y}(s)\big|_{s=-5} = -\frac{9}{20} \end{array} \right.$$

using the table, we get the time domain response

$$y(t) = \frac{1}{5} + \frac{1}{4}e^{-t} - \frac{9}{20}e^{-5t}$$



First order systems

standard form :

$$\hat{y}(s) = \frac{k}{\tau s + 1}\,\hat{e}(s)$$

where we define τ the *time constant* and k the *static gain*.

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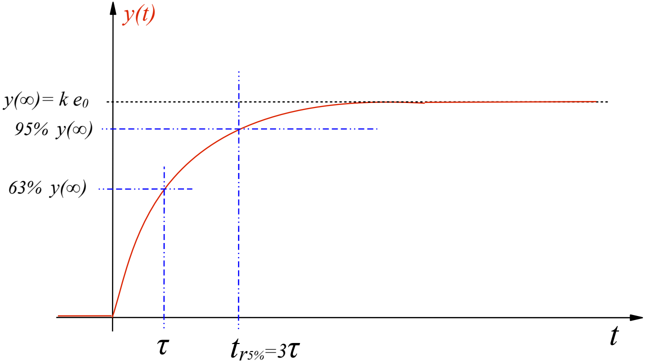
Time response to a step input e_0 : $y(t) = e_0 k\left(1 - e^{-\frac{1}{\tau}t}\right)$



Second order system

standard form :

$$\hat{y}(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\ \hat{e}(s)$$

where we define ζ the *damping ratio*, ω_n the *natural frequency* and K the *static gain*. For the step response, 3 cases :

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- case $\zeta > 1$: overdamped response

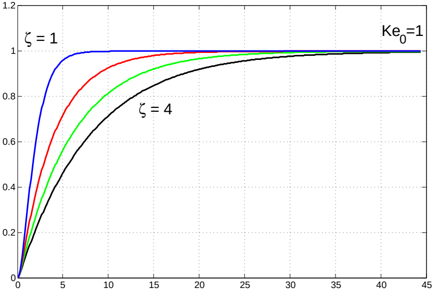
$$y(t) = Ke_0\left[1 + \tfrac{p_2}{p_1 - p_2}e^{-\zeta\omega_n t} + \tfrac{p_1}{p_2 - p_1}e^{-\zeta\omega_n t}\right]$$

- cas $\zeta = 1$: critically damped response

$$y(t) = Ke_0\left[1 - \Big(1 - p_1 t\Big)e^{-\zeta\omega_n t}\right]$$

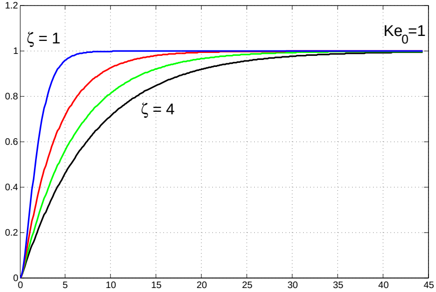
- cas $0 < \zeta < 1$: underdamped response ($\omega_p = \omega_n\sqrt{1-\zeta^2}$)

$$y(t) = Ke_0\left[1 - e^{-\zeta\omega_n t}\left(\cos(\omega_p t) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_p t)\right)\right]$$



Case $\zeta \geq 1$

- (here : $e_0 = 1$, $K = 1$ and $\omega_n = 1$)
- no overshoot
- when $\zeta \searrow$ or $\omega_n \nearrow$, settling time $\searrow$



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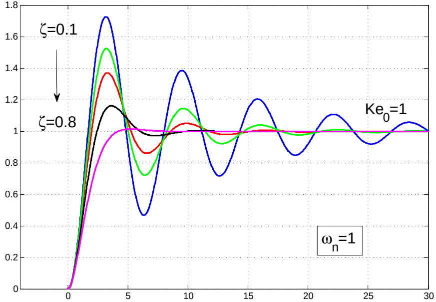
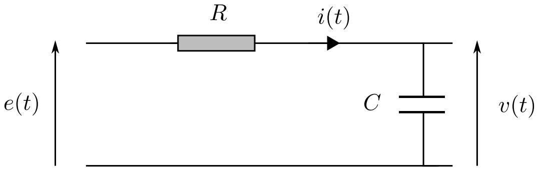


Case $0 < \zeta < 1$

- (here : $e_0 = 1,\ K = 1$ and $\omega_n = 1$)
- overshoot : $D_1 = 100 e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$
- at $t = \dfrac{\pi}{\omega_p}$
- settling time : $t_{r5\%} \simeq \dfrac{3}{\zeta\omega_n}$

Frequency response

First Example : RC circuit



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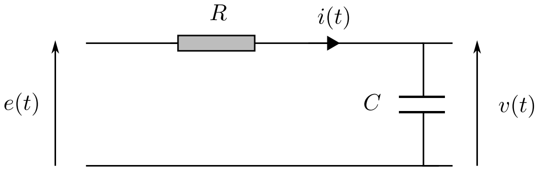
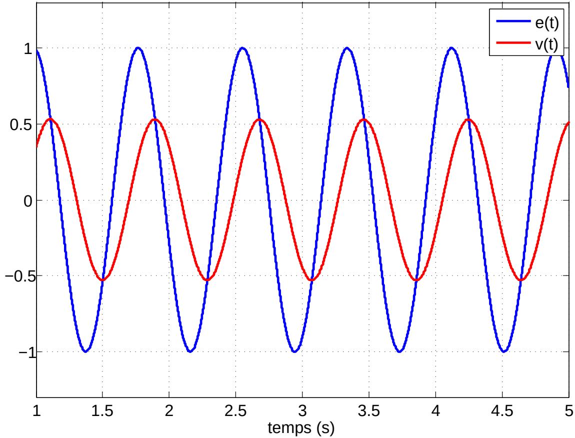


Sinusoidal steady state :

$$\begin{cases} e(t) = e_m \cos(\omega t + \phi_e) \\ v(t) = v_m \cos(\omega t + \phi_v) \end{cases} \quad \Rightarrow \quad \begin{cases} \underline{e} = e_m e^{j\phi_e} \\ \underline{v} = v_m e^{j\phi_v} \end{cases}$$

For $R = 1k\Omega$ and $C = 200\mu F$, let's apply the voltage input $e(t) = \cos(8t)$.



Ohm's law : $\underline{u} = \underline{Z}\underline{i}$

$$\underline{Z}_R = R \qquad \text{and} \qquad \underline{Z}_C = \frac{1}{j\omega C}$$

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Let's apply the voltage divider formula :

$$\underline{v} = \frac{\underline{Z}_C}{\underline{Z}_C + \underline{Z}_R}\underline{e}$$

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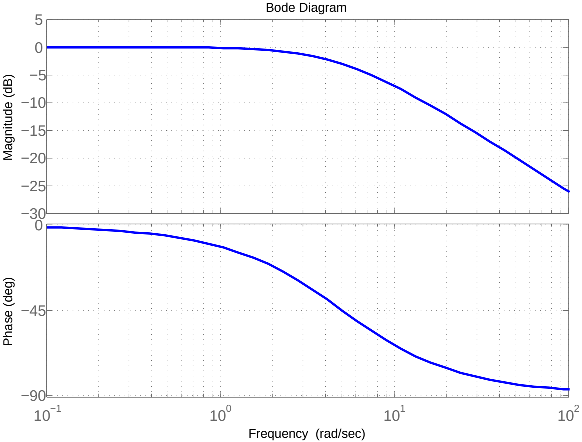
Let's apply the voltage divider formula :

$$\underline{v} = \frac{\underline{Z}_C}{\underline{Z}_C + \underline{Z}_R}\underline{e}$$

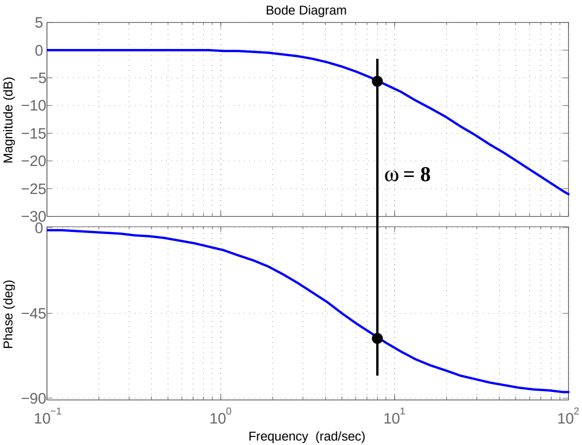
Then, the transfer from $e(t)$ to $v(t)$ is :

$$\underline{T} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{j\omega RC + 1}.$$

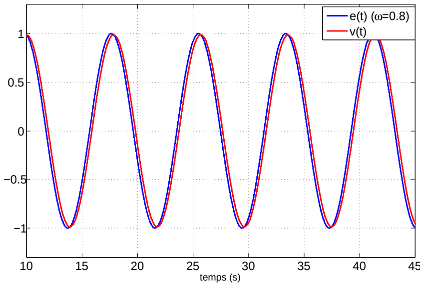
Bode diagram of the transfer function



Bode diagram of the transfer function



Circuit response for $\omega = \{0.8, 4, 8, 40\}$



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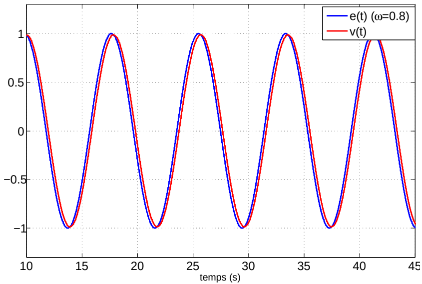

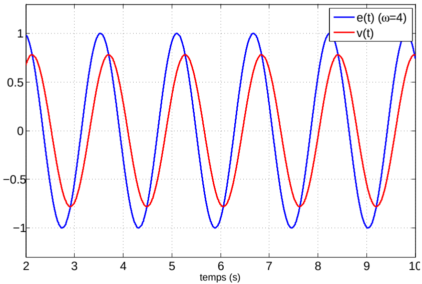

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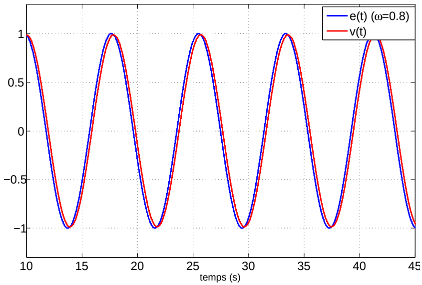

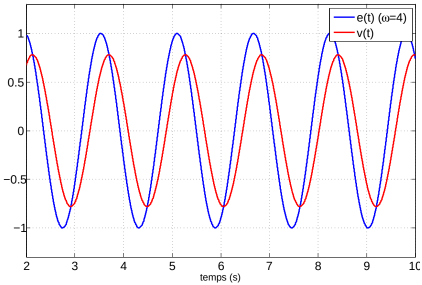

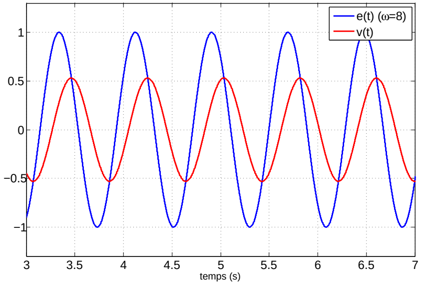

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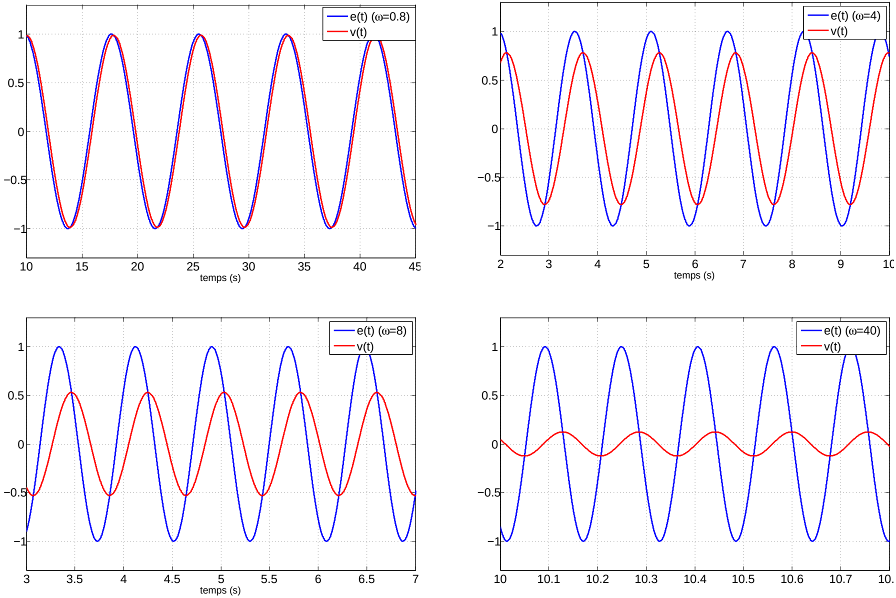
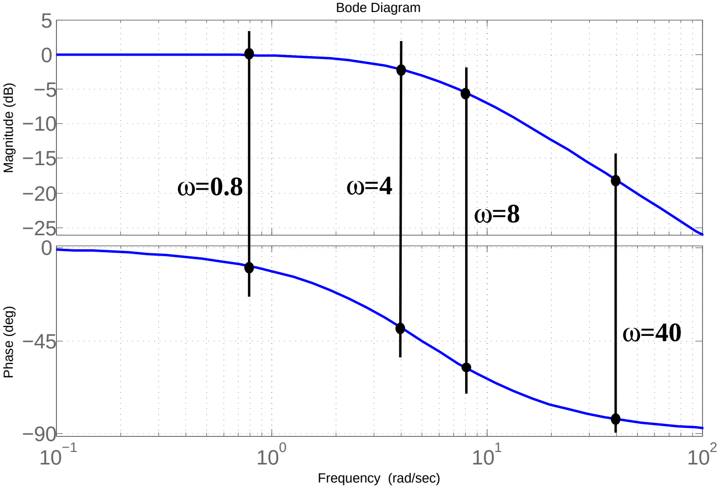
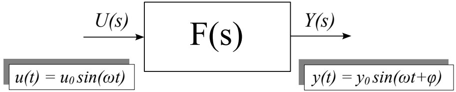



Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
ω=0.8
ω=4
ω=8
ω=40
5
0
−5
−10
−15
−20
−25
0
−45
−90
10^−1
10^0
10^1
10^2

General case

Frequency analysis ⇒ system response for sinewave input with various frequencies



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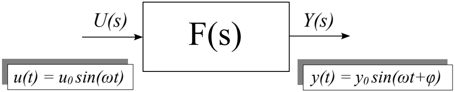

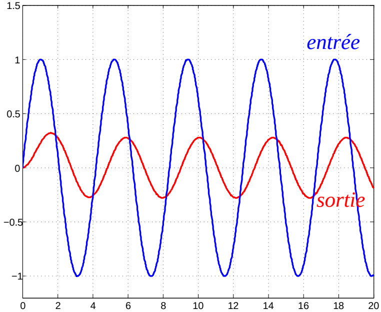

The output signal is also a sinewave, with the same frequency, but with a different magnitude and a phase shift.

Example : Let consider the system

$$F(s) = \frac{0.5}{s+1}$$

Let's have a look at the system response for 3 frequencies :

$$u_1 = \sin(0.05\,t)$$
$$u_2 = \sin(1.5\,t)$$
$$u_3 = \sin(10\,t)$$

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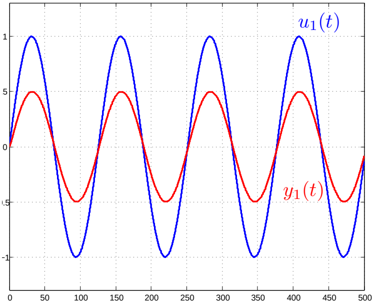

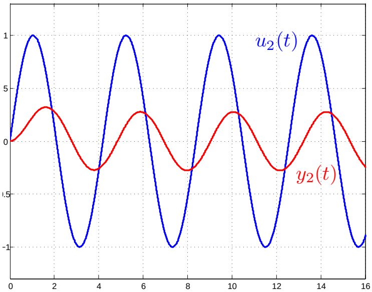

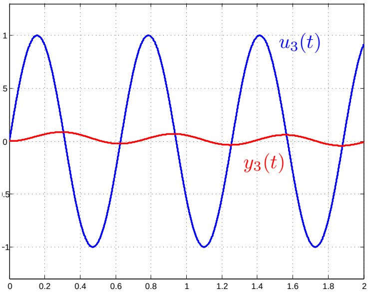

A system can be characterized by its

$$\text{gain} = |F(j\omega)| \qquad \text{phase shift} = arg\Big(F(j\omega)\Big)$$

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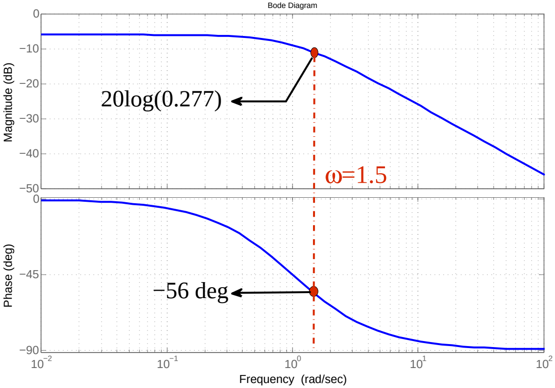

Basic plots

$$\textbf{Constant gain :} \qquad F(s) = k, \quad (k > 0)$$

transfer w.r.t. frequency : $F(j\omega) = k$

$$\begin{cases} |F(j\omega)|_{db} & = & 20log(k) \\ \phi & = & 0 \end{cases}$$

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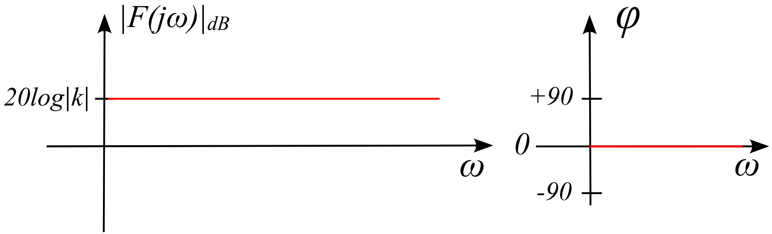

Basic plots

Derivator : $F(s) = s$

transfer w.r.t. frequency : $F(j\omega) = j\omega$

$$\begin{cases} |F(j\omega)|_{db} &= 20log(\omega) \\ \phi &= +90^{\circ} \end{cases}$$

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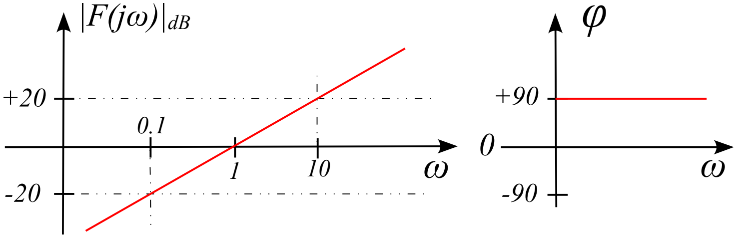

Basic plots

$$\textbf{Integrator :} \qquad F(s) = \frac{1}{s}$$

transfer w.r.t. frequency : $\qquad F(j\omega) = \dfrac{1}{j\omega}$

$$\begin{cases} |F(j\omega)|_{db} & = & -20log(\omega) \\ \\ \phi & = & arg(-j\,\frac{1}{\omega}) = -90^{\circ} \end{cases}$$

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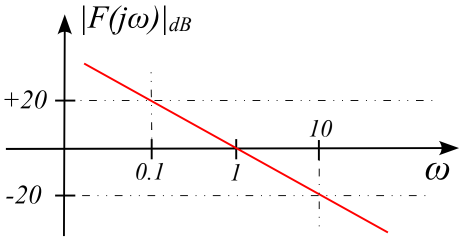

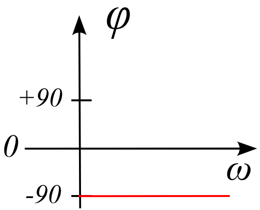

First order system

$$F(s) = \frac{1}{1+\tau s}$$

transfer w.r.t. frequency : $$F(j\omega) = \frac{1}{1+j\tau\omega}$$

- Module :

$$|F(j\omega)| \;=\; \frac{1}{|1+j\tau\omega|} \;=\; \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

- Argument :

$$arg\big(F(j\omega)\big) \;=\; arg\big(1\big) - arg\big(1+j\tau\omega\big) \;=\; -arctan\frac{\tau\omega}{1}$$

$$\left\{\begin{array}{rcl} |F(j\omega)|_{db} & = & -20log\big(\sqrt{1+\tau^2\omega^2}\big) \\ \phi & = & -arctan\big(\tau\omega\big) \end{array}\right.$$

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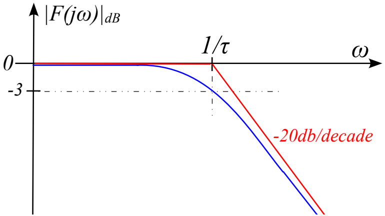

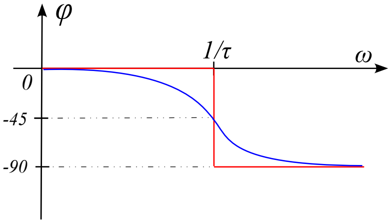

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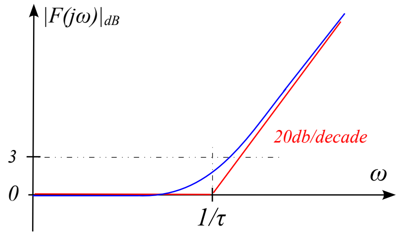

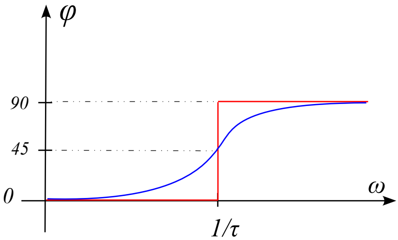

Example

Bode diagram of the transfer function $F(s) = \frac{20(10s+1)}{(100s+1)(s+1)}$

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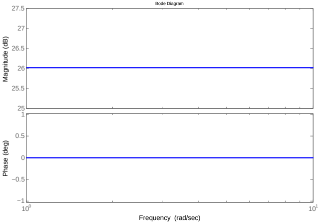
Transfer function : 20



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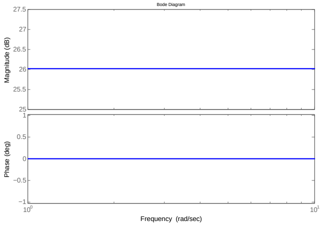
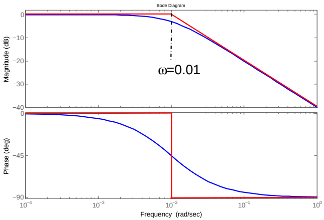


Transfer function : $\frac{1}{100s+1}$



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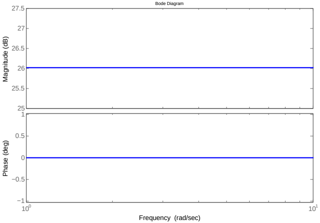

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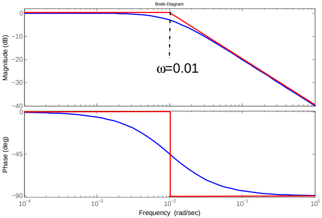

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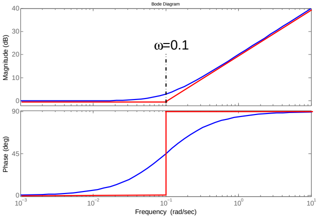

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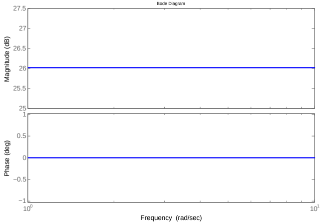

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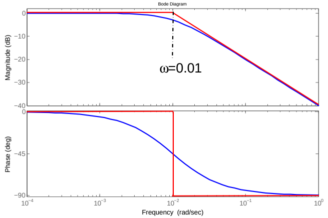

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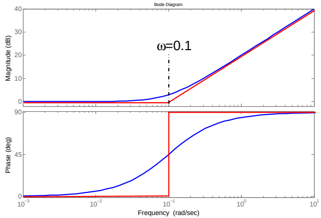

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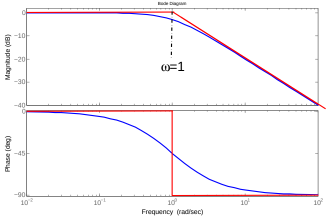

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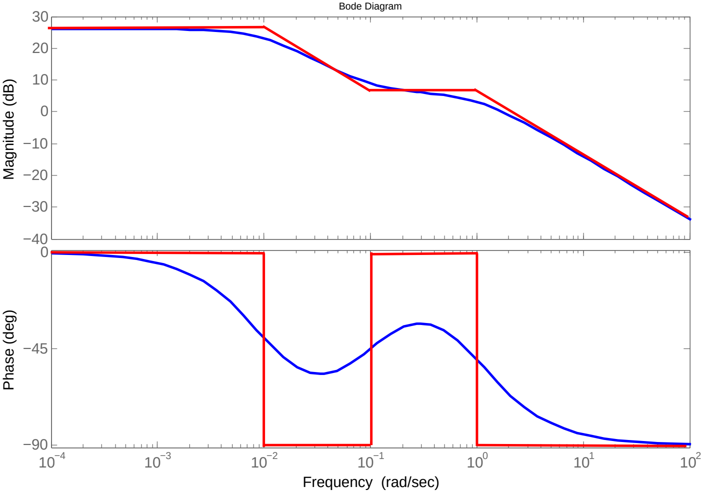
Somme des tracés

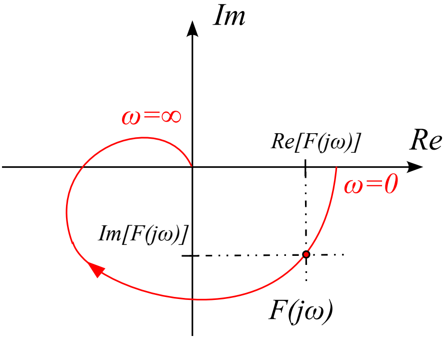


Nyquist diagram

Plot the transfert $F(j\omega)$ in the complex plane.

$$F(j\omega) = Re\Big[F(j\omega)\Big] + j\ Im\Big[F(j\omega)\Big]$$

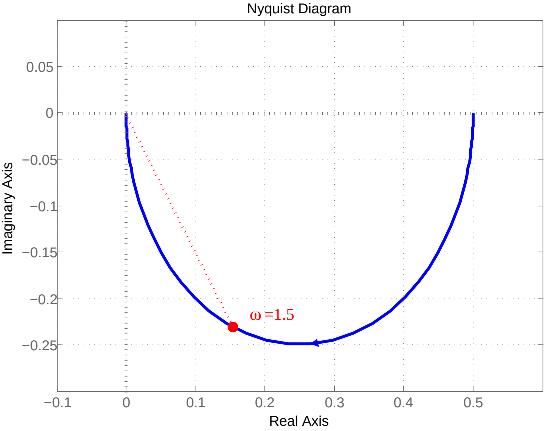
It is a parametric plot of the frequency ω.



Example :

$$F(s) = \frac{0.5}{s+1}$$

The graph may be sketched based on the Bode diagram.



Notion of stability

Let's consider the following transfer function

$$\hat{y}(s) = \frac{1}{s+a}\hat{u}(s).$$

Its unit step response is given by

$$y(t) = \frac{1}{a}\left(1 - e^{-at}\right).$$

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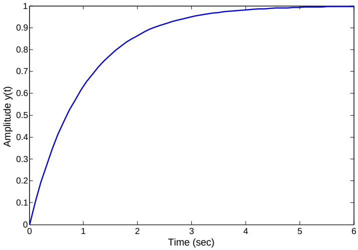

e^{-at} converges to 0

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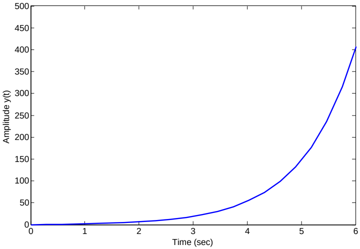

e^{-at} converges to $+\infty$

$\Rightarrow$ **Stability** property

Definition

A system is defined to be **stable** if every bounded input to the system results in a bounded output.

Theorem

A transfer function $F(s)$ is **stable** if and only if every pole of $F(s)$ has a negative real part, that is they are located in the right-half of the complex plane.

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Examples

$$\frac{1}{s-2} \qquad \frac{4}{s+0.5} \qquad \frac{3}{(s+1)(s+3)}$$

$$\frac{1}{s(5s+1)} \qquad \frac{s-2}{s^2+2s+2} \qquad \frac{2s-1}{(s+1)(s+2)(s-6)}$$

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$$\frac{1}{s-2} \quad \Rightarrow \text{Instable} \qquad \frac{4}{s+0.5} \quad \Rightarrow \text{Stable} \qquad \frac{3}{(s+1)(s+3)} \quad \Rightarrow \text{Stable}$$

$$\frac{1}{s(5s+1)} \quad \Rightarrow \text{Instable} \qquad \frac{s-2}{s^2+2s+2} \quad \Rightarrow \text{Stable} \qquad \frac{2s-1}{(s+1)(s+2)(s-6)} \quad \Rightarrow \text{Instal}$$

Routh criterion

Build a table with the polynomial coefficient (from the denominator)

$$D(s) = a_n s^n + \cdots + a_1 s + a_0$$

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2. Build the table

$$\begin{array}{c|cccc}
p^n & a_n & a_{n-2} & a_{n-4} & \dots \\
p^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \dots \\
p^{n-2} & b_1 & b_2 & b_3 & \dots \\
p^{n-3} & c_1 & c_2 & \dots & \\
\vdots & \vdots & \vdots & \vdots & \\
p^0 & \alpha & & &
\end{array}$$

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$$b_1 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \quad b_2 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} \quad b_3 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}$$

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where

$$b_1 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \quad b_2 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix} \quad b_3 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}$$

$$c_1 = -\frac{1}{b_1} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix} \quad c_2 = -\frac{1}{b_1} \begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}$$

3. The system is stable if and only if all the coefficients from the 1^{st} column have the same sign.

Example 1 :

Let's consider the transfer function $F(s) = \dfrac{s+4}{s^4+s^3+4s^2+2s+1}$, stable ?

Example 2 :

Let's consider the transfer function $F(s) = \dfrac{7}{3s^3+s^2+2s+4}$, stable ?

Example 1 :

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$$\begin{array}{c|cccc} s^4 & 1 & 4 & 1 & 0 \\ s^3 & 1 & 2 & 0 & 0 \\ s^2 & 2 & 1 & 0 & \\ s^1 & \frac{3}{2} & 0 & & \\ s^0 & 1 & & & \end{array}$$

$$\begin{aligned} b_1 &= -\tfrac{1}{1}(2-4) = 2 \\ b_2 &= -\tfrac{1}{1}(0-1) = 1 \\ c_1 &= -\tfrac{1}{2}(1-4) = \tfrac{3}{2} \\ c_2 &= 0 \\ \alpha &= -\tfrac{2}{3}(0-\tfrac{3}{2}) = 1 \end{aligned}$$

$\Rightarrow$ Stable.

Example 2 :

Let's consider the transfer function $F(s) = \dfrac{7}{3s^3+s^2+2s+4}$, stable ?

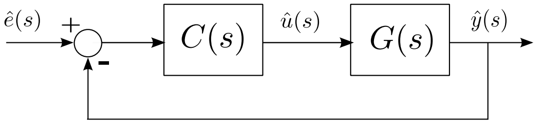
$$\begin{array}{c|ccc} s^3 & 3 & 2 & 0 \\ s^2 & 1 & 4 & 0 \\ s^1 & -10 & 0 & \\ s^0 & 4 & & \end{array}$$

$$\begin{aligned} b_1 &= -\tfrac{1}{1}(12-2) = -10 \\ \alpha &= -\tfrac{1}{-10}(0+40) = 4 \end{aligned}$$

$\Rightarrow$ Unstable.

Stability of a closed-loop system

How do we analyze the stability of a closed-loop system ?



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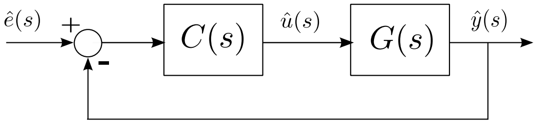

Methodes :

- Write the equivalent global transfer function $T(s) = \dfrac{C(s)G(s)}{1 + C(s)G(s)}$,

  $\Rightarrow$ Apply one of the two methods seen previously.

- Revers criterion (graphical condition).

  If the open-loop system is stable, and if all zeros have a negative real part, then the closed-loop system is stable if and only if, on the Nyquist diagram of the open-loop transfer function, the critical point $(-1\ ,\ 0)$ is on the left when the Nyquist plot is crossing the real-axis as ω is increasing.

Example 1 :

The transfer functions of the plant and the controller are

$$G(s) = \frac{10}{10s^2 + 18s + 8} \quad \text{and} \quad C(s) = \frac{4}{s^2 + 3s + 2}.$$

Let's plot the Nyquist diagram of the open-loop transfer function $C(s)G(s)$.

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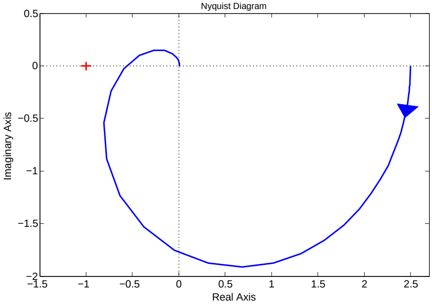

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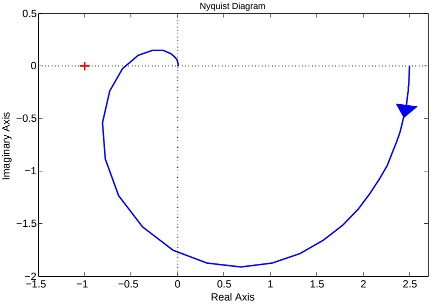

$\Rightarrow$ the closed-loop system is stable.

Example 2 :

The transfer functions of the plant and the controller are

$$G(s) = \frac{5}{s^2 + 0.4s + 1} \quad \text{et} \quad C(s) = \frac{s+3}{10s+1}.$$

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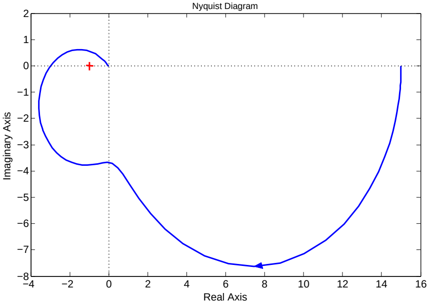

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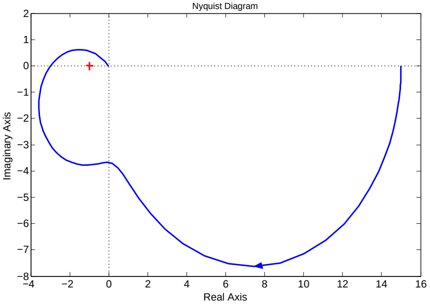

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Summary

Modelisation $\Longrightarrow$ What mathematical model best describes the system ?

- differential equation / transfer function
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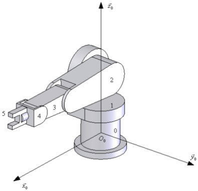
Stability $\Longrightarrow$ Is the system converging or not, bounded or not ?

- transfer function poles / Routh criterion
- Revers criterion (only for closed-loop systems)

Back to our example

Consider again the robotic arm.

Goal : control the angular position around the Z-axis



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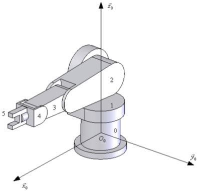

$$u(t) \longrightarrow \boxed{\text{système}} \longrightarrow \theta(t)$$

A relationship between the voltage $u(t)$ (motor input) and the position $\theta(t)$ is given by

$$\ddot{\theta}(t) + \dot{\theta}(t) = u(t)$$

The corresponding transfer function is

$$G(s) = \frac{\hat{\theta}(s)}{\hat{u}(s)} = \frac{1}{(s+1)s}$$

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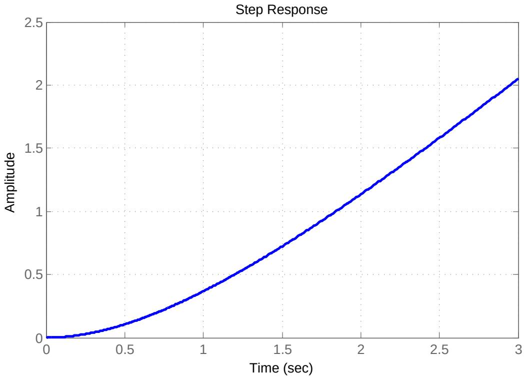
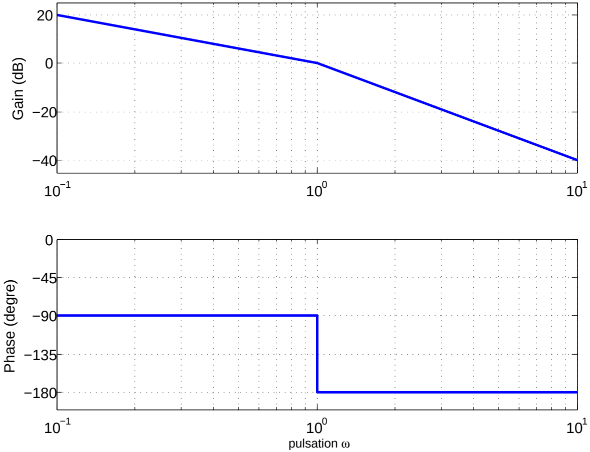
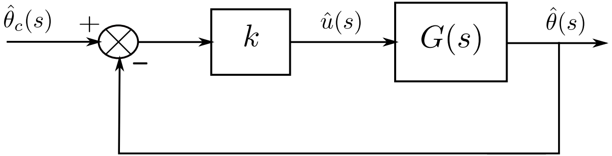
Its step response is diverging



Bode diagram approximation



Feedback control of the system



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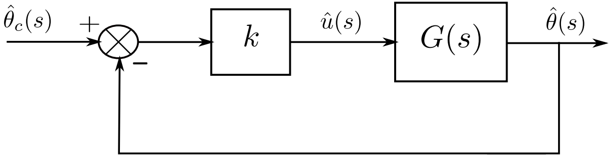
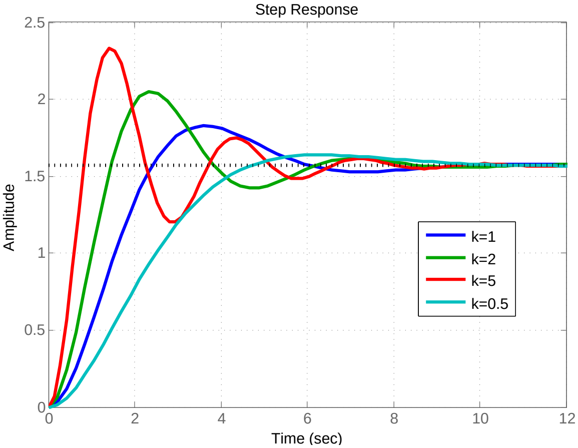


The closed-loop transfer function is

$$F(s) = \frac{k}{s^2 + s + k}$$

from the Routh criterion : $F(s)$ is stable $\forall k > 0$.

Time response $\theta(t)$ for a step input for the reference signal $\theta_c(t) = \frac{\pi}{2}$



Quick analysis of the closed-loop system

Tracking error : $\varepsilon(t) = \theta_c(t) - \theta(t)$

$$\hat{\varepsilon}(s) = \frac{s^2 + s}{s^2 + s + k}\hat{\theta}_c(s)$$

The static error is zero : $\varepsilon_s = \lim_{s \to 0} s\,\hat{\varepsilon}(s) = 0$ (with $\hat{\theta}_c(s) = \frac{1}{s}$)

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Standard form :

$$F(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \Rightarrow \qquad \left\{ \begin{array}{rcl} K & = & 1, \\ \omega_n & = & \sqrt{k} \\ \zeta & = & 1/2\sqrt{k} \end{array} \right.$$

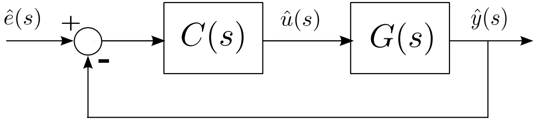
We can state that

- when $k \nearrow$, damping $\zeta \searrow$ and oscillations $\nearrow$;
- the settling time is about $t_{5\%} \approx \frac{3}{\zeta\omega_n} = 6s$.

Contents

Performances

Several criteria are used to assess feedback system performances.



Besides stability, other properties may be interesting :

- **precision**.
- **responsiveness**.
- **stability margin**.

Precision

Precision is assessed by the **steady state tracking error** :

$$\varepsilon(t) = e(t) - y(t) \text{ when } t \to \infty$$

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- **static error** ε_s

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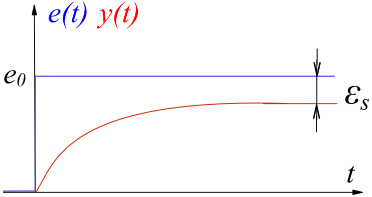
$$e(t) = e_0, \ \forall t \geq 0$$



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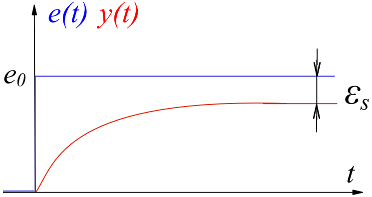
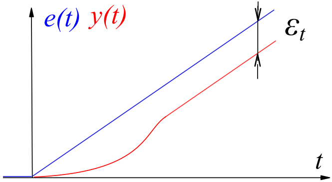


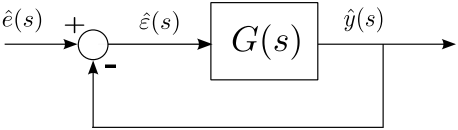
- **tracking error** ϵ_t

when the reference input is a ramp signal

$$e(t) = e_0 t, \ \forall t \geq 0$$



The steady state error is given by $\lim_{t \to \infty} e(t) - y(t)$.



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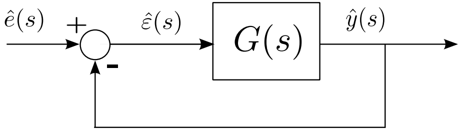

Using the final value theorem, we have : $\lim_{t\to\infty} \varepsilon(t) = \lim_{s\to 0} s\, \hat{\varepsilon}(s)$.

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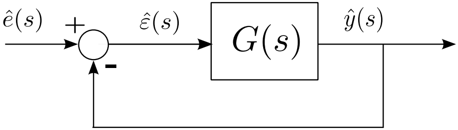

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Calculations are then easier :

$$\hat{\varepsilon}(s) \;=\; \hat{e}(s) - \hat{y}(s) \;=\; \Big(1 - F(s)\Big)\hat{e}(s)$$

where $F(s) = \dfrac{G(s)}{1+G(s)}$ is the closed-loop transfer function.

Example 1 : $G(s) = \dfrac{10}{s+20}$

After a few calculations : $F(s) = \dfrac{10}{s+30}$ and $\hat{\varepsilon}(s) = \dfrac{s+20}{s+30}\hat{e}(s).$

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Then we have :

$$\text{error for step input } \varepsilon_s = \lim_{s\to 0} s \, \frac{s+20}{s+30} \, \frac{e_0}{s} = \frac{2}{3} e_0,$$

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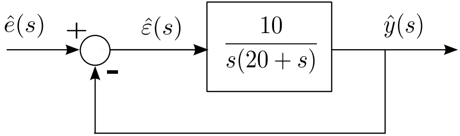

After few calculus : $F(s) = \dfrac{10}{10 + s(20+s)}$ and $\hat{\varepsilon}(s) = \dfrac{s(20+s)}{10 + s(20+s)}\hat{e}(s).$

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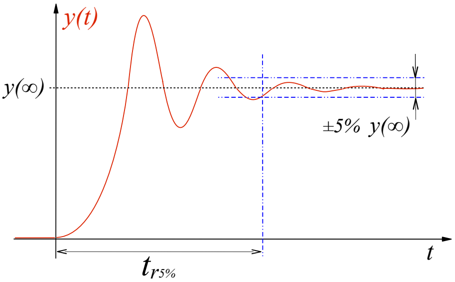
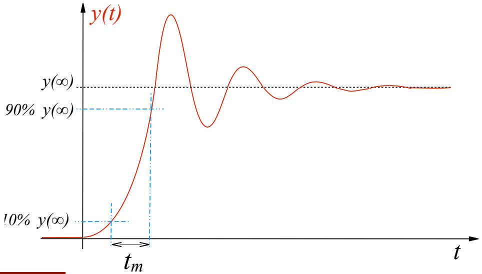
Responsiveness

Performances in terms of responsiveness : settling time and rising time.

The **settling time** is the time interval from the start to the time at which the output signal has entered and remained within a specified error band ($\pm5\%$).



The **rising time** is the time elapsed when the output signal changes from 10% to 90% of its final value.



Stability margins

A system to be controlled : $G(s) = \dfrac{k}{s(s+1)(s+2)}$.

Stability of the closed-loop system $F(s) = \dfrac{k}{s(s+1)(s+2)+k}$?

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s^0	k		

$\Rightarrow$ stable if $0 < k < 6$.

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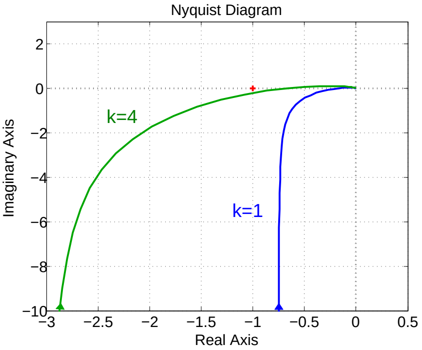


★ Notion of **stability margin** : measure of the "distance" to the instability threshold.

Gain margin

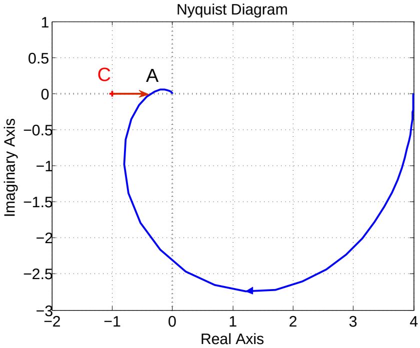
The gain margin is defined as the value of the gain multiplying the open-loop system to make the closed-loop system unstable.

Measure :

The gain margin is given by the formula :

$$\Delta G \; = \; -20 \log |G(j\omega_{-\pi})|$$

where $\omega_{-\pi}$ is the frequency for which the phase shift of the OLTF $G(j\omega)$ is -180°.



Phase margin

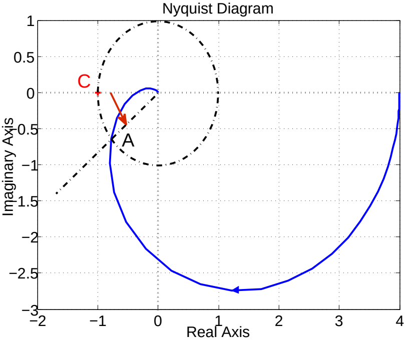
The phase margin is defined as the value of the phase shift to be applied to the open-loop system to make the closed-loop system unstable.

Measure :

The phase margin is given by the formula :

$$\Delta\phi \ = \ \pi + argG(j\omega_{0dB})$$

where ω_{0dB} is the frequency for which the gain of the OLTF $G(j\omega)$ is 1 (or 0 in dB).



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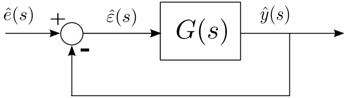

with $G(s) = \dfrac{5}{s^3 + 3.5s^2 + 3.5s + 1}$. Is the closed-loop system stable ?

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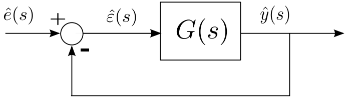

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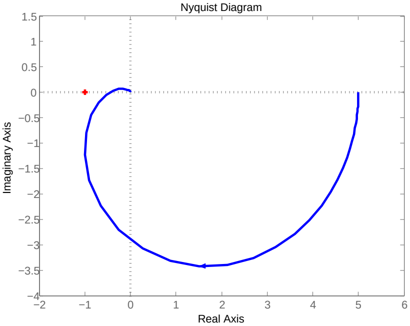
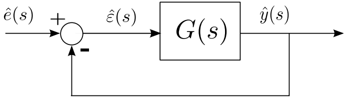


Example : Let's consider the following system



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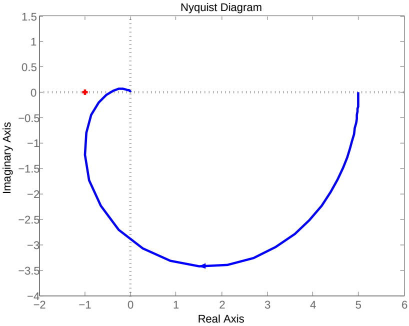

$\Rightarrow$ Invoking the Revers criteria : the feedback system is stable

What is the stability margin ?

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Some measures on the Bode diagram (or Nyquist) of $G(s)$ show that

- $|G(j\omega)| = 1$ at the frequency $\omega = 1.24\ rad/s$,
- $\arg\Big(G(j\omega)\Big) = -180^\circ$ at the frequency $\omega = 1.87\ rad/s$.

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Calculation of margins

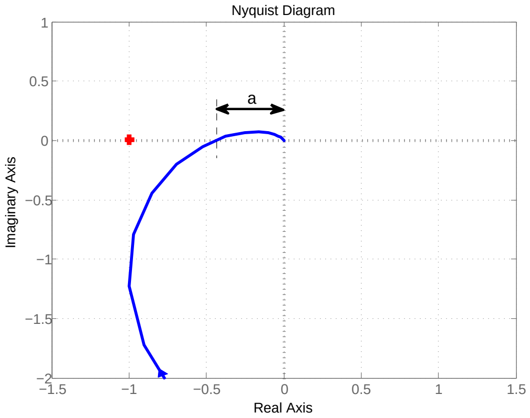
- gain margin :

$$\begin{aligned}\Delta G &= -20\log\frac{5}{|(jw)^3 + 3.5(jw)^2 + 3.5(jw) + 1|}, \text{ with } \omega = 1.87\\ &= 7.04\ dB\end{aligned}$$

- phase margin :

$$\begin{aligned}\Delta\phi &= \pi + \arg\frac{5}{(jw)^3 + 3.5(jw)^2 + 3.5(jw) + 1}, \text{ with } \omega = 1.24\\ &= 0.51\ rad \quad (29.2\ deg)\end{aligned}$$

Margins could have been directly measured on the Nyquist plot of $G(s)$



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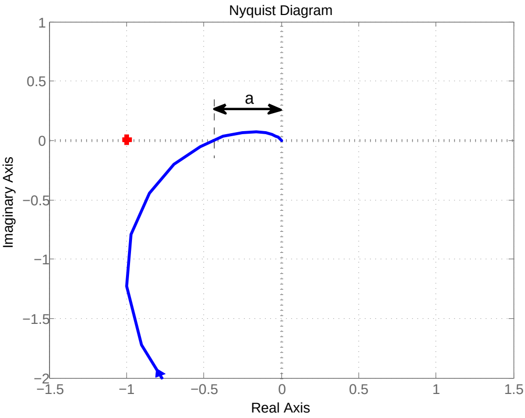

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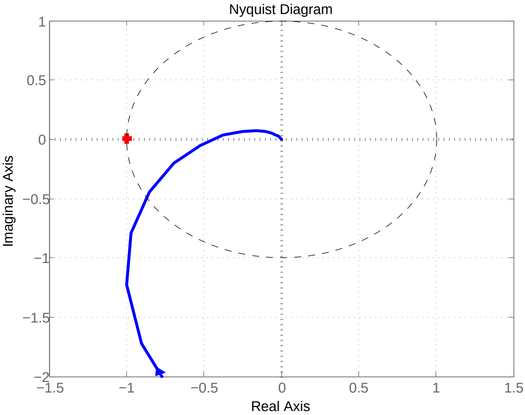

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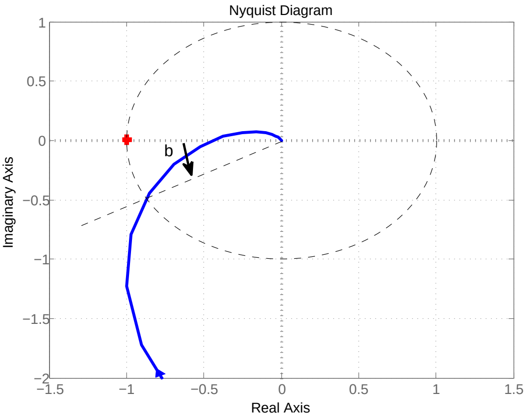

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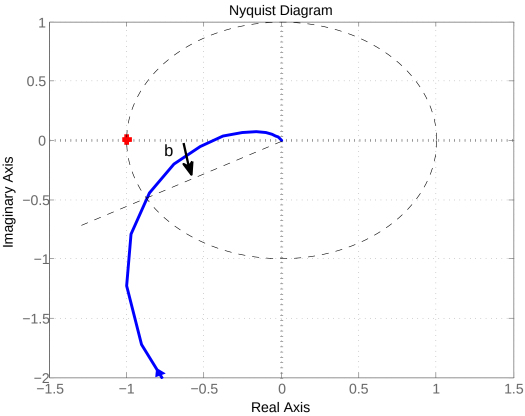

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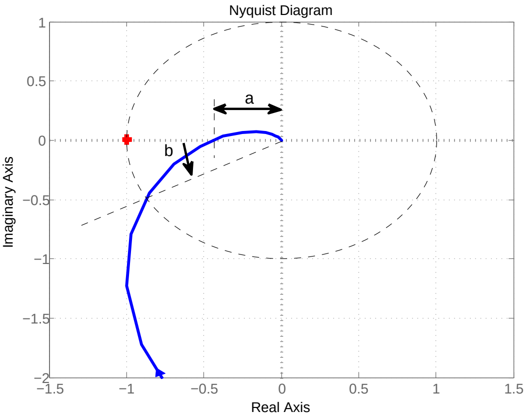

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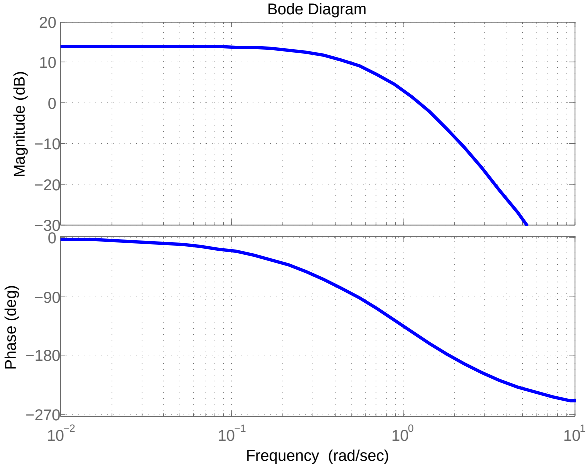
Or, on the Bode plot of $G(s)$



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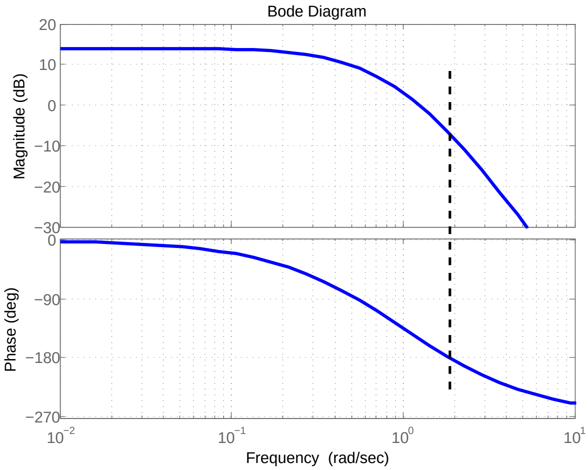

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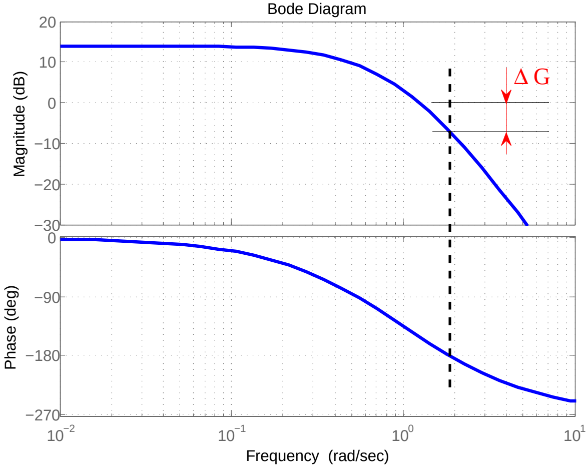

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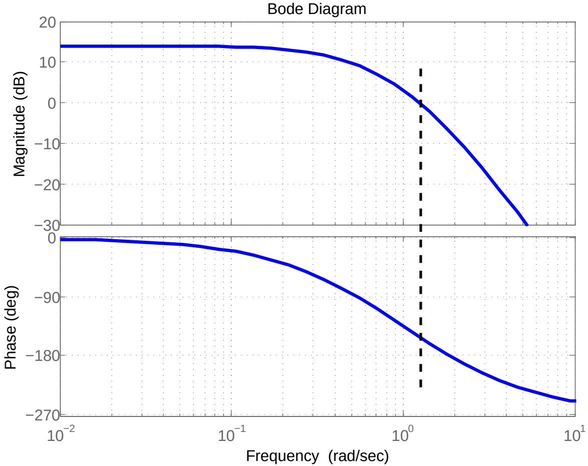

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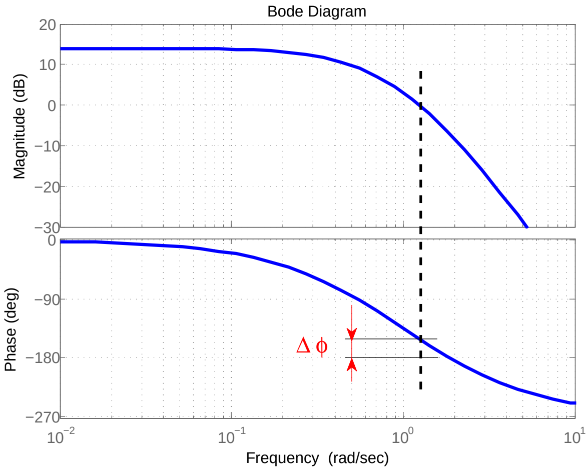

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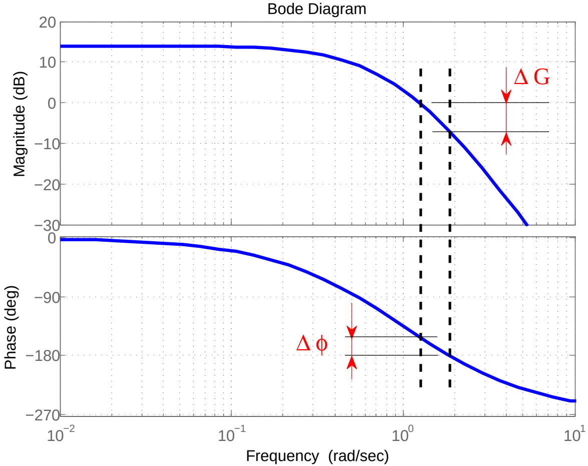

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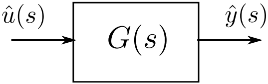

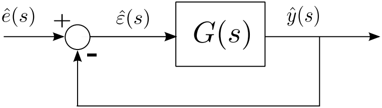

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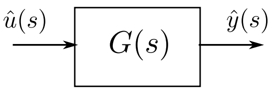

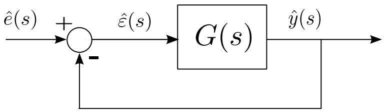

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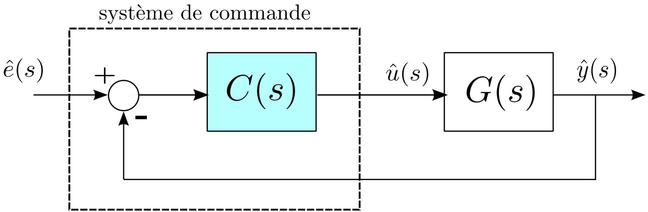
$\Rightarrow$ what you have seen so far.

Control design :



Design a control system.

- find a controller
- stabilization
- performance improvement

$\Rightarrow$ The closed-loop system must satisfy some specifications.

A control system is designed to control/improve/ensure :

- the feedback system stability,
- the responsiveness (transient regime),
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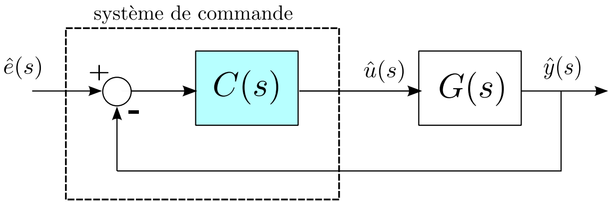
In this class, only classical frequency based methods will be considered. Widely used in industries because of

- their practical aspect,
- easy to design and often based on experience,
- sometimes parameters can be tuned in an intuitive way.

Here are some notations considered in this chapter :



- $C(s)$ is the controller.
- $G(s)$ is the system to be controlled.
- $\hat{u}(s)$ is the control signal.
- $F(s)$ is closed-loop transfer function.
- $C(s)G(s)$ is the open-loop transfer function
- The tracking error is defined by $\hat{\varepsilon}(s) = \hat{e}(s) - \hat{y}(s)$.

Synthèse directe : modèle imposé

Une $1^{\text{ière}}$ méthode directe et pratique consiste à imposer un modèle pour la FTBF.

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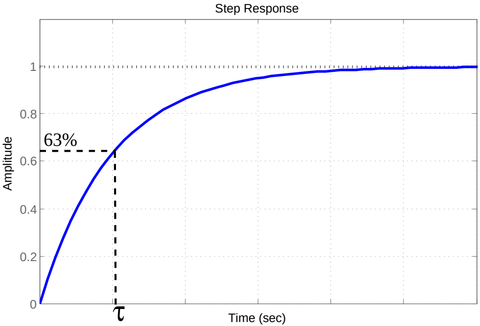
Une 1$^{\text{ière}}$ méthode directe et pratique consiste à imposer un modèle pour la FTBF.

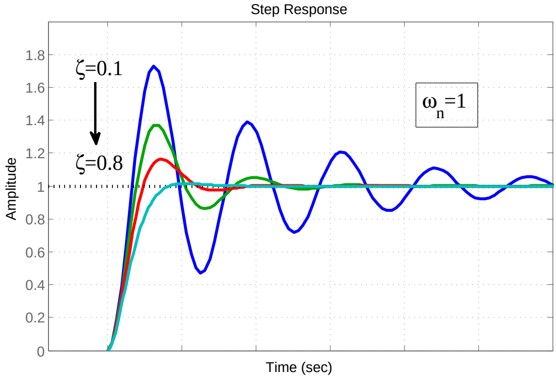
Qu'est-ce qu'un modèle "idéal" ?

pour un premier ordre : $$F_d(s) = \frac{1}{\tau s + 1}$$



la constante de temps τ définit la rapidité : $t_{r5\%} = 3\tau$

pour un second ordre : $$F_d(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1}$$



- l'amortissement ζ définit le dépassement : $D_1 = 100\, e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$
- la pulsation propre et l'amortissement définissent la rapidité $t_{r5\%} \simeq \frac{3}{\zeta\omega_n}$

Méthode :

1. Spécifier une FTBF ayant une dynamique souhaitée "idéale".

   - type premier ordre

   $$F_d(s) = \frac{1}{\tau s + 1}$$

   - type second ordre

   $$F_d(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1}$$

   - pour les ordres supérieurs on peut envisager : un pôle multiples ou un second ordre dominant.

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2. En identifiant la FTBF avec la fonction désirée, calculer le correcteur :

$$F_d(p) = F(p) \qquad \Rightarrow \quad F_d(p) = \frac{C(p)G(p)}{1 + C(p)G(p)}$$

$$\Rightarrow \quad C(p) = \frac{F_d(p)}{G(p)\Big(1 - F_d(p)\Big)}$$

3. Le correcteur doit être propre pour être réalisable.
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Remarques :

- L'approche est valide seulement si le modèle du système est bien connu.
- Le système à commander doit posséder des pôles et zéros stables.
- L'approche est adaptée plutôt pour des systèmes d'ordre faible.

Exemple

Soit le système à commander de fonction de transfert :

$$G(s) = \frac{6}{s^2 + 2s + 8}.$$

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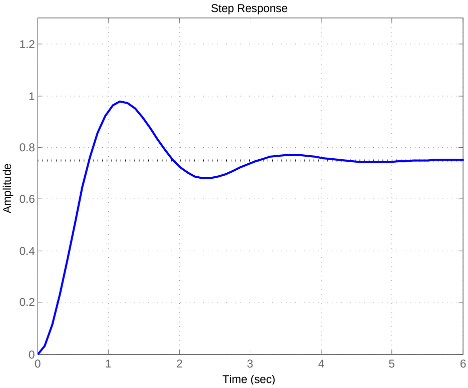
Simulons sa réponse indicielle :



Dépassement $= 30\%$; temps de réponse à $5\% = 2.78s$; erreur de position $= 25\%$.

Choisissons un modèle de référence du second ordre :

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- L'amortissement ζ est fixé à 0.8 de sorte que le dépassement soit inférieur à 2%.
- La pulsation propre ω_n est fixée à 1.87 de sorte que le temps de réponse soit de l'ordre de $2s$.
  (formule approximative : $t_{r5\%} \simeq \frac{3}{\zeta\omega_n}$)
- La FTBF désirée s'écrit donc :

$$F_d(s) = \frac{1}{0.284s^2 + 0.853s + 1}.$$

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Enfin, nous pouvons en déduire le correcteur :

$$C(s) = \frac{s^2 + 2s + 8}{\frac{6}{\omega_n^2}s^2 + \frac{12\zeta}{\omega_n}s} = \frac{s^2 + 2s + 8}{1.707s^2 + 5.12s}$$

Simulons la réponse indicielle du système asservi par notre correcteur :

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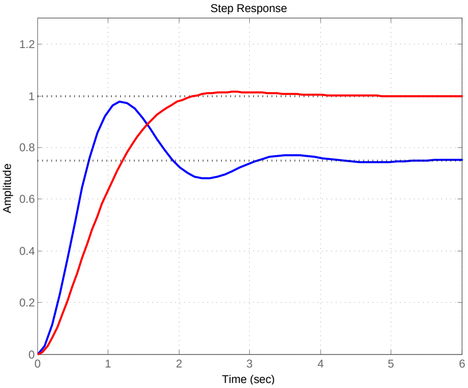
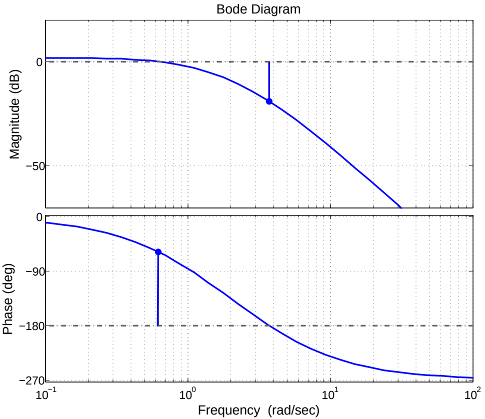


Dépassement $= 1.51\%$; temps de réponse à $5\% = 1.81s$; erreur de position $= 0\%$.

Action proportionnelle

Ce type de correcteur est un simple amplificateur de gain réglable :

$$u(t) = k_p \epsilon(t) \quad \Rightarrow \quad C(s) = k_p.$$



Avantages et Inconvénients :

- Si $k_p \nearrow$, le système répond plus rapidement, l'erreur de position diminue.
- Si $k_p \searrow$, les marges de stabilité augmentent.
- Si $k_p \nearrow$, peut entrainer des oscillations et un dépassement préjudiciable.

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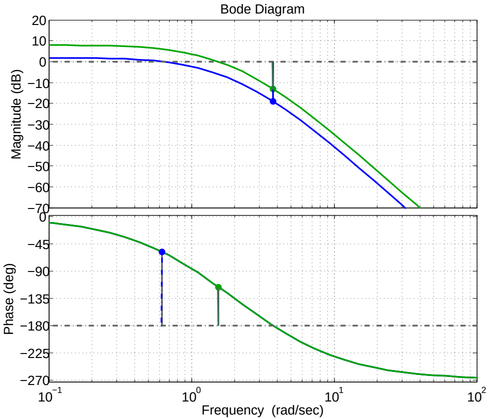

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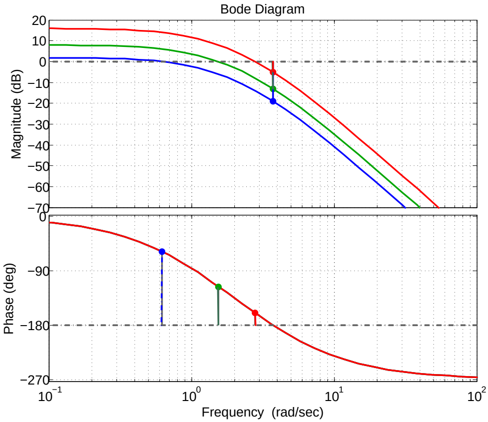

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Soit le système à commander $G(s) = \frac{1}{s^2+s+1}$ et le système de commande $C(s) = k_p$. Le système en boucle fermée s'exprime par

$$F(s) = \frac{k_p}{s^2 + s + 1 + k_p}.$$

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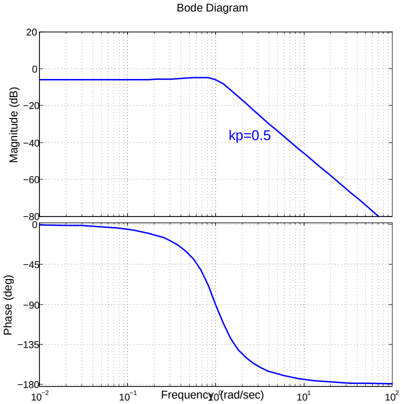

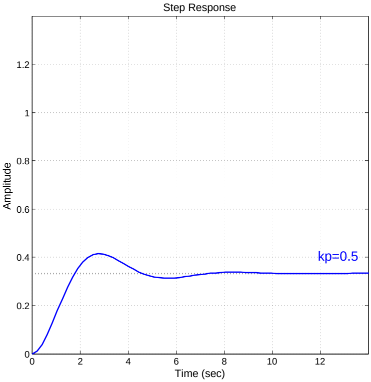

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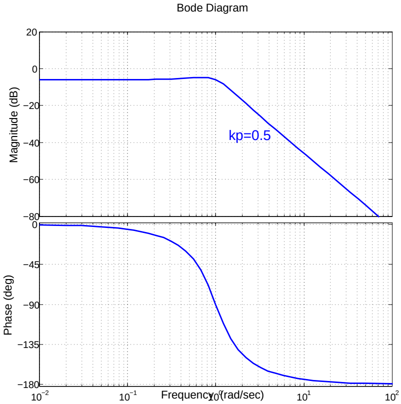

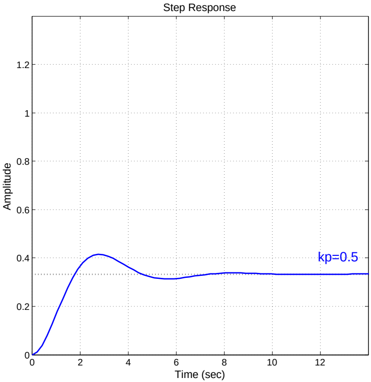

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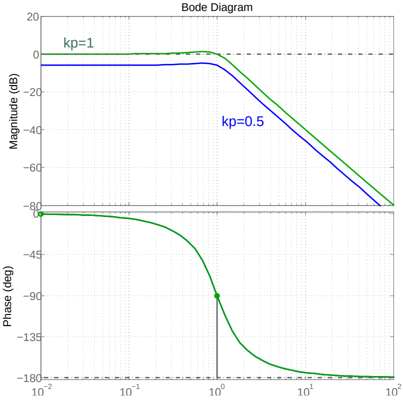

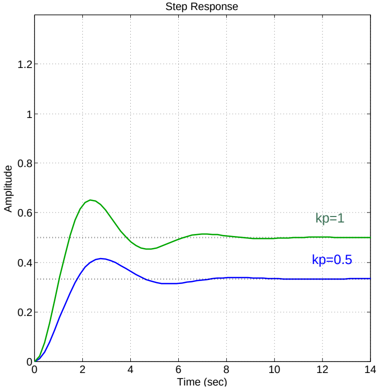

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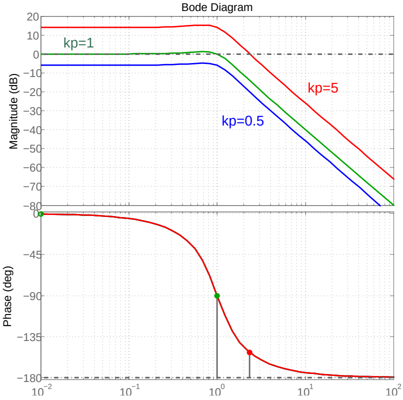
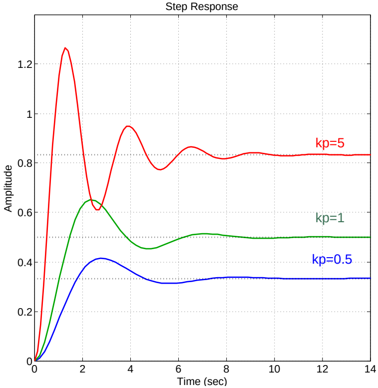
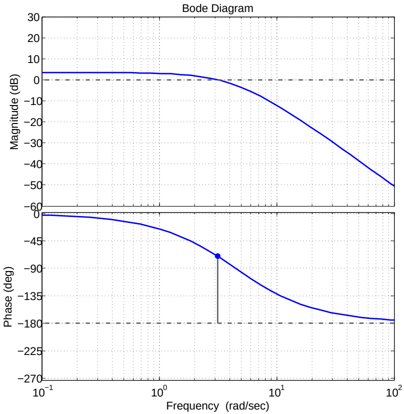




Action intégrale

Ce correcteur introduit un intégrateur qui ajoute un pôle nul à la fonction de transfert en boucle ouverte :

$$u(t) = \int_0^t \epsilon(t)dt \quad \Rightarrow \quad C(s) = \frac{1}{s}.$$



Avantages et Inconvénients :

- Erreur statique nulle en boucle fermée (BF),
- Diagramme de phase décalé de -90° : marges de stabilités dégradées,

Action intégrale

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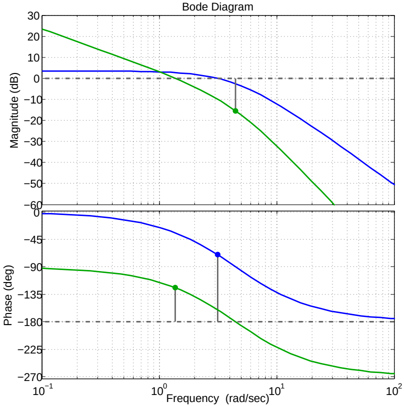

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Exemple :
Soit le système à commander $G(s) = \frac{0.8}{s^2+s+1}$ et le système de commande $C(s) = 1/s$. Le système en boucle fermée s'exprime par

$$F(p) = \frac{0.8}{s^3 + s^2 + s + 0.8}.$$

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F est un ordre 3 : la marge de gain est finie,
Courbe plus proche du point critique : marges dégradées, oscillations et dépassement,
Pas d'erreur statique. schema de gauche : bode de la BO ; schema de droite : réponse indicielle de la BF.

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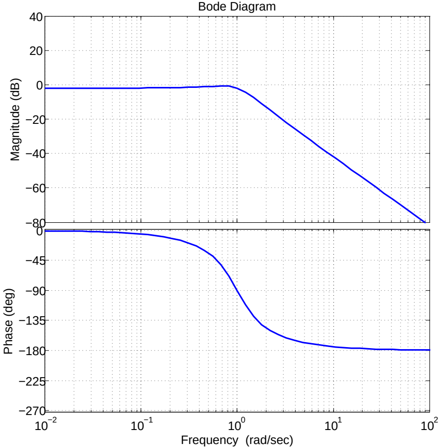

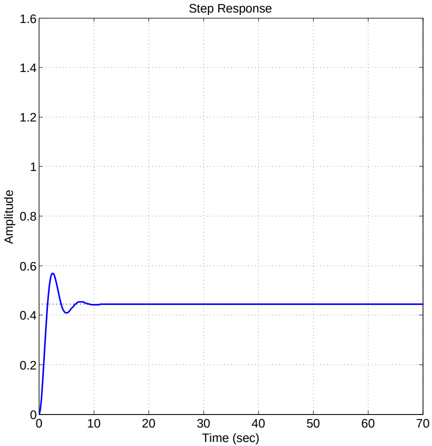

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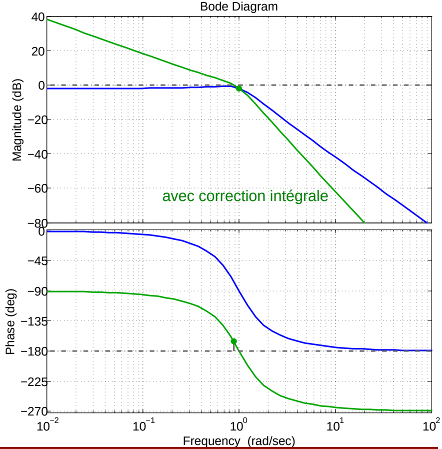
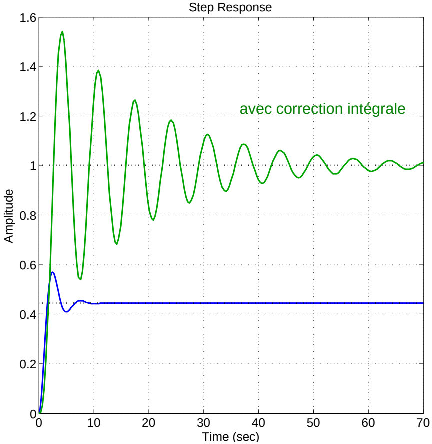
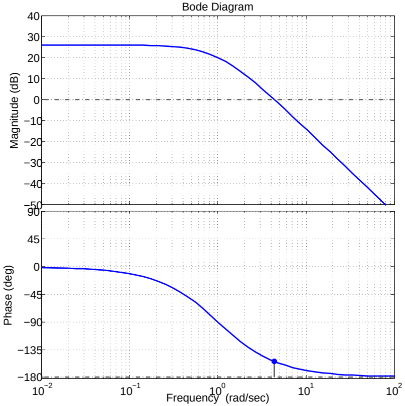




Action dérivée

Ce correcteur introduit un dérivateur qui ajoute un zero nul à la fonction de transfert en boucle ouverte :

$$u(t) = \frac{d\epsilon(t)}{dt} \quad \Rightarrow \quad C(s) = s.$$



Avantages et Inconvénients :

- Ajoute de la phase : marges susceptibles d'être augmentées,
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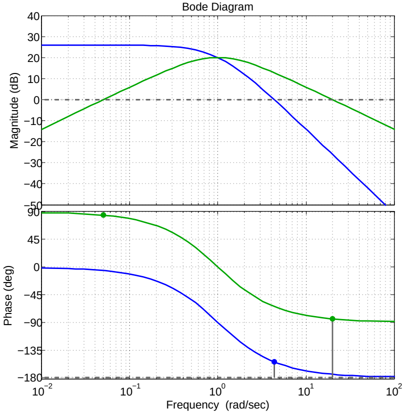

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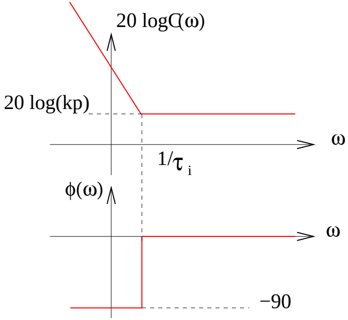
Correcteur Proportionnel-Intégral (PI)

Ce correcteur a pour objectif de tirer profit des avantages de l'effet de I sans ses inconvénients :

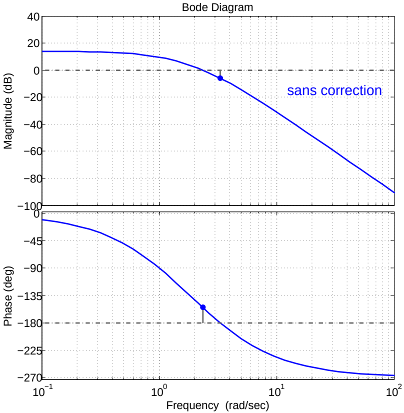
$$C(s) = k_p \frac{1 + \tau_i s}{\tau_i s}.$$



Idée du correcteur :

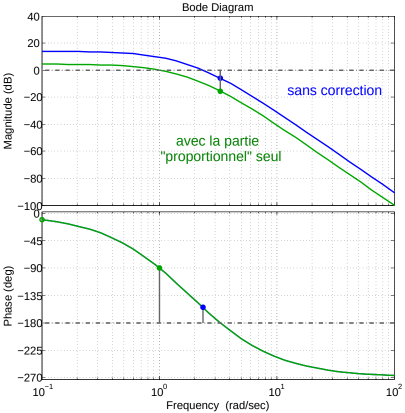
- Utiliser l'avantage de l'intégrateur en basses fréquences : précision infinie,
- L'action intégrale ne doit plus avoir d'effet dans les fréquences élevées, en particulier dans la région du point critique,

Réglage intuitif du correcteur :



Réglage intuitif du correcteur :

- Ajuster le gain proportionnel en fonction de l'objectif et des caractéristiques du système avant correction :
  - le diminuer pour augmenter les marges de stabilité,
  - l'augmenter pour améliorer la rapidité du système.



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  - le diminuer pour augmenter les marges de stabilité,
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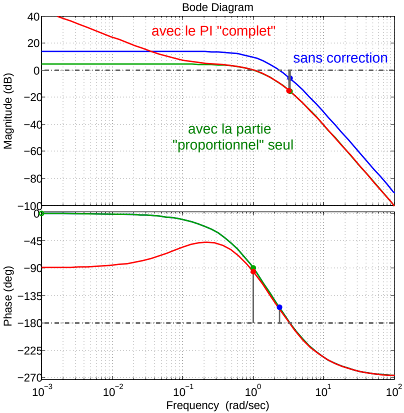
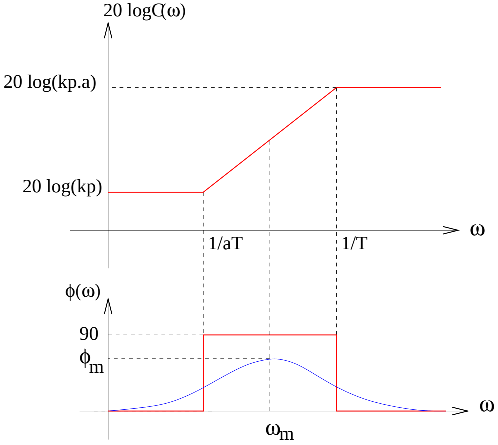
- Ensuite, la partie I est ajoutée en réglant le zero $1/\tau_i$ de façon à ce que la correction ne se fasse qu'en basses fréquences.



Correcteur à Avance de Phase

Ce correcteur a pour objectif d'apporter de la phase autour du point critique afin d'augmenter les marges de stabilité

$$C(s) = k_p \frac{1 + aTs}{1 + Ts}, \quad a > 1.$$



Le principe repose sur le réglage de a et T tels que le correcteur apporte de la phase (plage $[\frac{1}{aT}, \frac{1}{T}]$) autour du point critique.

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Méthode de réglage du correcteur :

- Régler le gain k_p pour ajuster la précision ou la rapidité ou les marges.

- Mesurer la marge de phase (après la correction prop. k_p) et en déduire la quantité de phase nécessaire
$$\phi_m = \Delta\phi_{\text{désirée}} - \Delta\phi.$$

- A partir de ϕ_m, a peut être calculé
$$a = \frac{1 + sin\phi_m}{1 - sin\phi_m}$$

- Enfin, nous avons la relation
$$\omega_m = \frac{1}{T\sqrt{a}},$$
il s'agit alors de calculer T tel que ω_m coïncide avec ω_{0db} (après correction prop.) : $T = \dfrac{1}{\omega_{0db}\sqrt{a}}$.

Exemple :

Soit la fonction de transfert

$$G(s) = \frac{100}{(1+s)^2}$$

On souhaite que le système en boucle fermée ait une marge de phase $M\phi = 45^\circ$:

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- Calculons la marge de phase avant correction

$$G(\omega) = \frac{100}{(1+\omega_{0db}^2)} = 1 \quad \Rightarrow \quad \omega_{0db} = 9.95 rad/s$$

$$M\phi = \pi - 2 arctan \omega_{0db} = 11^\circ$$

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- La marge est insuffisante, il faut donc remonter la phase de $\phi_m = 34^\circ$ à la pulsation ω_{0db}

$$a = \frac{1 + sin\phi_m}{1 - sin\phi_m} = \frac{1 + sin34^\circ}{1 - sin34^\circ} = 3.54$$

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- Puis, on règle T afin d'ajouter la quantité ϕ_m au bon endroit

$$\frac{1}{T\sqrt{a}} = \omega_{0db} \quad \Rightarrow \quad T = 0.053$$

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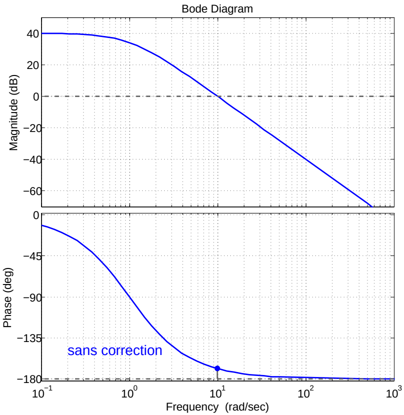
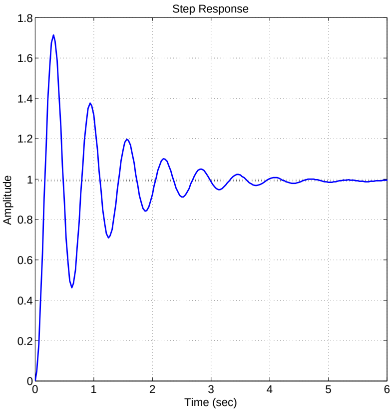
- Souvent, on choisit $\phi_m = 1.2\phi_{necessaire}$ pour compenser le décalage de ω_{0db} après correction.

Schema de gauche : bode de la BO ; schema de droite : réponse indicielle de la BF





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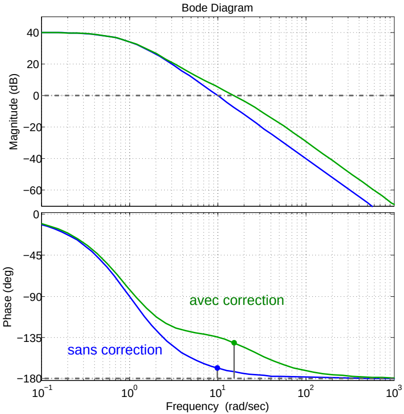
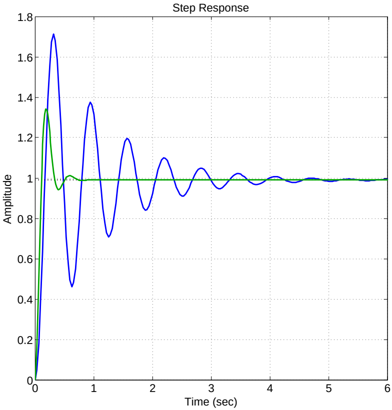
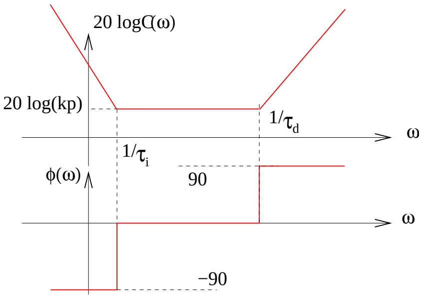




Proportionnel-Intégral-Dérivé (PID)

Ce correcteur PID est une combinaison des actions PI et PD

$$C(s) = k_p \left(\frac{1 + T_i s}{T_i s} \right) (1 + T_d s).$$



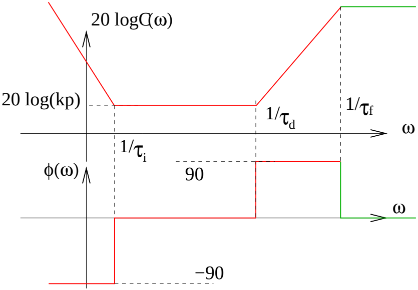
Idée du correcteur :

- Ajouter du gain en bf (précision $\nearrow$),
- Ajouter de la phase près du point critique ($M\phi \nearrow$).

Proportionnel-Intégral-Dérivé (PID)

Ce correcteur PID est une combinaison des actions PI et PD + un filtre

$$C(s) = k_p \left(\frac{1 + T_i s}{T_i s} \right) (1 + T_d s) \frac{1}{(1 + T_f s)}.$$



Idée du correcteur :

- Ajouter du gain en bf (précision $\nearrow$),
- Ajouter de la phase près du point critique ($M\phi \nearrow$).
- Le correcteur est propre,
- Atténuer l'effet du bruit (moins de gain en hf).

Proportionnel-Intégral-Dérivé (PID)

Méthode de réglage du correcteur :

- Etudier le système en BO et ses caractéristiques

  (marges, précision, rapidité...).

- Régler le gain proportionnel k_p afin d'obtenir un premier asservissement satisfaisant

  (en termes de marges, dépassement, oscillations, rapidité).

- Régler la constante de temps de l'action intégrale de sorte qu'elle n'agisse qu'en bf

  (pour ne pas pénaliser les marges).

- Régler la constante de temps de l'action dérivée de sorte qu'elle n'agisse qu'autour du point critique

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- Régler la constante de temps de l'action dérivée de sorte qu'elle n'agisse qu'autour du point critique

  (pour ne pas pénaliser la précision).

- Régler la constante de temps du filtre de sorte qu'il n'enlève pas de phase près du point critique,

- L'ordre du filtre peut être augmenté afin de mieux atténuer le bruit en hf.

Exemple : Soit le système $G(s) = \dfrac{5}{s^2 + s + 1}$.

- Une étude de ce système nous montre qu'il est stable en BF avec une erreur de position $\epsilon_s = 16.7\%$, un dépassement de 51% et une marge de phase $M_\phi = 27.8°$ à $\omega_{0db} = 2.33 rad/s$.

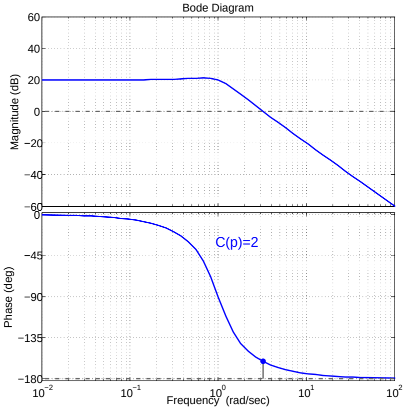
- On choisit un premier correcteur proportionnel afin de baisser l'erreur et accélérer le système : on prend $k_p = 2$ pour avoir une erreur de 9%. Il en résulte une nouvelle marge de phase $M_\phi = 18.9°$ à $\omega_{0db} = 3.23 rad/s$ et un dépassement de 61.9%.

- On place la constante de temps du I avant le point critique $T_i = 3 \gg 1/3.23$. Le correcteur devient $C(s) = 2\dfrac{1 + 3s}{3s}$.

- L'erreur en régime permanent est maintenant nul mais la nouvelle marge de phase est de $M_\phi = 12.9°$ à $\omega_{0db} = 3.24 rad/s$. Il s'agit ensuite de placer la constante de temps de la partie D avant le point critique de façon à laisser le temps au correcteur de remonter la phase jusqu'a obtenir une marge satisfaisante :$T_i \gg T_d = 0.4 > 1/3.24$. On obtient $M_\phi = 70.4°$.

- Pour finir, afin de synthétiser un correcteur réalisable et pour mieux filtrer le bruit, on ajoute un filtre avec une constante de temps relativement faible $T_f = 0.1 < 1/3.24$. Il en résulte $M_\phi = 45.9°$, ce qui reste satisfaisant.

Diagramme de Bode de la fonction de transfert en boucle ouverte ($C(s)G(s)$)



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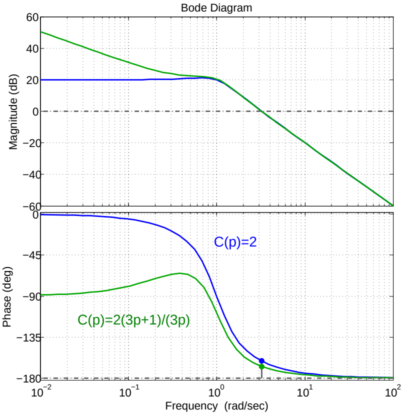

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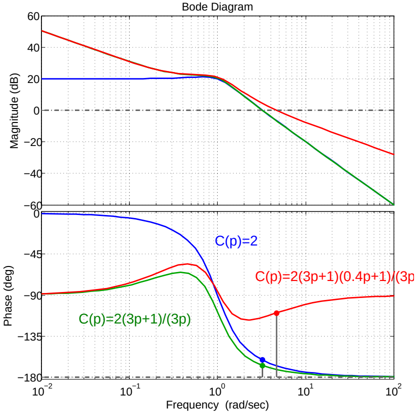

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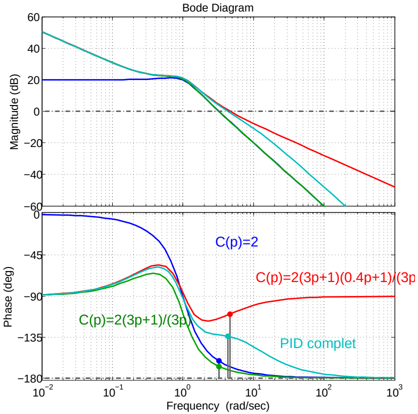

Réponses temporelles de la BF à un échelon unité