LMI optimization for fixed-order H_∞ controller design

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Abstract

A general H_∞ controller design technique is proposed for scalar linear systems, based on properties of positive polynomial matrices. The order of the controller is fixed from the outset, independently of the order of the plant and weighting functions. A sufficient LMI condition is used to overcome non-convexity of the original design problem. The key design step, as well as the whole degrees of freedom are in the choice of a central polynomial, or desired closed-loop characteristic polynomial.

Keywords

H_∞ control, fixed-order controller design, polynomials, LMI, computer-aided control system design.

1 Introduction

This paper is a continuation of our research work initiated in [HSK02], where a linear matrix inequality (LMI) method was described to design a fixed-order controller robustly stabilizing a linear system affected by polytopic structured uncertainty. A convex LMI approximation of the stability domain in the space of coefficients of a polynomial was obtained there, based on recent results on positive polynomials and strictly positive real (SPR) functions. As explained in [HSK02], the key ingredient in the design procedure resides in the choice of a central polynomial, or desired nominal closed-loop characteristic polynomial.

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Paper [HSK02] focused only on polynomial polytope stabilization, which we believe is an interesting research problem, but remains obviously very far from an actual engineering design problem. So in this paper we try to build upon the ideas of [HSK02] to derive a more practical controller design methodology. Standard design specifications are formulated in the frequency domain, on peak values of Bode magnitude plots of (possibly weighted) system transfer functions, this is the so-called H_∞ optimization framework surveyed e.g. in [K93].

The main characteristics of our approach, and its contribution with respect to existing work in the area are as follows:

- The order of the controller is fixed from the outset, which overcomes the standard limitation of state-space H_∞ techniques that the order of the controller must be at least the same as the order of the plant. Note that, as a consequence, with our techniques using weighting functions does not entail increasing the controller order;
- We use convex optimization over positive polynomials and SPR rational functions, just as in [DFT92, RM94, D00]. The main distinction is that we do not use the infinite dimensional Youla-Kučera parametrization of all stabilizing controllers as in [DFT92, RM94, D00], or analytical (stable) rational functions in H_∞ [HM98], so that it is not necessary to resort to model reduction techniques to derive a low-order controller;
- As in our previous work [HSK02], all the degrees of freedom in the design procedure are captured in the choice of the so-called central polynomial, or desired closed-loop characteristic polynomial. Note however that, contrary to the design procedure of [LL99] or [WC00], the central polynomial will not necessarily be the actual characteristic polynomial, but only a reference polynomial around which the design is carried out. Influence of the central polynomial on closed-loop performance is generally easy to predict. A general rule of thumb is that open-loop stable poles must be mirrored in the central polynomial, completed by sufficiently fast additional dynamics. This is in contrast with the recent work in [N02], where fixed order H_∞ controller design is carried out with the help of Nevanlinna-Pick interpolation, but the influence of design parameters (the so-called spectral zeros) cannot be easily characterized.

Complementary features of our approach are as follows:

- We can enforce LMI structural constraints on the controller coefficients. For example, we can enforce the controller to be strictly proper, or a PID. We can also minimize the Euclidean norm of controller coefficients if suitable;
- Contrary to standard H_∞ techniques, there are no assumptions on open-loop dynamics, presence of zeros along the imaginary axis, properness of weighting functions etc.;
- Continuous-time and discrete-time systems are treated in a unified way, as well as pole location in arbitrary half-plane or disks;

Finally, here is a list of current limitations of our H_∞ design technique:

- As in [HSK02], we use a sufficient convex (LMI) conditions that ensure closed-loop specifications, possibly at the price of some conservatism. We are not aware of any reliable method for measuring the amount of conservatism of our method, even though the many numerical examples we have treated seem to indicate that the approach generally performs at least as well as other design techniques;
- Contrary to well-established state-space H_∞ techniques, the numerical behavior and performance of LMI or semidefinite programming solvers on these optimization problems over polynomials is still unclear. In the conclusion we mention some research directions and some recent references focusing on this important issue;
- Similarly to standard H_∞ techniques, our design technique is iterative, and a trial-and-error approach cannot be avoided to choose appropriately the central polynomial.

2 Problem statement

The scalar H_∞ design problem to be solved in this paper can be formally stated as follows, based on Kučera's algebraic polynomial formulation [K79].

Problem 1 *Given a set of polynomials $n_i^k(s)$, $d_i^k(s)$ for $i = 1, 2, \ldots$, $k = 1, 2, \ldots$, as well as a set of positive real numbers γ^k, seek polynomials $x_i(s)$ of given degrees such that*

$$\left\| \frac{\sum_i n_i^k(s)x_i(s)}{\sum_i d_i^k(s)x_i(s)} \right\|_\infty < \gamma^k, \quad k = 1, 2, \ldots \tag{1}$$

In the above inequalities

$$\|S\|_\infty = \sup_{s\in\partial\mathcal{D}} |S(s)|$$

denotes the peak value of the magnitude of rational transfer function S when evaluated along the one-dimensional boundary $\partial\mathcal{D}$ of a given stability region

$$\mathcal{D} = \{s \in \mathbb{C} : \begin{bmatrix} 1 \\ s \end{bmatrix}^\star \begin{bmatrix} d_{11} & d_{12} \\ d_{12}^\star & d_{22} \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} < 0\}$$

of the complex plane, where the star denotes transpose conjugate and Hermitian matrix

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{12}^\star & d_{22} \end{bmatrix}$$

has one strictly positive eigenvalue and one strictly negative eigenvalue. Standard choices for $\mathcal{D}$ are the left half-plane ($d_{11} = 0, d_{12} = 1, d_{22} = 0$) and the unit disk ($d_{11} = -1, d_{12} = 0, d_{22} = 1$). Other choices of scalars d_{11}, d_{12} and d_{22} correspond to arbitrary half-planes and disks.

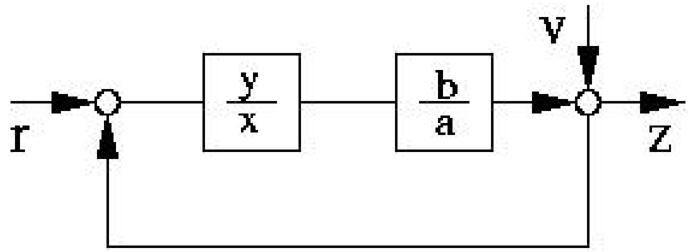

Figure 1: Standard feedback configuration.

The above H_∞ design paradigm covers all the standard frequency domain specifications arising in scalar control problems. For example, in the feedback system of figure 1 the sensitivity of the control system output z to disturbances v is characterized by the sensitivity function

$$S = \frac{1}{1 + \frac{b}{a}\frac{y}{x}} = \frac{ax}{ax + by}$$

where plant polynomials a and b are given, and controller polynomials x and y must be found, see [K93]. As shown in [DFT92], robustness of the closed-loop plant to model uncertainty may be characterized by the complementary sensitivity function

$$T = 1 - S = \frac{by}{ax + by}$$

which is also the closed-loop system transfer function. As recalled in [APH98], simplified yet sensible design specifications for a control law can be formulated as

$$\|S| \ < \gamma_S, \quad \|T\| < \gamma_T$$

where typical values of γ_S range between 1.2 and 2.0 and typical values of γ_T range between 1.0 and 1.5. This H_∞ control problem, as well as many others, can be formulated using the general paradigm proposed above.

3 H_∞ design technique

Dropping for convenience the dependence on polynomial indeterminate s and index k, the H_∞ design inequality of problem 1 on polynomials x_i

$$\left\| \frac{\sum_i n_i x_i}{\sum_i d_i x_i} \right\|_\infty < \gamma$$

can be written equivalently as

$$\mathrm{Re}\left(\frac{\gamma + \frac{\sum_i n_i x_i}{\sum_i d_i x_i}}{\gamma - \frac{\sum_i n_i x_i}{\sum_i d_i x_i}} \right) = \mathrm{Re}\left(\frac{\gamma(\sum_i d_i x_i) + \sum_i n_i x_i}{\gamma(\sum_i d_i x_i) - \sum_i n_i x_i} \right) > 0 \tag{2}$$

where Re denotes the real part of a complex number. In the above inequalities it is implicit that polynomial indeterminate s describes the stability boundary, so that all the polynomials are frequency-dependent complex numbers when $s \in \partial\mathcal{D}$.

In order to simplify notations, define

$$n = \gamma(\sum_i d_i x_i) + \sum_i n_i x_i, \quad d = \gamma(\sum_i d_i x_i) - \sum_i n_i x_i \tag{3}$$

and notice that strict positive realness requirement (2)

$$\mathrm{Re}\,\frac{n}{d} = \frac{1}{2}\left(\frac{n}{d} + \frac{n^\star}{d^\star}\right) = \frac{\mathrm{Re}\,n\,\mathrm{Re}\,d + \mathrm{Im}\,n\,\mathrm{Im}\,d}{\|d\|_2^2} > 0$$

is equivalent to the geometric argument condition

$$\cos{(n,d)} = \frac{\mathrm{Re}\,n\,\mathrm{Re}\,d + \mathrm{Im}\,n\,\mathrm{Im}\,d}{\|n\|_2\|d\|_2} > 0$$

or

$$|(n,d)| < \frac{\pi}{2} \tag{4}$$

where (n,d) denotes the angle between complex numbers n and d.

Now introduce an auxiliary polynomial c, referred to as the central polynomial for reasons that should become clear later on.

Lemma 1 *Geometric condition (4) is equivalent to the existence of a central polynomial c such that*

$$|(n,c)| < \frac{\pi}{4}, \quad |(d,c)| < \frac{\pi}{4}$$

or equivalently

$$2\cos^2(n,c) > 1, \quad 2\cos^2(d,c) > 1. \tag{5}$$

Proof: Suppose first that polynomial c exists such that inequalities (5) are satisfied. Geometrically, it follows that the angle between n and d never exceeds $\pi/2$, which is inequality (4). Conversely, just choose $c = n + d$ as a valid central polynomial, and then inequalities (5) hold. □

Lemma 2 *Inequalities (5) hold if and only if*

$$\begin{bmatrix} \mathrm{Re}\,n^\star c & \mathrm{Im}\,n^\star c \\ \mathrm{Im}\,n^\star c & \mathrm{Re}\,n^\star c \end{bmatrix} \succeq 0, \quad \begin{bmatrix} \mathrm{Re}\,d^\star c & \mathrm{Im}\,d^\star c \\ \mathrm{Im}\,d^\star c & \mathrm{Re}\,d^\star c \end{bmatrix} \succeq 0 \tag{6}$$

where $\succeq 0$ means positive semidefinite.

Proof: The first inequality in (5) can be written explicitly as

$$2(\mathrm{Re}\,n\,\mathrm{Re}\,c + \mathrm{Im}\,n\,\mathrm{Im}\,c)^2 > (\mathrm{Re}^2 n + \mathrm{Im}^2 n)(\mathrm{Re}^2 c + \mathrm{Im}^2 c)$$

or equivalently

$$(\mathrm{Re}\,n\,\mathrm{Re}\,c + \mathrm{Im}\,n\,\mathrm{Im}\,c)^2 > (\mathrm{Re}\,n\,\mathrm{Im}\,c - \mathrm{Im}\,n\,\mathrm{Re}\,c)^2.$$

Using a Schur complement argument, this can be reformulated as a 2-by-2 positive semidefiniteness constraint

$$\begin{bmatrix} \mathrm{Re}\, n\, \mathrm{Re}\, c + \mathrm{Im}\, n\, \mathrm{Im}\, c & \mathrm{Re}\, n\, \mathrm{Im}\, c - \mathrm{Im}\, n\, \mathrm{Re}\, c \\ \mathrm{Re}\, n\, \mathrm{Im}\, c - \mathrm{Im}\, n\, \mathrm{Re}\, c & \mathrm{Re}\, n\, \mathrm{Re}\, c + \mathrm{Im}\, n\, \mathrm{Im}\, c \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\, n^\star c & \mathrm{Im}\, n^\star c \\ \mathrm{Im}\, n^\star c & \mathrm{Re}\, n^\star c \end{bmatrix} \succeq 0.$$

The second matrix inequality in (6) is obtained similarly. □

Defining the 2-by-2 polynomial matrices

$$N(s) = \begin{bmatrix} n(s) & 0 \\ 0 & n(s) \end{bmatrix}, \quad D(s) = \begin{bmatrix} d(s) & 0 \\ 0 & d(s) \end{bmatrix}, \quad C(s) = \begin{bmatrix} c(s) & c(s) \\ -c(s) & c(s) \end{bmatrix}$$

inequalities (6) can also be written as

$$N^\star(s)C(s) + C^\star(s)N(s) \succeq 0, \quad D^\star(s)C(s) + C^\star(s)D(s) \succeq 0 \tag{7}$$

for $s \in \partial\mathcal{D}$. Inequalities (7) are positivity conditions on polynomial matrices.

Controller parameters, i.e. coefficients of polynomials $x_i(s)$, enter linearly in polynomials $n(s)$ and $d(s)$, as well as in polynomial matrices $N(s)$ and $D(s)$. So it means that as soon as central polynomial c is given, positivity conditions (7) are linear in design parameters. Positivity conditions on polynomial matrices depending linearly on design parameters can be formulated as LMIs as follows.

Let

$$N = \begin{bmatrix} N_0 & N_1 & \cdots & N_\delta \end{bmatrix}, \quad D = \begin{bmatrix} D_0 & D_1 & \cdots & D_\delta \end{bmatrix}, \quad C = \begin{bmatrix} C_0 & C_1 & \cdots & C_\delta \end{bmatrix}$$

denote matrix coefficients of powers of indeterminate s in polynomial matrices $N(s)$, $D(s)$ and $C(s)$ respectively, where δ is the highest degree arising in polynomials $n(s)$, $d(s)$ and $c(s)$. Define

$$\Pi = \begin{bmatrix} I_2 & & & 0 \\ & \ddots & & \vdots \\ & & I_2 & 0 \\ 0 & I_2 & & \\ \vdots & & \ddots & \\ 0 & & & I_2 \end{bmatrix}$$

as a matrix of size 4δ-by-$2(\delta+1)$, together with the linear mapping

$$H(P) = \Pi^T(H \otimes P)\Pi = \Pi^T \begin{bmatrix} aP & bP \\ b^\star P & cP \end{bmatrix} \Pi$$

where square matrix P has dimension 2δ and $\otimes$ denotes the Kronecker product. Then the following result is a corollary of lemma 2 in [HPA02].

Lemma 3 *Given polynomial matrix $C(s)$, polynomial matrices $N(s)$ and $D(s)$ satisfy positivity conditions (7) if and only if there exist matrices $P_n = P_n^\star$ and $P_d = P_d^\star$ such that*

$$N^\star C + C^\star N - H(P_n) \succ 0, \quad D^\star C + C^\star D - H(P_d) \succ 0. \tag{8}$$

Repeating this argument on all the frequency domain specifications (1), we obtain the following central result:

Theorem 1 *Given polynomials $n_i^k(s)$, $d_i^k(s)$, positive scalars γ^k and central polynomials $c^k(s)$ for $k = 1, 2, \ldots$, there exist polynomials $x_i(s)$ solving H_∞ design problem 1 if problem (8) is feasible. This is a convex LMI problem in coefficients of polynomials $x_i(s)$.*

Based on the discussion of [HSK02], central polynomial c plays the role of a target closed-loop characteristic polynomial around which the design is carried out. In particular, setting the degree of central polynomial c also sets the degree of polynomials n and d in (3), as well as the degree of controller polynomials x_i. Sensible strategies for the choice of central polynomial c are discussed in [HSK02], but a general rule of thumb is that open-loop stable poles must be mirrored in the central polynomial, completed by sufficiently fast additional dynamics.

Note finally that, since LMI (8) is a sufficient condition to enforce H_∞ specifications (1), generally these specifications will be satisfied with a certain amount of conservatism, i.e. γ^k is always an upper bound on the actual H_∞ norm in (1) achieved by feedback.

4 Numerical examples

The H_∞ design technique of theorem 1 has been implemented in a documented Matlab 6.5 m-file available at

`http://www.laas.fr/~henrion/software/hinfdes`

that will be included to the next release 3.0 of the commercial Polynomial Toolbox [P00] for Matlab. An alternate code that does not require the Polynomial Toolbox can be obtained by contacting the author.

The numerical examples were treated with the help of Matlab 6.5 running under SunOS release 5.8 on a SunBlade 100 workstation. Operations on polynomials were performed with the Polynomial Toolbox 2.5 [P00]. The LMI problems were solved with SeDuMi 1.05 [S99] with default tuning parameters, interfaced with a beta version 1.0 of LiMaCOS [LPH02].

4.1 Optimal robust stability

Consider the optimal robust stability problem of section 11.1 in [DFT92], where the open-loop plant in figure 1 is given by

$$\frac{b}{a} = \frac{s-1}{(s+1)(s-0.5)}$$

and we seek a controller y/x minimizing γ_T under the following weighted H_∞ constraint on the closed-loop transfer function

$$\|WT\|_\infty = \left\| \left(\frac{s+0.1}{s+1}\right) \left(\frac{by}{ax+by}\right) \right\|_\infty < \gamma_T.$$

The following Matlab code seeks a first order controller for $\gamma_T = 1.9$:

```
a = (s+1)*(s-0.5); b = (s-1); gammaT = 1.9;
c = (s+0.1)*(s+1)^2*(s+3); % central polynomial
lmi = hinfdes([],'init',[1 1]); % seek first order controller
lmi = hinfdes(lmi,(s+0.1)*[0 b],(s+1)*[a b],c,gammaT); % H-inf spec
out = hinfdes(lmi,'solve'); % solve LMI
x = out(1); y = out(2);
```

Central polynomial c is the key design parameter, and together with upper bound γ_T they capture the whole degrees of freedom. Roots in c are just an indication on where closed-loop poles should be located: generally, roots of characteristic polynomial $ax + by$ will be located around roots of c, but they may also differ significantly due to structural constraints. The H_∞ design procedure then consists in iteratively playing with the roots of c, while lowering upper bound γ_T.

In table 1 we show different choices of roots for c, denoted by $\sigma(c)$ (4 roots = 2 for the open-loop system, 1 for the weighting function, 1 for the controller), together with actual poles of closed-loop transfer function T (3 roots) denoted by $\sigma(ax + by)$, upper bounds γ_T and the actual weighted norms $\|WT\|_\infty$ achieved by the computed controllers. Each design requires about 1 second of CPU time on our computer.

$\sigma(c)$	$\sigma(ax+by)$	γ_T	$\|WT\|_\infty$
-1,-1,-1,-1	$-1.04 \pm i1.08$, -0.230	2.9	2.11
-1,-1,-1,-0.1	$-0.731 \pm i0.566$, -0.118	2.3	1.74
-2,-1,-1,-0.1	$-1.133 \pm i0.586$, -0.114	2.1	1.54
-3,-1,-1,-0.1	$\mathbf{-1.383 \pm i0.642}$, **-0.0932**	**1.9**	**1.47**
-10,-1,-1,-0.1	-6.775, -1.063, -0.1059	1.8	1.31
-500,-1,-1,-0.1	-1700, -0.992, -0.103	1.7	1.21

Table 1: Optimal robust stability. Roots of central polynomial, characteristic polynomial, H_∞ upper bound and achieved H_∞-norm.

We can see that a good strategy is to start with a central polynomial with all its roots in -1, and a loose upper bound on γ_T. Decreasing γ_T, some closed-loop poles move away from -1, which gives indications on how to move roots of the central polynomial. At the bottom of the table, we can see that by allowing a very fast root in the central polynomial, γ_T can be decreased significantly close to the theoretical infimum of 1.20. Yet the closed-loop system also features a very fast pole, and the resulting controller $y/x = (-2046.2 - 2039.7s)/(3744.0 + s)$ results impractical.

A good tradeoff here is indicated in boldface letters in table 1, where a weighted H_∞-norm of 1.47 is achieved with the first-order controller

$$\frac{y}{x} = \frac{-3.0456 - 3.2992s}{5.6580 + s}.$$

Note however that the sensitivity function has very poor norm $\|S = 1 - T\|_\infty = 13.1$, due to the fact that no specifications were enforced on S. As a result, the above controller can be very sensitive to perturbations, or fragile, as pointed out in [KP97].

A more sensible design approach would then enforce an additional H_∞ specification on S, such as

$$\|S\|_\infty = \left\| \frac{ax}{ax + by} \right\|_\infty < \gamma_S$$

for some suitable value of γ_S. However, as shown in [A00], for this numerical example the ratio between the unstable open-loop pole and zero is small so there is no controller that will give a reasonably robust closed-loop system.

Adding a line to the above Matlab code to enforce an additional specification on $\|S\|_\infty$, we obtain (after about 2 seconds of CPU time) with $c(s) = (s+1)^3(s+100)$, $\gamma_T = 4$ and $\gamma_S = 4$ the following first-order controller

$$\frac{y}{x} = \frac{-873.30 - 816.37s}{1202.4 + s}$$

producing $\|S\|_\infty = 3.44$ and $\|WT\|_\infty = 2.24$.

4.2 Flexible beam

Consider the flexible beam example of section 10.3 in [DFT92]. The open-loop plant in figure 1 is given by

$$\frac{b}{a} = \frac{-6.4750s^2 + 4.0302s + 175.7700}{5s^4 + 3.5682s^3 + 139.5021s^2 + 0.0929s}.$$

For the closed-loop plant to approximate a standard second-order system with settling time at 1% of 8 seconds and overshoot less then 10%, the following frequency domain specification on the weighted sensitivity function is enforced in [DFT92]:

$$\|WS\|_\infty = \left\| \left(\frac{s^2 + 1.2s + 1}{s(s+1.2)} \right) \left(\frac{ax}{ax + by} \right) \right\|_\infty < \gamma_S.$$

Suppose we are looking for a second-order controller. The open-loop plant has poles 0, $-0.6660 \cdot 10^{-3}$, and $-0.3565 \pm i5.270$, and the weighting function has poles at 0 and -1.2. The central polynomial must mirror open-loop stable poles, so an initial choice of central polynomial features roots $-0.6660 \cdot 10^{-3}$, $-0.3565 \pm i5.270$, -1.2 plus two roots at -10^{-2} corresponding to the open-loop plant integrator and the weighting function integrator, plus two roots at -1 (arbitrary) corresponding to the controller poles. With this choice of central polynomial and $\gamma_S = 5$ the H_∞ LMI problem is solved in 15 seconds but the resulting step response is too slow.

After a series of attempts, an acceptable step response was obtained with the roots $\sigma(c) = \{-0.6660 \cdot 10^{-3}, -10^{-2}, -0.3565 \pm i5.270, -0.1, -1, -1, -1\}$ corresponding to the central polynomial $c(s) = 0.1858 \cdot 10^{-4} + 0.3000 \cdot 10^{-1}s + 3.178s^2 + 37.33s^3 + 94.06s^4 + 90.50s^5 + 33.45s^6 + 3.824s^7 + s^8$. With $\gamma_S = 5$ function `hinfdes` returns the controller

$$\frac{y}{x} = \frac{0.77489 \cdot 10^{-4} + 0.16572 \cdot 10^{-1}s + 0.36537s^2}{0.41025 \cdot 10^{-1} + 1.0437s + s^2}$$

producing

$$\|S\|_\infty = 1.27, \quad \|T\|_\infty = 1.01$$

and a step response with settling time at 1% of 11.3 seconds and overshoot of 4%. Bode magnitude plots of S and T are given in figure 2, and the step response is shown in figure 3. Note that a similar performance was obtained in [DFT92] with a controller of eighth order.

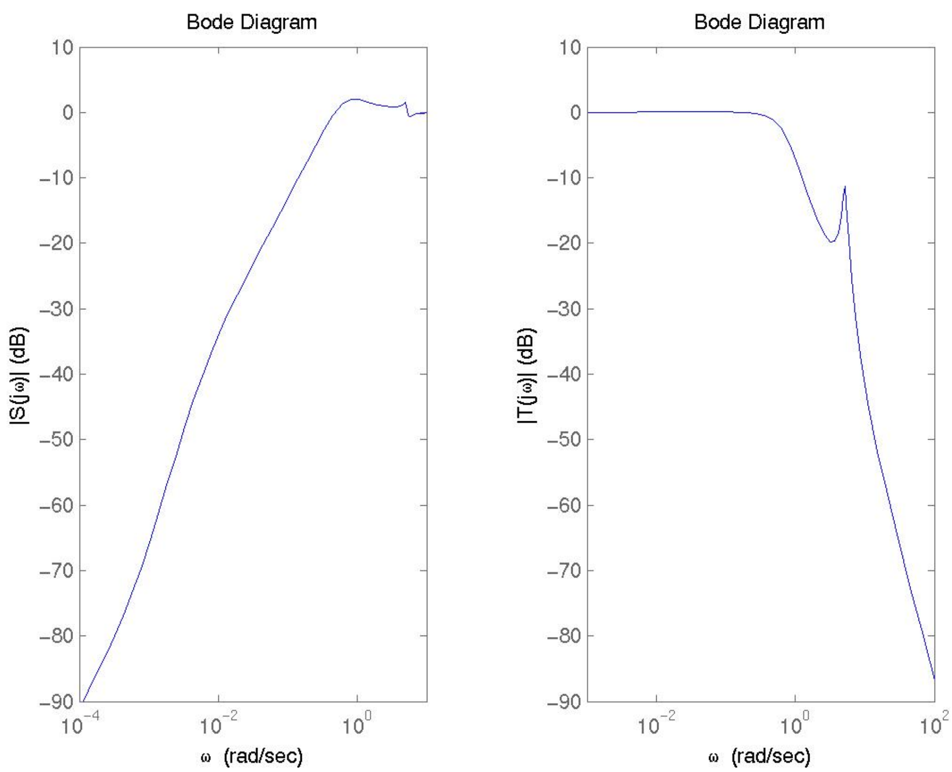

Figure 2: Flexible beam. Bode magnitude plots of S and T.

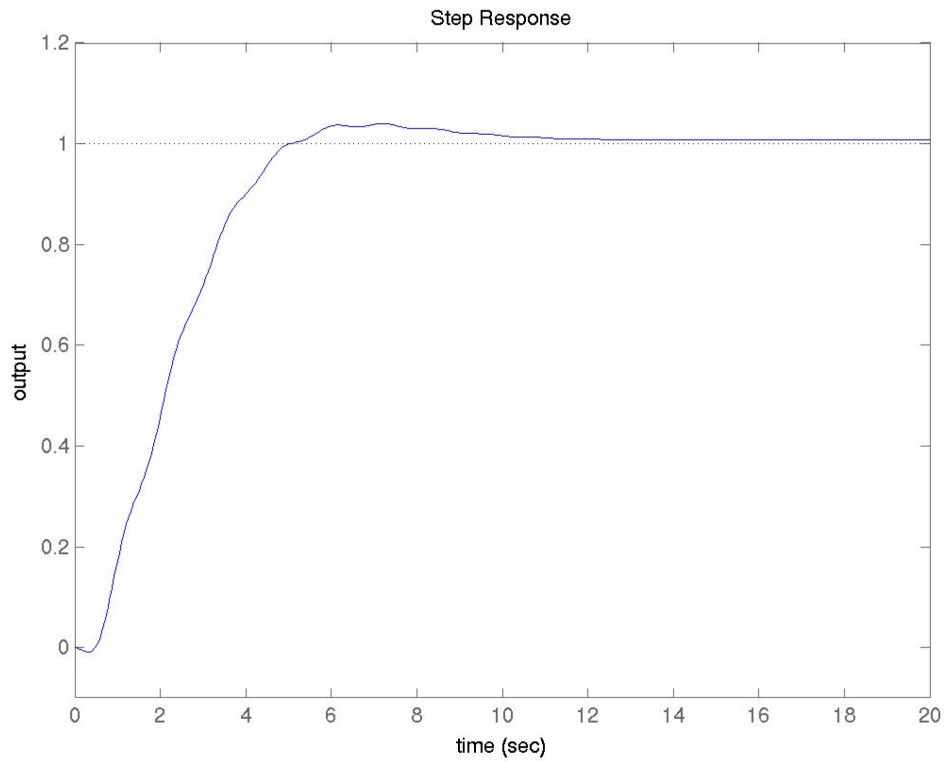

Figure 3: Flexible beam. Step response.

5 Conclusion

We have proposed an iterative H_∞ design technique where all the degrees of freedom are on the choice of a central polynomial, or desired closed-loop characteristic polynomial around which the design is carried out. Contrary to most of the existing H_∞ optimization techniques, the order of the controller is fixed from the very outset, independently of the order of the plant or weighting functions. Our H_∞ design method is based on results on positive polynomial matrices and convex optimization over LMIs. As a result, it could be easily implemented in a Matlab and SeDuMi framework.

Our work can be extended without major technical difficulty to multivariable systems. Related results will be published elsewhere. Combined with closed-loop structural requirements such as decoupling, we believe that it should result in a practical, efficient H_∞ design methodology for linear systems.

Finally, we believe that a promising research direction may be the study of numerical properties (computational complexity, numerical stability) of algorithms tailored to solve the structured LMI problems arising from the theory of positive polynomials and polynomial matrices. As shown in [GHN02], the Hankel or Toeplitz structure can be exploited to design fast algorithms to solve Newton steps in barrier schemes and interior-point algorithms. Numerical stability is also a concern, since it is well-known for example that

Hankel matrices are exponentially ill-conditioned. Alternative polynomial bases such as Chebyshev or Bernstein polynomials may prove useful.

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