A Branch and Cut Algorithm for the Minimum Connected Dominating Set Problem

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1 Introduction

Given an undirected graph $G = (V, E)$, a subset W of V is a dominating set of G if, for every $i \in V \setminus W$, there exists an edge $e \in E$, adjacent to i, with an endpoint $j \in W$. The dominating set is connected if the subgraph it induces in G is connected and the Minimum Connected Dominating Set Problem (MCDSP) is to find a connected dominating set with as few vertices as possible. MCDSP and the very closely related Maximum Leaf Spanning Tree Problem (MLSTP) have been suggested in the literature as a model to, among others, ad-hoc wireless networks and fiber optics networks where regenerators must be placed.

In our presentation we emphasize the close links that exist between MCDSP and MLSTP and review the applications suggested for the two problems. We also present a new formulation for MCDSP and describe a branch and cut algorithm based on this formulation. Computational results are presented, showing this algorithm to be competitive with the best exact solution algorithms available for MCDSP and MLSTP.

2 A formulation for MCDSP

The idea behind our formulation is to identify a minimum connected dominating set $W \subset V$ by exhibiting a spanning tree for the subgraph of G induced by W. Accordingly, associate variables $\{y_i : i \in V\}$ and $\{x_e : e \in E\}$ respectively to the vertices and edges of G and consider a polyhedral region $\mathcal{R}_0$ defined as

$$\sum_{e \in E} x_e = \sum_{i \in V} y_i - 1 \tag{1}$$

$$\sum_{e \in E(S)} x_e \leq \sum_{i \in S \setminus \{j\}} y_i, \quad S \subset V, j \in S \tag{2}$$

$$y_i + \sum_{j \in \Gamma_i} y_j \geq 1, \ \ i \in V \tag{3}$$

$$x_e \geq 0, \ \ e \in E \tag{4}$$

$$0 \leq y_i \leq 1, \ \ i \in V. \tag{5}$$

A formulation for MCDSP is then given by

$$\left\{ \min \sum_{i \in V} y_i : (\mathbf{x}, \mathbf{y}) \in \mathcal{R}_0 \cap (\mathbb{R}_+^{|E|}, \mathbb{Z}^{|V|}) \right\}, \tag{6}$$

where inequalities (3) guarantee that every vertex selected for W covers itself and that every vertex in $V \setminus W$ is covered by a vertex of W. Constraints (1),(2), (4) and (5) with an additional condition that ensures the variables are integers characterize the tree polytope.

Inequalities (3) could be strengthened if one restricts oneself to connected dominating sets involving at least 2 vertices. This could indeed be done without loss of generality since existence of dominating sets of cardinality one could be efficiently checked. Assuming this to be the case, one could work with a substantially stronger formulation $\mathcal{R}_1$ where inequalities (3) are replaced in (1)-(6) by inequalities

$$\sum_{j \in \Gamma_i} y_j \geq 1, \ \ i \in V. \tag{7}$$

Our Branch and Cut algorithm for MCDSP exploits the structures embedded in $\mathcal{R}_1$ and also uses cuts separated from the associated first Chvatal closure.