Solving a two-level lot sizing problem with bounded inventory

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We consider a two-level lot sizing problem where the first level consists of N end products competing for a single type of raw material (second level), which is supposed to be critical. In particular, the storage capacity for the raw material is limited and must be carefully managed. The goal is to simultaneously determine an optimal replenishment plan for the raw material and optimal production plans for the end products on a horizon of T periods.

The problem is first modeled with two integer linear programs, based on the well-known aggregate and disaggregate formulations of lot-sizing problems. By relaxing, in the aggregate model, the constraints linking the raw material inventory and the production plan of the end products, a Lagrangian relaxation model can be derived. The relaxed problem is decomposed into N classical single-item Uncapacitated Lot-Sizing problem (ULS) and a particular single item problem that can be solved analytically. Each ULS sub-problem is solved using an $O(T \log T)$ dynamic programming algorithm ([1]). Some valid inequalities are proposed to improve the Lagrangian based lower bound. A heuristic is then presented to construct a feasible solution at each iteration of the iterative algorithm used to determine the Lagrangian dual.

Extensive numerical tests are performed on randomly generated instances to compare the straightforward resolution of the integer linear programming models with a standard solver with our Lagrangian heuristic. The execution time of the Lagrangian heuristic being always less than 0.1 sec., we set the maximum execution time of the standard solver to 10 sec. The numerical results show that the Lagrangian heuristic gives better (smaller) gaps than the aggregate formulation, with an average gap of 9.3% for the Lagrangian approach and 12.0% for the standard solver. The worst gaps are obtained with the disaggregate formulation (with an average gap of 50.89%). These results show that the Lagrangian relaxation approach is promising, and new ideas are studied for further improvements.

Références

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