Robust Ordinal Regression for Outranking Methods

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1 Introduction

Ordinal regression (or disaggregation-aggregation approach) is a method used in Multiple Criteria Decision Aiding (MCDA) for inducing some parameters of a preference model, like a value function, able to represent a set of holistic preference information provided by the Decision Maker (DM). Usually, among the many sets of parameters of a preference model representing the preference information, only one specific set is used to give a recommendation on a set of actions. For example, among many value functions representing pairwise comparisons of some actions made by the DM, only one value function is finally used to recommend the best choice, or sorting, or ranking of actions. Since the choice of one among many sets of parameters compatible with the preference information is rather arbitrary, *robust ordinal regression* has been recently proposed with the aim of taking into account all the sets of parameters compatible with the preference information given by the DM [1, 4, 3, 2]. In this presentation, we show how to apply the robust ordinal regression to MCDA methods which use an outranking relation as a preference model. The outranking relation has been used in the well-known family of ELECTRE methods [6, 7]. We consider the case of a single DM and the case of a group of DMs.

2 Problem Statement and the Principle of Robust Ordinal Regression

We consider an MCDA problem concerning a finite set of actions $A = \{a, b, \ldots, m\}$, evaluated on n criteria from a consistent family $F = \{g_1, \ldots, g_n\}$, with $g_i\colon A \to \mathbb{R}, i = 1, \ldots, n$. With respect to set A, a single DM, or a group of DMs, may wish to get recommendation on one of the following questions : (i) what is the subset of best actions in A, (ii) how to assign actions from A to pre-defined and preference ordered classes, or (iii) how to rank the actions from A from the best to the worst. It is well known that the only objective information coming out from the above problem statement is a dominance relation in set A. Being a partial pre-order, the dominance relation is poor, however, that is it leaves many actions incomparable. In order to "enrich" this relation, the analyst must learn more about a value system of the DM (DMs), so as to be able to construct the DM's preference model. This model can then be used in order to work out a recommendation with respect to one of the above mentioned questions.

Among many preference models considered in the literature, there is an *outranking relation*, which has been used in the family of ELECTRE methods. Like other models, also the outranking relation model is defined using a set of parameters among which there are importance weights, and indifference, preference and veto thresholds. A direct elicitation of these parameters by a DM requires, however, a hard cognitive effort from the DM. This is why research has been focused on an indirect elicitation of preference model parameters via decision examples provided by the DM [5]. Such a reverse search of preference model parameters from decision examples corresponds to ordinal regression.

The robust ordinal regression approach extends the simple ordinal regression by taking into account not a single instance of the preference model compatible with DM's preference information, but the whole set of compatible instances of the preference model. In result of considering the whole set of compatible instances of the preference model, one gets two kinds of results with respect to each pair of actions $a, b \in A$:

– *necessary preference relation* $a \succsim^N b$, if and only if a is at least as good as b according to all instances of the preference model compatible with the preference information,
– *possible preference relation* $a \succsim^P b$, if and only if a is at least as good as b according to at least one instance of the preference model compatible with the preference information.

The necessary preference relation can be considered as *robust* with respect to the preference information. The robustness of the necessary preference relation refers to the fact that a given pair of actions compares in the same way whatever the instance of the preference model compatible with the preference information. Indeed, when no preference information is given, the necessary preference relation boils down to the dominance relation, and the possible preference relation is a complete relation. Every new item of the preference information, e.g., a pairwise comparison of some reference actions for which the dominance relation does not hold, is enriching the necessary preference relation and it is impoverishing the possible preference relation, so that they converge with the growth of the preference information. Such an approach gives also space for *interactivity* with the DM.

From computational point of view, the robust ordinal regression for the outranking methods requires to solve a series of small-size 0-1 mixed linear programs.

Références

[1] S. Greco, V. Mousseau, R. Słowiński. Ordinal regression revisited : multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):415–435, 2008.

[2] S. Greco, V. Mousseau, R. Słowiński. The possible and the necessary for multiple criteria group decision. [In] : F.Rossi, A.Tsoukias (eds.), *Algorithmic Decision Theory.* LNAI 5783, Springer-Verlag, Berlin, 2009, pp. 203-214.

[3] S. Greco, R. Słowiński, J. Figueira, V. Mousseau. Robust ordinal regression. [In] : M. Ehrgott, J. Figueira, S. Greco, (eds.), *New Advances in Multiple Criteria Decision Analysis.* Springer-Verlag, Berlin, 2009, to appear.

[4] J. Figueira, S. Greco, R. Słowiński. Building a set of additive value functions representing a reference preorder and intensities of preference : GRIP method. *European Journal of Operational Research*, 195(2):460–486, 2009.

[5] V. Mousseau, R. Słowiński. Inferring an ELECTRE TRI model from assignment examples. *Journal of Global Optimization* 12(2):157–174, 1998.

[6] B. Roy. *Méthodologie Multicritère d'Aide à la Décision*, Economica, Paris, 1985.

[7] B. Roy, D. Bouyssou. *Aide Multicritère à la Décision : Méthodes et Cas*, Economica, Paris, 1993.