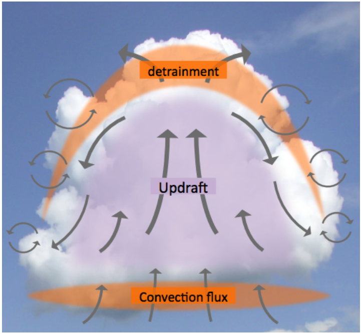
# **Adaptive Sampling of Clouds with a Fleet of UAVs**: Improving Gaussian Process Regression by Including Prior Knowledge

# Motivation: SkyScanner Project

**Adaptive Sampling of Cumulus Clouds with a Fleet of UAVs:**

Clouds remain an uncertainty in current atmospherical models:

- Characterize the evolution of parameters (3D wind, liquid water content, etc.)



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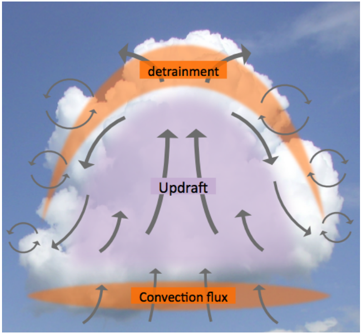

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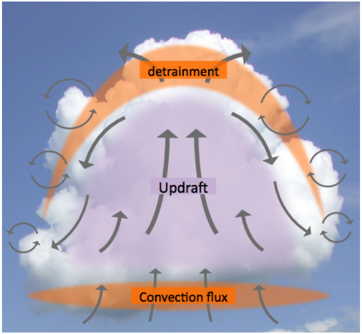

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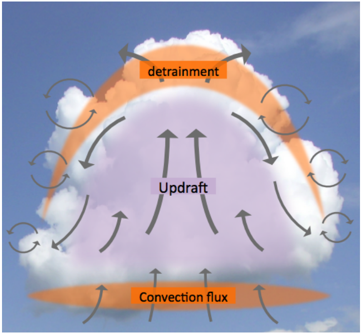
    $\longrightarrow$ quantification of uncertainty
  - Energy efficiency
    $\longrightarrow$ mapping and exploiting vertical wind.



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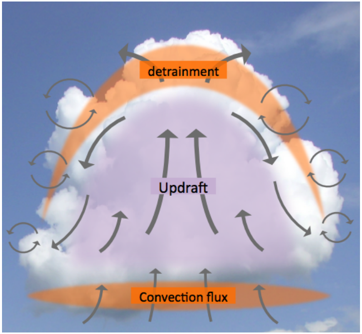


$\rightarrow$ **Gaussian Process Regression**

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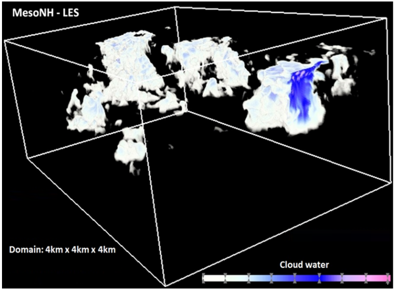


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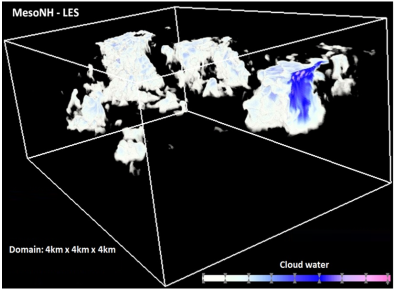


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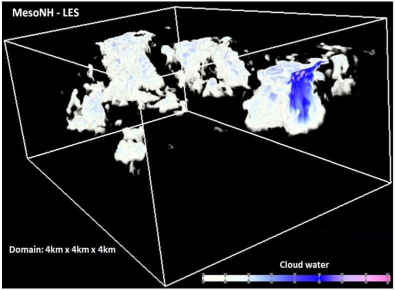


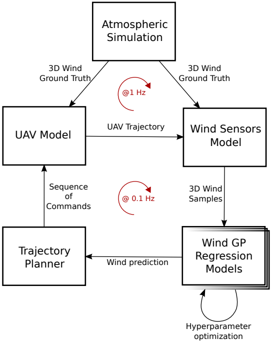


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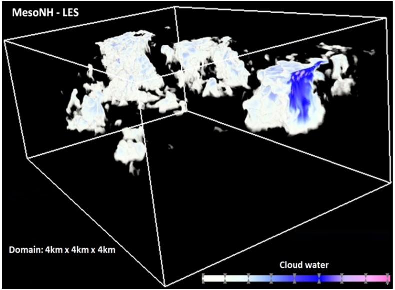


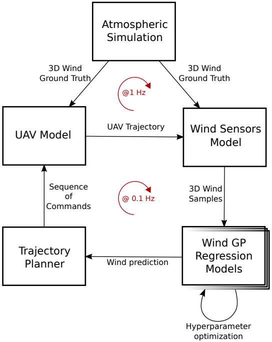


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$\rightarrow$ Wind predictions needed under real-time constraints

# *Introduction to Gaussian Process Regression*

- Bayesian Machine Learning framework
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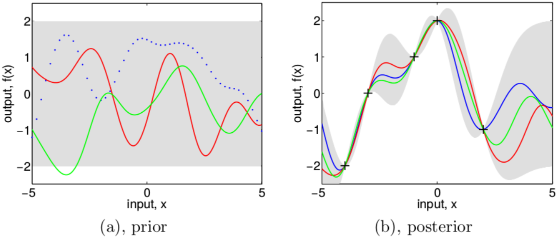

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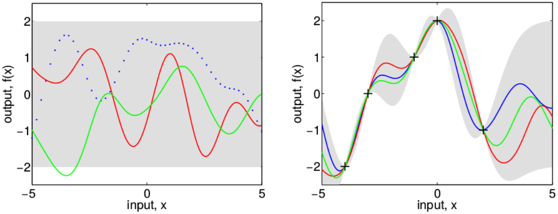


(a), prior (b), posterior

**Two key ingredients**

- Mean function $m(\mathbf{x})$: Center for the distribution of functions
- Covariance function, matrix $k(\mathbf{x}, \mathbf{x}')$, $\Sigma$:
  Defines smoothness and variability. Quantifies similarity.
  If $\mathbf{x}$,$\mathbf{x}'$ similar $\longrightarrow$ outputs similar

**Making predictions**

With training data: $\mathbf{X}, \mathbf{Y}$ | new input vector $\mathbf{x}_\star$| mean function $m(\mathbf{x})$ | covariance matrices $\Sigma_{\mathbf{X},\mathbf{X}} = [k(\mathbf{x}_i, \mathbf{x}_i)]\,,\; i, j = 1, ..., n$ | $\Sigma_{\mathbf{x}_\star,\mathbf{X}} = [k(\mathbf{x}_\star, \mathbf{x}_i)]\,,\; i = 1, ..., n$ |

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$$p\left(y_\star|\mathbf{x}_\star, \mathbf{X}, \mathbf{Y}\right) = \mathcal{N}\left(\bar{y}_\star, \mathbb{V}[y_\star]\right), \quad (1)$$

$$\bar{y}_\star = m(\mathbf{x}_\star) + \Sigma_{\mathbf{x}_\star,\mathbf{X}}\Sigma_{X,X}^{-1}(\mathbf{Y} - m(\mathbf{X})), \quad (2)$$

$$\mathbb{V}[y_\star] = k(\mathbf{x}_\star, \mathbf{x}_\star) - \Sigma_{\mathbf{x}_\star,\mathbf{X}}\Sigma_{X,X}^{-1}\Sigma_{\mathbf{x}_\star,\mathbf{X}}^{T} \quad (3)$$

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**Advantages of GPR**

- Inbuilt estimation of uncertainty adapted to test inputs

**Limitations**

- Mean function and covariance function are parameterized
  $\longrightarrow$ Expensive optimization, usually Bayesian Marginal Log-Likelihood (several iterations of $\mathcal{O}(n^3)$)
- With no prior knowledge about process, "off-the-shelf":
  $\longrightarrow m(\mathbf{x}) = 0, \ k(\mathbf{x}, \mathbf{x}') = \sigma^2 \exp\left(\frac{-0.5|\mathbf{x} - \mathbf{x}'|^2}{l^2}\right)$

**Types of prior knowledge to improve GPR:**

1. Determining the mean function $m(\mathbf{x})$
2. Determining type and parameter distribution of covariance function $k(\mathbf{x}, \mathbf{x}')$
3. If output multidimensional, then determine and exploit correlations between outputs

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**Approaches to determine prior knowledge**

- *Brute Force*:
  - Cross-validate implementations that combine several mean-functions, covariance functions and output-correlation structures
    $\longrightarrow$ No real understanding about the process
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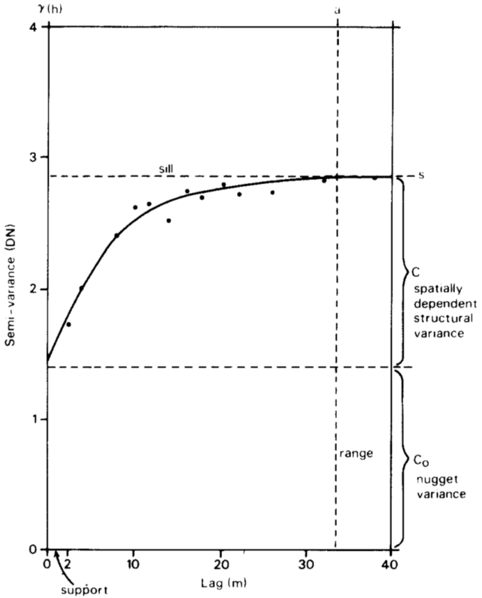
- *Spatial Statistics, Geostatistics*:
  - Estimate statistics from data and do regular curve fitting on these statistics to infer the priors
    $\longrightarrow$ Computational complexity: statistics $\mathcal{O}(n)$, curve fitting $\mathcal{O}(m^3)$, $m << n$

# Spatial Statistics: The Variogram

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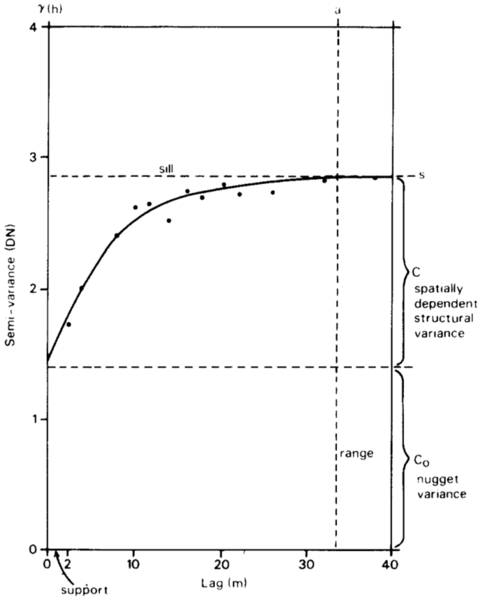


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- With assumptions *Stationarity* and *Isotropy*: $2\gamma(h)$, distance $h = |\mathbf{x} - \mathbf{x}'|$
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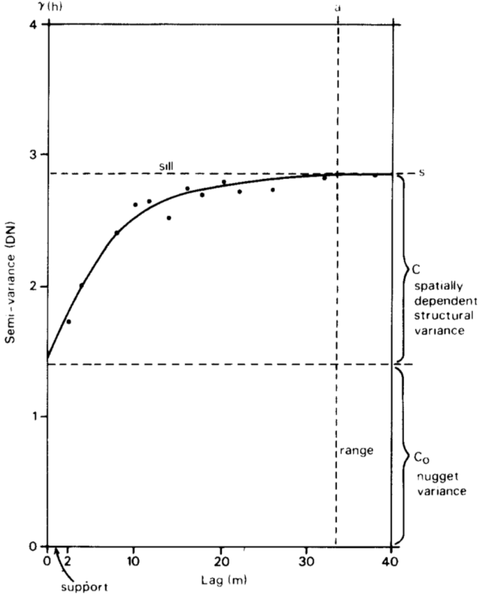


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- Estimated from the data and then fitted with a model
- Basis for spatial prediction in Geostatistics, i.e. *Kriging*
- Non-converging empirical variograms indicate problems with *Stationarity*

**Estimating**

$$2\hat{\gamma}(\mathbf{h}) \equiv \frac{1}{|N(\mathbf{h})|} \sum_{N(\mathbf{h})} (Z(\mathbf{s}_i) - Z(\mathbf{s}_j))^2, \mathbf{h} \in \mathbb{R}^d, \tag{4}$$

$$N(\mathbf{h}) \equiv \{(\mathbf{s}_i, \mathbf{s}_j) : \mathbf{s}_i - \mathbf{s}_j = \mathbf{h}; i, j = 1, ..., n\} \tag{5}$$

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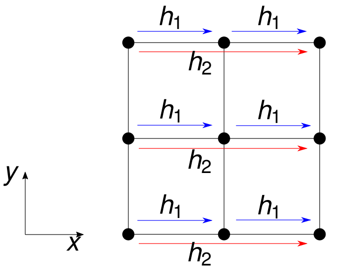

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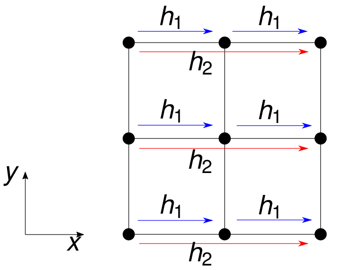


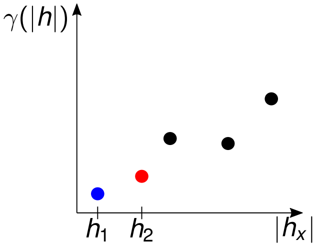

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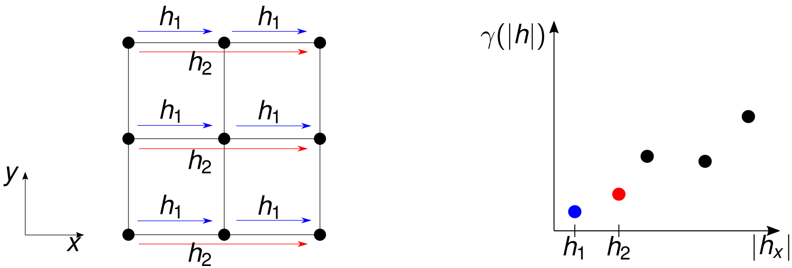


**Fitting**

$$\sum_{j=1}^{k} |N(h(j))| \left(\frac{\hat{\gamma}(h(j))}{\gamma(h(j);\theta)} - 1\right)^2 \quad (6)$$

# Spatial Statistics: The Variogram and Gaussian Process Regression

Converging variogram models and stationary covariance functions are related:

$$\gamma(\mathbf{h}) = k(\mathbf{0}) - k(\mathbf{h}), \tag{7}$$

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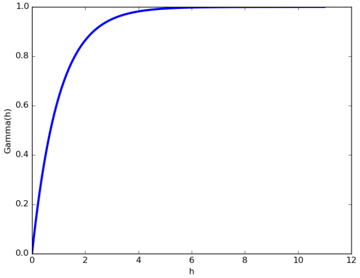
**Examples**:

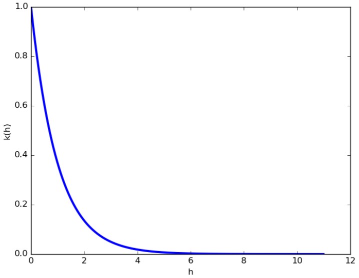
Exponential Variogram

$$\gamma(h) = \sigma^2(1 - \exp(\frac{-|h|}{l}))$$



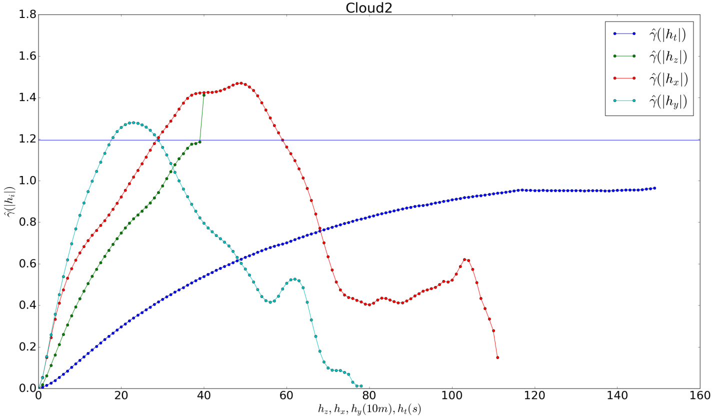

Exponential Covariance Function

$$k(h) = \sigma^2 \exp(\frac{-|h|}{l})$$



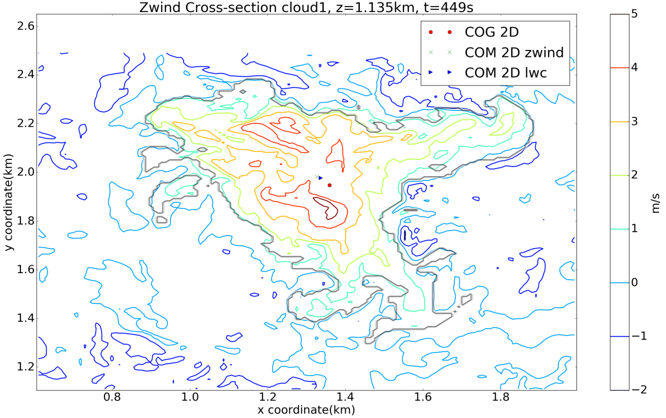
# Implementation: Vertical Wind Empirical Variograms

- 5 Clouds were segmented, and used to estimate variograms in $t, z, x, y$




- Values at big distances are very similar in $x, y$
- Variograms continue to grow over theoretical sill in $x, y$
  $\longrightarrow$ Non-stationarity, mean function?

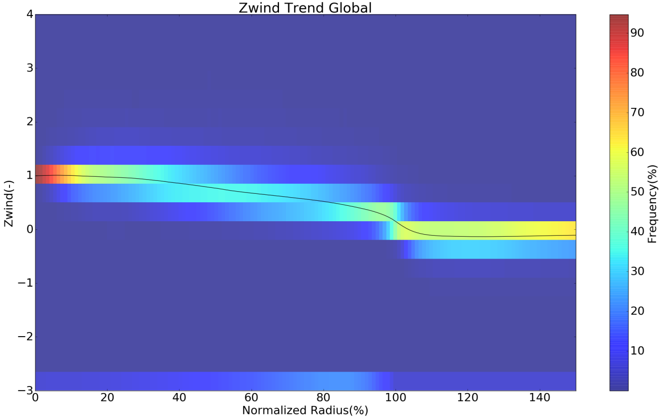
# Implementation: New coordinates




- Polar coordinates based on center of LWC more “natural”
- Vertical winds near the center are higher, near boundaries lower
  $\longrightarrow$ Radial mean function?

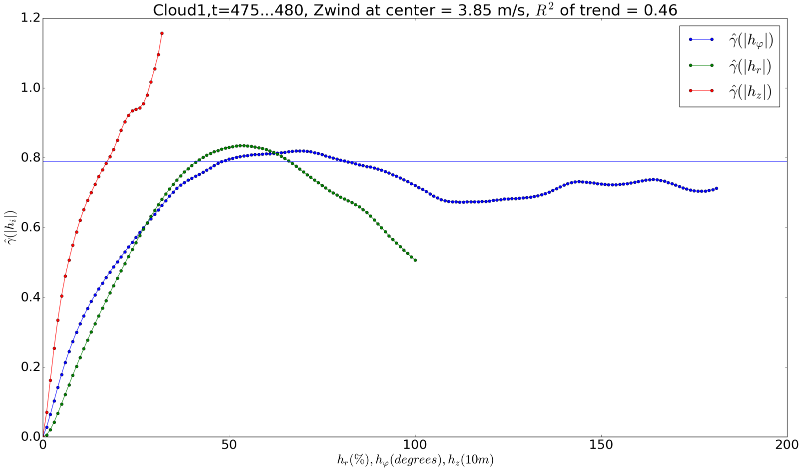
# Implementation: Estimating the Mean Function

- Normalization of radius and vertical wind at center
- Over 300.000 radial trends to estimate the median



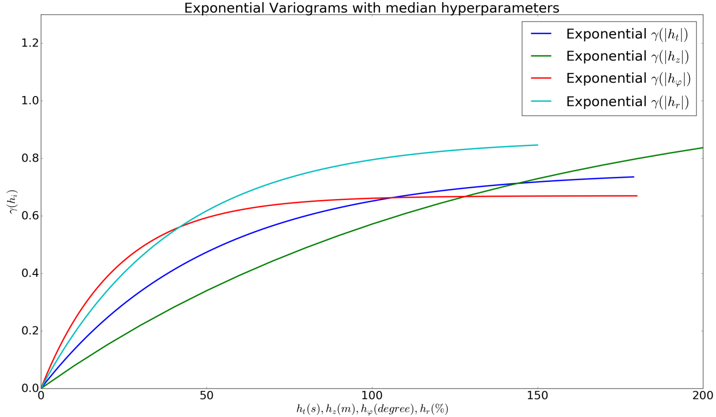
# Implementation: Detrended Empirical Variograms

- Clouds were detrended with the mean function
- New variograms were computed in the four polar directions $t, z, \varphi, r$

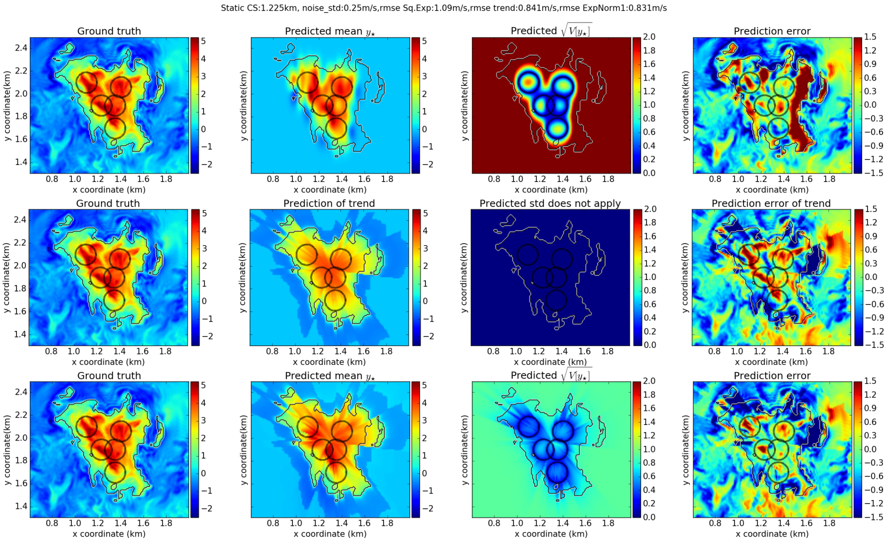


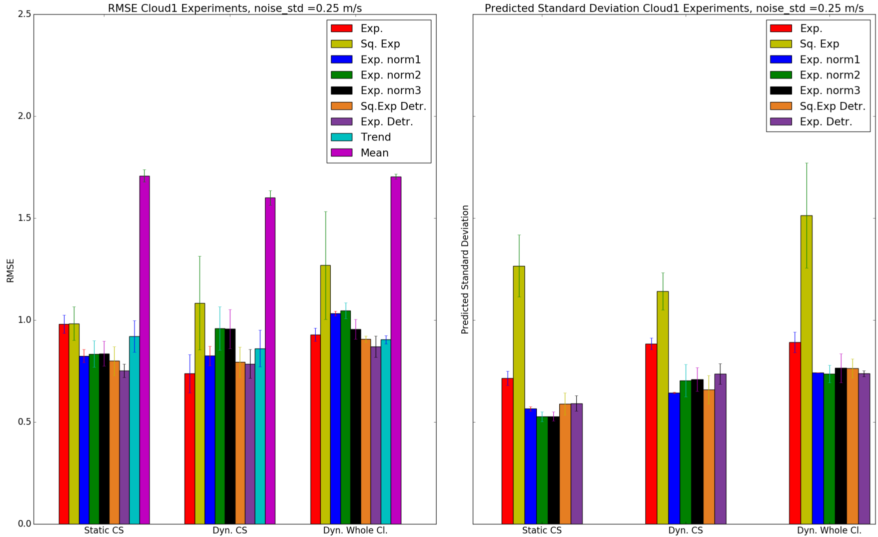
# Implementation: Best Fit Detrended Variograms

- Around 20-30 variograms with detrended vertical wind were fitted
  $\longrightarrow$ Parameters of covariance function
- Out of four possible models tested, Exponential Variogram best fit
- Similarity in sills suggests that range anisotropy is more accentuated
  $\longrightarrow \gamma(|r|), \quad r^2 \equiv \mathbf{h}^T\mathbf{M}\mathbf{h}, \quad \mathbf{M} = diag(1/l_{x_i}^2)$



# Implementation: Testing the new GPR




RMSE Cloud1 Experiments, noise_std =0.25 m/s
RMSE
0.0
0.5
1.0
1.5
2.0
2.5
Static CS
Dyn. CS
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**Summary**

- Prior on mean function ✓
- Prior on covariance function ✓
- Improved performance vs. “off-the-shelf” GPR ✓

**Outlook**

- Repeat line of analysis on other variables, e.g. liquid water content(LWC)
- Exploit correlations between LWC and vertical wind
- Integrate polar coordinates preprocessing to current adaptive sampling scheme

Questions?