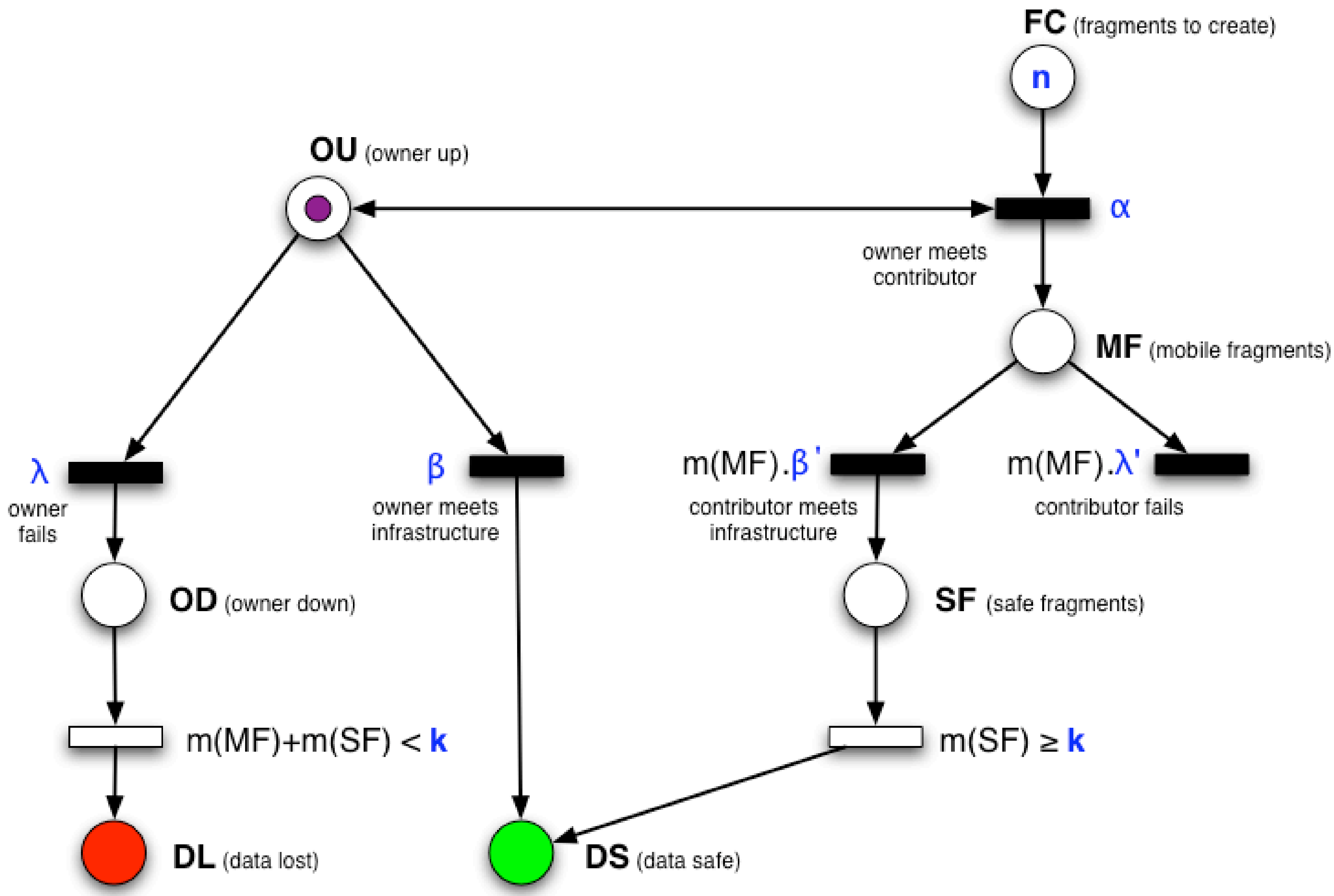
MoSAIC Evaluation

LAAS-CNRS

GSPN Model for (k,n) Erasure Code



Probability of data loss

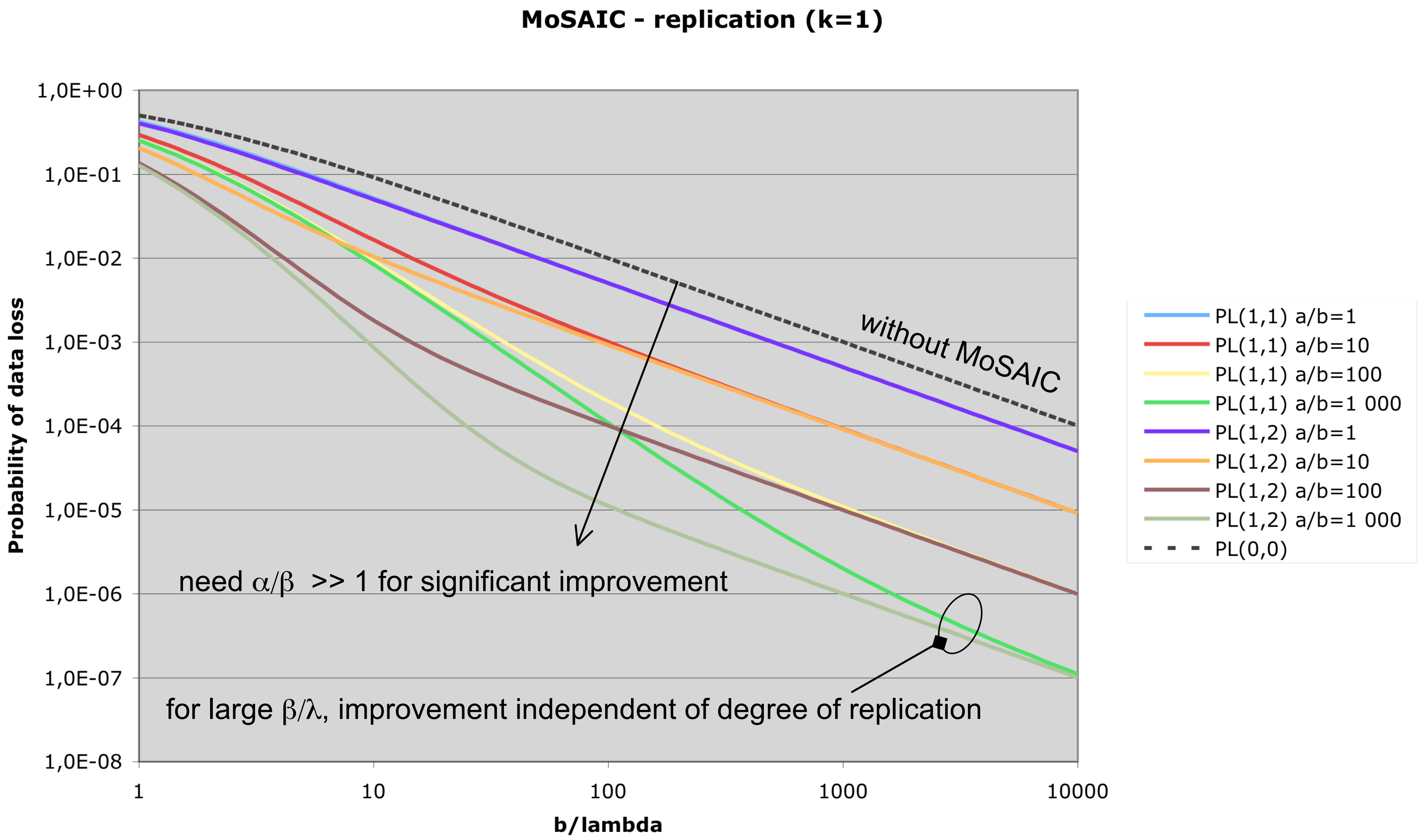
- PL(k,n) : probability of data being lost
- Without MoSAIC

$$PL(0,0) = \frac{\lambda}{\lambda + \beta}$$

- With MoSAIC

$$PL(1,1) = \frac{\lambda}{\lambda + \beta + \alpha} + \frac{\alpha}{\lambda + \beta + \alpha}\left(\frac{\lambda}{\lambda + \beta}\right)^2$$

$$PL(1,2) = \frac{\lambda}{\lambda + \beta + \alpha} + \frac{\alpha^2}{(\lambda + \beta + \alpha)(\lambda + 2\beta + \alpha)}\left[\frac{\lambda}{\alpha}\left(\frac{\lambda}{\lambda + \beta + \alpha} + \frac{\lambda}{\lambda + \beta}\right) + \frac{\lambda\alpha}{\lambda + \beta + \alpha}\left(\frac{\lambda}{\lambda + \beta}\right)^2 + \alpha\left(\frac{\lambda}{\lambda + \beta}\right)^3\right]$$



MoSAIC - replication (k=1)
Probability of data loss
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
1,0E-05
1,0E-06
1,0E-07
1,0E-08
1
10
100
1000
10000
b/lambda
without MoSAIC
need α/β >> 1 for significant improvement
for large β/λ, improvement independent of degree of replication
PL(1,1) a/b=1
PL(1,1) a/b=10
PL(1,1) a/b=100
PL(1,1) a/b=1 000
PL(1,2) a/b=1
PL(1,2) a/b=10
PL(1,2) a/b=100
PL(1,2) a/b=1 000
PL(0,0)

Loss probability improvement

- Let IPL(k,n)=PL(0,0)/PL(k,n) : reduction in probability of data being lost provided by MoSAIC

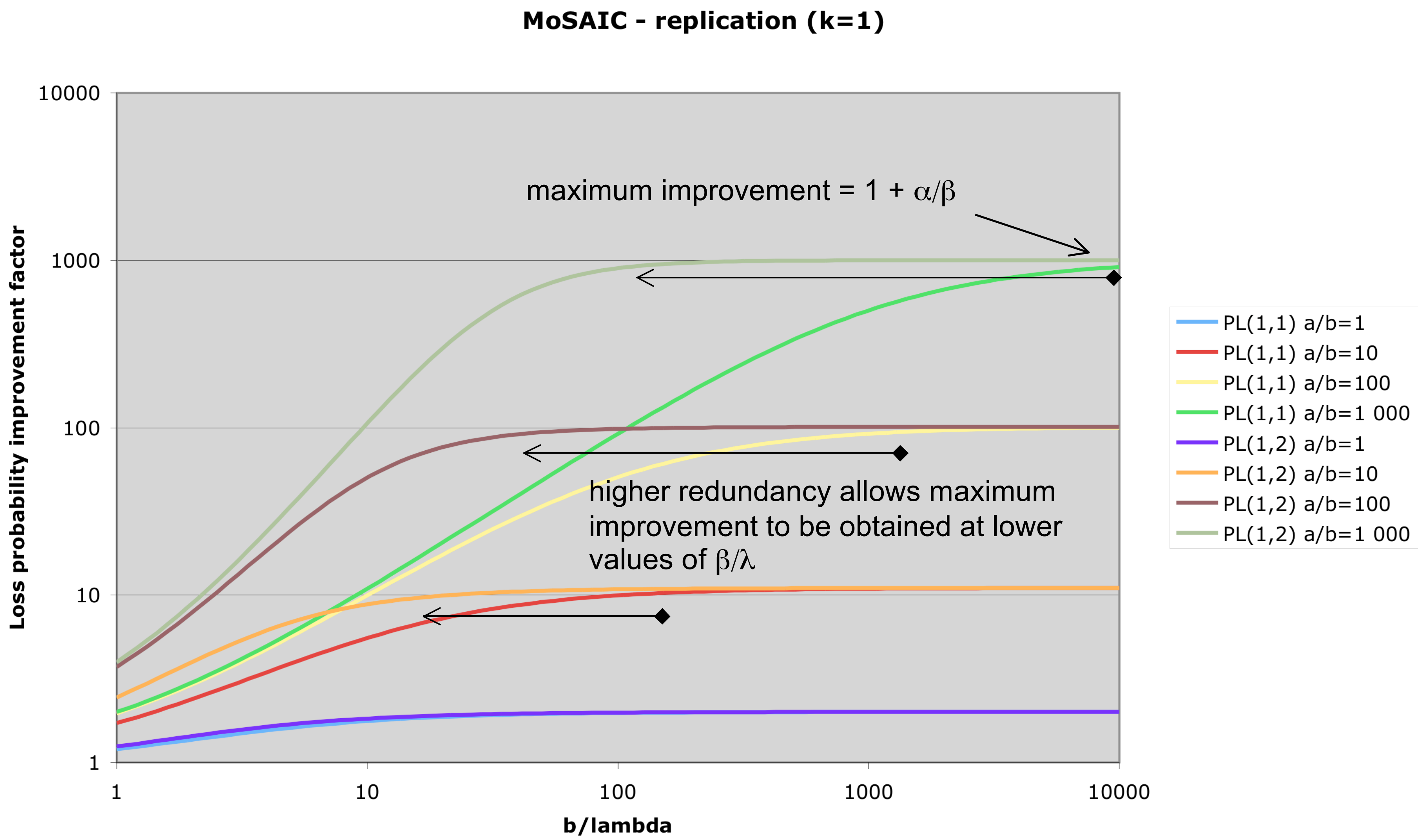
$$IPL(1,1) = \left[\left(\frac{\lambda}{\lambda+\beta+\alpha} + \frac{\alpha}{\lambda+\beta+\alpha} \left(\frac{\lambda}{\lambda+\beta} \right)^2 \right) \times \left(\frac{\lambda+\beta}{\lambda} \right) \right]^{-1}$$

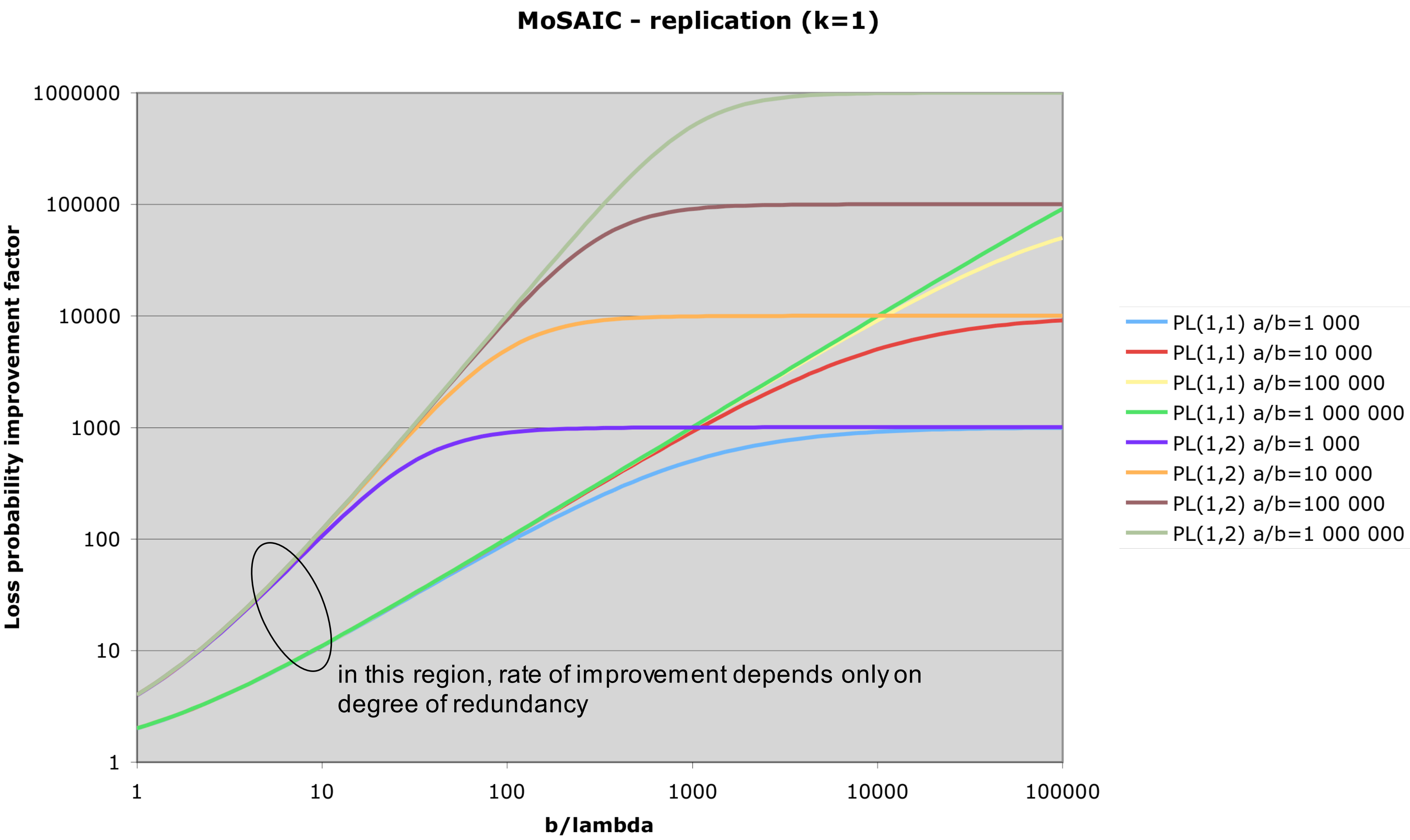
$$= \left[\frac{\lambda+\beta}{\lambda+\beta+\alpha} + \frac{\alpha}{\lambda+\beta+\alpha} \left(\frac{\lambda}{\lambda+\beta} \right) \right]^{-1}$$

$$= \left[\frac{\lambda/\beta+1}{\lambda/\beta+1+\alpha/\beta} + \frac{\alpha/\beta}{\lambda/\beta+1+\alpha/\beta} \left(\frac{\lambda/\beta}{1+\lambda/\beta} \right) \right]^{-1}$$

$$\lim_{\lambda/\beta \to 0} \left(IPL(1,1) \right) = \left(\frac{1}{1+\alpha/\beta} \right)^{-1} = 1+\alpha/\beta$$

- Conjecture that (can demonstrate via Markov graph approximation): $\forall n, \lim_{\lambda/\beta \to 0} \left(IPL(1,n) \right) = 1 + \frac{\alpha}{\beta}$


MoSAIC - replication (k=1)
maximum improvement = 1 + α/β
higher redundancy allows maximum improvement to be obtained at lower values of β/λ
Loss probability improvement factor
b/lambda
10000
1000
100
10
1
1
10
100
1000
10000
PL(1,1) a/b=1
PL(1,1) a/b=10
PL(1,1) a/b=100
PL(1,1) a/b=1 000
PL(1,2) a/b=1
PL(1,2) a/b=10
PL(1,2) a/b=100
PL(1,2) a/b=1 000



MoSAIC - replication (k=1)
Loss probability improvement factor
1000000
100000
10000
1000
100
10
1
1
10
100
1000
10000
100000
b/lambda
in this region, rate of improvement depends only on degree of redundancy
PL(1,1) a/b=1 000
PL(1,1) a/b=10 000
PL(1,1) a/b=100 000
PL(1,1) a/b=1 000 000
PL(1,2) a/b=1 000
PL(1,2) a/b=10 000
PL(1,2) a/b=100 000
PL(1,2) a/b=1 000 000

IPL limit for large α

- Consider case of $\alpha \to \infty$
- Change of variable: $\alpha/\beta = (\alpha/\lambda)(\lambda/\beta)$

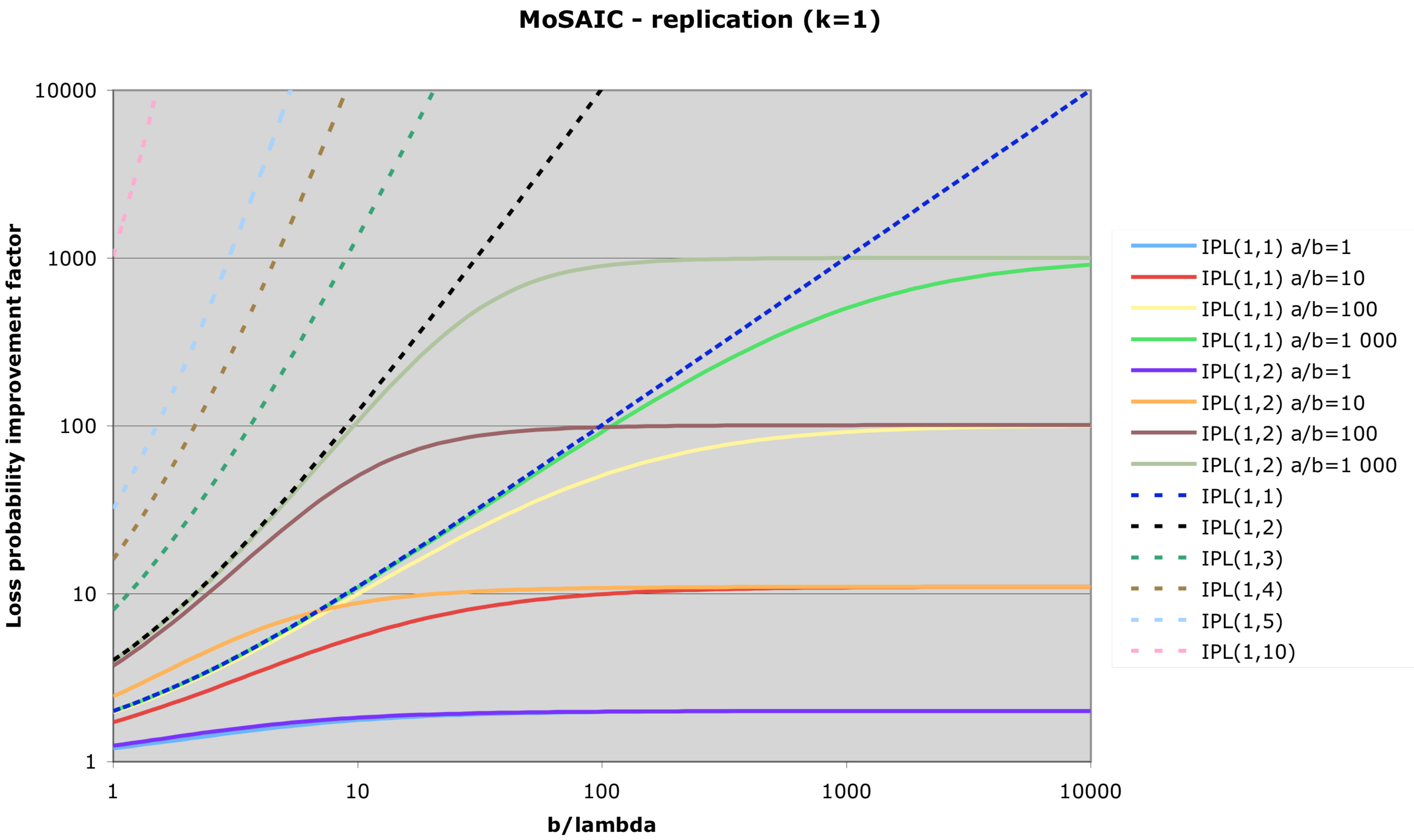
$$IPL(1,1) = \left[\frac{\lambda/\beta+1}{\lambda/\beta+1+(\alpha/\lambda)(\lambda/\beta)} + \frac{(\alpha/\lambda)(\lambda/\beta)}{\lambda/\beta+1+(\alpha/\lambda)(\lambda/\beta)}\left(\frac{\lambda/\beta}{1+\lambda/\beta}\right)\right]^{-1}$$

$$= \left[\frac{(\lambda/\alpha)(\lambda/\beta+1)}{(\lambda/\alpha)(\lambda/\beta)+\lambda/\alpha+\lambda/\beta} + \frac{(\lambda/\beta)}{(\lambda/\alpha)(\lambda/\beta)+\lambda/\alpha+\lambda/\beta}\left(\frac{\lambda/\beta}{1+\lambda/\beta}\right)\right]^{-1}$$

$$\lim_{\lambda/\alpha\to 0} IPL(1,1) = \left(\frac{\lambda/\beta}{1+\lambda/\beta}\right)^{-1} = \left(\frac{1+\lambda/\beta}{\lambda/\beta}\right) = \left(1+\frac{\beta}{\lambda}\right)$$

- Conjecture that (can demonstrate via Markov graph approximation):

$$\lim_{\lambda/\alpha\to 0} IPL(1,n) = \left(1+\frac{\beta}{\lambda}\right)^{n}$$


MoSAIC - replication (k=1)
Loss probability improvement factor
b/lambda
10000
1000
100
10
1
1
10
100
1000
10000
IPL(1,1) a/b=1
IPL(1,1) a/b=10
IPL(1,1) a/b=100
IPL(1,1) a/b=1 000
IPL(1,2) a/b=1
IPL(1,2) a/b=10
IPL(1,2) a/b=100
IPL(1,2) a/b=1 000
IPL(1,1)
IPL(1,2)
IPL(1,3)
IPL(1,4)
IPL(1,5)
IPL(1,10)

Preliminary conclusions

- Have shown limiting expressions for pure replication:

$$\forall n, \lim_{\lambda/\beta \to 0} \left(IPL(1,n) \right) = 1 + \frac{\alpha}{\beta}$$

$$\lim_{\lambda/\alpha \to 0} IPL(1,n) = \left(1 + \frac{\beta}{\lambda} \right)^n$$

- Future work:
  - study k>1
  - consider $\lambda_{contributor} > \lambda_{owner}$ (contributors may be malicious)
  - take into account energy and other resource limitations