ALGORITHMS FOR THE BI-OBJECTIVE MULTIMODAL VIABLE SHORTEST PATH

1. Problem statement and literature
2. Algorithms
3. Computational results
4. Conclusion and future work
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CONTEXT

- Urban Multimodal Transportation Networks
  - Cars, Bus, Metro, Walking, Bike, …

- Alternative itineraries for users
  - User preferences and/or constraints on modes

- Various objectives
  - Travel time
  - Number of modal transfers
  - Cost
  - Resource consumption
  - …
PROBLEM UNDER STUDY

- Multimodal Transportation Network
  - Several modes: bus, metro, walk, private vehicle, ...
  - Modal transfer:
    - Switch from a given mode to another one

- Shortest path
  - Origin → Destination

- Constraints on Modes
  - Viable Path:
    - Feasible Path / constraints on modes
**Problem under study**

- **Two objectives:**
  - Minimize travel time
  - Minimize number of modal transfers

![Graph showing non-dominated paths](TRISTAN - VII - June 2010)
Problem modeling

- **Graph**
  - Each mode:
    - Set of Nodes
    - Set of Arcs
      - Travel Time: $tt(i,j)$
  - Transfer Arcs
    - connecting two modes
PROBLEM MODELING

- Viability
  - Finite State Automaton (FSA)
    - State = sequence of used modes
    - Transitions = feasible evolutions of the mode sequence

- Initial state(s) $I$ and Final state(s) $F$

- Depending on each application
PROBLEM MODELING

- Feasible path from O to D

- State(O) ∈ I
- State(D) ∈ F

Min Travel Time
Min Number of Transfers

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MultiModal Shortest Path
- Pallottino and Scutellà (1998)
- Topological algorithm:
  - min travel time
  - Increasing the number of transfers

MultiModal Viable Shortest Path
- Lozano and Storchi (2001)
  - Extension of the previous algorithm

Label-Constrained Shortest Path
- Barett et al. (2000)
  - An alphabet, Graph / labeled by the alphabet, regular language
  - SP / word formed by labels is included into the regular language
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**Label Setting**

- **Label** \((i, s, k)\)
  
  *i*: Node, *s*: State, *k*: Transfer

- Corresponds to the couple of values:
  - \(T(i,s,k)\): **arrival time** at node *i* in state *s* with *k* transfers
  - \(P(i,s,k)\): **predecessor** label of \((i, s, k)\)

- Only one couple of values for given *i*, *s* and *k*

- **Polynomial algorithms**
**Label Setting**

- **Principle**
  - Select label with the minimum arrival time \((i, s, k)\)

- **Compute/Update labels of successors**
  - if \(\text{Mode}(i) = \text{Mode}(j)\) then \(k' \leftarrow k\); else \(k' \leftarrow k+1\)
  - \(s' \leftarrow \text{Transition in Finite State Automaton, } s, \text{Mode}(i), \text{Mode}(j)\)
  - \(T(j,s',k') \leftarrow \text{Min} \left( T(j, s', k') ; T(i, s, k) + tt(i,j) \right)\)
  - \(P(j, s', k') \leftarrow (i, s, k)\)
**DOMINANCE RULES**

- **Basic Dominance Rule**
  - Two labels: \((i, s, k)\) and \((i, s, k')\)
  
  If \(k \leq k'\) and \(T(i, s, k) \leq T(i, s, k')\) then discard \((i, s, k')\)

- **State-based Dominance Rule**
  - New Binary relation on states \(s \leq s'\):
    - \(s\) yields more extension possibilities than \(s'\)
  
  - Two labels: \((i, s, k)\) and \((i, s', k')\)
  
  If \(k \leq k'\) and \(T(i, s, k) \leq T(i, s, k')\) and \(s \leq s'\) then discard \((i,s',k')\)
**Topological Label-Setting (TLS)**

- Slight adaptations of algorithms
  - Pallotino and Scutellà / Lozano and Storchi
  - State-based dominance rule

- **Principle**
  - Increasing order of Transfers
  - Decreasing order of Travel Time

- **Two Priority Queues**
  - $Q_{\text{now}}$ : labels with $k$ transfers
  - $Q_{\text{next}}$ : labels with $k+1$ transfers

- **LastLabel(i,s)**
  - Best travel time from 0 to $i$ in $s$ (*shortest path in increasing order of transfers*)
TLS: Example

- Shortest paths from 1 to 5

![Graph showing shortest paths from 1 to 5 with nodes labeled and edges marked with weights.]

$\text{k}=0$

![Graph showing the initial state with nodes 1, 2, 3, 4, and 5 with their connections and weights.]

![Graph showing the state after the first iteration, highlighting the updated Qnow and Qnext with new weights.]

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TLS : EXAMPLE

- Shortest paths from 1 to 5

$k=0$

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TLS : Example

- Shortest paths from 1 to 5

\[ k=1 \]

\[
\begin{array}{cc}
(2,1) & (4,6) \\
Q_{\text{now}} & Q_{\text{next}}
\end{array}
\]
TLS : EXAMPLE

- Shortest paths from 1 to 5

\[ k=2 \]

\[ (3,2) \]
\[ (5,7) \]

Qnow

Qnext

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**TLS : Example**

- Shortest paths from 1 to 5

\[ k=3 \]

![Diagram](image-url)
TLS: Example

- Shortest paths from 1 to 5

$k=4$

Q_{now} (5, 4)

Q_{next} (5, 4)

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**Multi-Queue Label-Setting (MQLS)**

- **Principle**
  - Increasing order of Travel Time
  - Decreasing order of Transfers

- **Set of Priority Queues**
  - $Q_0, Q_1, \ldots, Q_k, \ldots$: labels with 0, 1, \ldots k transfers
  - Only one label ($i, s, k$) in each $Q_k$
MQLS: Example

- Shortest paths from 1 to 5

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MQLS: Example

- Shortest paths from 1 to 5

![Graph showing shortest paths from 1 to 5 with associated nodes and paths labeled with distances.]

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MQLS : Example

Shortest paths from 1 to 5
MQLS: Example

Shortest paths from 1 to 5
MQLS : Example

- Shortest paths from 1 to 5
**Bidirectional MQLS (FB-MQLS)**

- **Principle**
  - **Forward Search**: MQLS based on
    - Graph $G$
    - Set $FQ = \{FQ_0, FQ_1, \ldots, FQ_p\}$
    - Finite State Automaton: $FA$
  - **Backward Search**: MQLS based on
    - Graph $G^{-1}$
    - Set $BQ = \{BQ_0, BQ_1, \ldots, BQ_h\}$
    - Finite State Automaton: $BA$

- **Key issues**:
  - Finite State Automaton for Backward Search
  - Connection conditions between labels in $FQ$ and $BQ$
**Bidirectionnal MQLS (FB-MQLS)**

- Finite State Automaton for Backward Search
  - Reverse Automaton used in Forward Search
    - Non-deterministic Automaton
    - Increase the number of labels

- Compute a deterministic Automaton
  - With minimum number of states
  - More efficient for algorithm

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**Bidirectionnal MQLS (FB-MQLS)**

- **Connection tests**
  - \((i,s,a)\) in \(FQ\) and \((i,s,b)\) in \(BQ\) → a path from O to D
    - \((i, s, a)\) in \(FQ\): travel time \(FT(i,s,a)\)
    - \((i, s, b)\) in \(BQ\): travel time \(BT(i,s,b)\)

- **Optimality Test (generalization of the classical test)**
  - if \(FT(i,s,a)+BT(i,s,b) \leq \text{Min}(FT(i',s',k')) + \text{Min}((BT(i',s',k'))\)
    - then → optimal path with \(a+b\) transfers
    - discard queues \(FQk\) and \(BQk\) / \(k \geq a+b\)
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RESULTS

- Multimodal network
  - Part of Toulouse (France)
    - Car, Walk, Bus, Métro
  - Timetables for bus and metro: average travel time
  - Private vehicle can only stop at parking nodes
  - Origin and destination are only in street nodes
  - Number of instances: 100 pairs (O-D)

<table>
<thead>
<tr>
<th>Modes</th>
<th>Nodes</th>
<th>Arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>6170</td>
<td>6646</td>
</tr>
<tr>
<td>Metro</td>
<td>75</td>
<td>72</td>
</tr>
<tr>
<td>Street</td>
<td>56774</td>
<td>146280</td>
</tr>
<tr>
<td>Transfer</td>
<td>-</td>
<td>6370</td>
</tr>
<tr>
<td>Parking</td>
<td>29</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>63048</strong></td>
<td><strong>159368</strong></td>
</tr>
</tbody>
</table>
RESULTS

- Comparison of TLS, MQLS, FB-MQLS
  - basic dominance / state-based dominance

- In terms of:
  - CPU time
  - Average number of enqueued nodes

- Solutions:
  - Average travel time: from 178 min to 818 min
  - Average Number of transfers: from 0 to 8
  - Average number of non-dominated solutions: 5.68 (from 5 to 7)
RESULTS

- In terms of CPU Time (milliseconds)

- Interest of FB-MQLS
  - Interest of State-based dominance rule / MQLS and FB-MQLS
RESULTS

- In terms of enqueued nodes

- Interest of MQLS and FB-MQLS
- Interest of State-based dominance rule

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CONCLUSION AND FUTURE WORK

- Bi-Objective Multimodal Viable Shortest Path
  - Minimal travel time and Minimal number of transfers
  - Viability expressed by Finite State Automaton (FSA)

- Polynomial algorithms
  - Dominance rule based on states of the FSA
  - 2 approaches
    - Paths in increasing order of transfers (TLS)
    - Paths in increasing order of travel time (MQLS)
  - Bidirectional variant (FB-MQLS)

- Experiments
  - Interest of the MQLS approach
  - Interest of the state-based dominance rule
CONCLUSION AND FUTURE WORK

- Integration of goal-oriented approach
  - A* principle
  - Still in progress: for TLS, MQLS and FB-MQLS

- Extension to time-dependent and multi-modal transportation network
  - Integration of results obtained by [Nannicini et al, 2008] for shortest path with dynamic travel time
  - Still in progress

- Taking into account other objectives
  - Non-polynomial algorithms
Thank you for your attention

Questions ?
In terms of visited nodes

- Interest of MQLS and FB-MQLS
- Interest of State-based dominance rule / MQLS and FB-MQLS