Column generation heuristic for a rich arc routing problem
Application to railroad track inspection routing

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10th Workshop on Algorithmic Approaches for Transportation
Modelling, Optimization, and Systems
1 Introduction
2 Literature review
3 Assumptions and model
4 Algorithm
5 Computational study
Rail Track Inspection Scheduling Problem (RTISP)

- real world optimisation
- scheduling network inspection tasks
- minimise total deadhead distance
- mixed integer formulation of the problem
- heuristic columns and rows generation
- a rich arc routing problem

“Real world”

- data from the French national railway company (SNCF)
- efficiency of the approach compared to a greedy algorithm
- results must be validated by humans
Introduction

What?
- ultrasonic defectoscopy
- inspection frequencies: 6 months → 20 years
- 2/3 of total inspection: 6 months ↔ 1 year

Why?
- failures in tracks = serious accidents
- network load increases
- hard to get good schedules
- new organisation study
New organisation: map of primary tracks

- primary tracks: national schedule
- secondary tracks: regional schedule
Vehicles

Three specialised rolling stock units:

- shift duration $\leq$ six hours
- deadhead speed and inspection speed
- reverberation analysis (need water)
- inspection duration $\leq$ water tank capacity
- tank can be refilled at special train stations (90/200)
- vehicle maintenance

... heterogeneous fleet ...
Introduction

The problem
- visit a given set of tracks
- avoid tracks outages
- park vehicles at special stations
- ensure vehicle maintenance satisfaction
- working and shift duration
- maximise performance

The model
A rich arc routing problem
- time windows on required arc
- time windows on deadhead arcs
- intermediate facilities
- heterogeneous fleet
- capacity constraint
- min total deadhead distance

... network special structure, and size ...
Literature review

Industrial arc routing problems
- Hasle and Kloster [2007] : IRP generalize academic works
- Perrier et al. [2006a,b, 2007a,b, 2008] : road maintenances
- Irnich [2008] : nation wide postal delivery

Arc routing problems
- Eiselt et al. [1995a,b] : survey
- Ghiani et al. [2001] : CARP with intermediate facilities
- Amaya et al. [2007] : CARP with refill points

... H-MCARP-IF-TW ...
Assumptions and model

Hypothesis

- Vehicle moves: modeled with arcs and edges (inspection tasks, deadhead traversals, switch back, station traversal)
- One shift per day
- One refill per shift (at the end)
- Unlimited network capacity

In short...

Each shift consists of a trip between two refill stations with a total distance to inspect smaller than the capacity of the water tank and a total trip duration smaller than the duration of a work shift. Given all the feasible shift pattern paths, the RTISP becomes the problem of selecting and scheduling them in order to satisfy all inspections at the lowest length.
Assumptions and model

Graph and vehicle representation

- A multigraph \( G = (V, A) \)
  - \( \tilde{A} \): tasks
  - \( \check{A} \): deadheads
  - \( \hat{A} \): waits
  - \( \bar{V} \): refill facilities
  - \( \tilde{V} \): communication

- One commodity per unit \( k \)
- \( w_k \): max working capacity
Assumptions and model
Graph model of railtracks around Bordeaux

- Une auscultations par an
- Deux auscultations par an

Sebastien Lannez (SNCF/LAAS)
Assumptions and model

Graph model of Bordeaux train station

- Arc raccourcis (1m)
- Arc rebroussement (12km)
- Arc remplissage eau (60km)
- Nœud gare
- Nœud voie

A (Bordeaux St-Jean)
B (Biganos Facture)
L (Langon)
C (Coutras)
Assumptions and model

Calendar

- $H$: set of periods (no non working days)
- $t$: a period in $H$
- $s$: duration of a shift
- $D \subseteq H$: first period of a shift
- $\bar{H}_{a,k} \subseteq H$: set of periods during which vehicle $k$ can not traverse arc $a$. 
Assumptions and model

Mathematical model: notation

- $c_a = l_a$, if $a \in \tilde{A}$, 0, else.
- $Q$: shift patterns ($Q^k$)
- $\mathcal{P}_q$: sequence of visited arcs
- $H_q$: set of valid periods
- $s$: duration of a shift
- $z^t_q$: equal 1 if shift pattern $q$ is performed during calendar day $t$.
- $A_{aq}$: equal 1 if arc $a$ is inspected
- $S_{aq}$: equal 1 if arc $a$ is the start of the shift
- $E_{aq}$: equal 1 if arc $a$ is the end of the shift
- $\delta^+ (\nu)$ ($\delta^- (\nu)$): set of outgoing (ingoing) arcs

$$c_q = \sum_{a \in \mathcal{P}_q} c_a, \ \forall k \in K, q \in Q^k. \quad (1)$$
Assumptions and model

Mathematical model : program

\[
\begin{align*}
\text{minimise} \quad & \sum_{q \in Q} \sum_{t \in D} c_q z_q^t \quad (2) \\
\sum_{t \in D} \sum_{q \in Q} A_{aq} z_q^t \geq 1, \quad & \forall a \in \bar{A} \quad (3) \\
\sum_{q \in Q^k} \sum_{a \in \delta^+(v)} S_{aq} z_q^{t+s} - \sum_{a \in \delta^-(v)} E_{aq} z_q^t = 0, \quad & \forall v \in \bar{V}, k \in K, t \in D \quad (4) \\
\sum_{q \in Q^k} z_q^t \leq 1, \quad & \forall k \in K, t \in D \quad (5) \\
z_q^t = 0, \quad & \forall t \notin H_q \quad (6) \\
z_q^t \in \{0, 1\}, \quad & \forall t \in D, q \in Q \quad (7)
\end{align*}
\]
Assumptions and model
Mathematical model: relaxation 1/2

Remarks
- large number of binary variables
- space and time distribution of inspection tasks are correlated

Approach
- relax constraints which tie together shifts
  - sum over the vehicles
  - sum over the periods
- ... but keep shifts feasibility
- ... and generate cuts
minimise \( \sum_{q \in Q} c_q z^f_q \) \hspace{1cm} (8)

\[
\sum_{q \in Q} A_{aq} z^f_q \geq 1, \quad \forall a \in \bar{A}, \quad (9)
\]

\[
\sum_{k \in K} \sum_{q \in Q^k} \sum_{a \in \delta^+ (v)} S_{aq} z^f_q - \sum_{a \in A} \sum_{a \in \delta^- (v)} E_{aq} z^f_q = 0, \quad \forall v \in \bar{V}, \quad (10)
\]

\[
z^f_q \in [0, 1], \quad \forall q \in Q. \quad (11)
\]
Algorithm
Overall view

1. Generate “good shifts”
2. Select a good coverage
3. Schedule the selected shifts
Algorithm
Details

Generate “good shifts”

Select a good coverage

Schedule the selected shifts
Generate “good shifts”
- Shortest paths with resource constraints and time windows
- Feillet et al. [2004] : adaptation of their dynamic program
- one subproblem per month per vehicle
- parallel computation

Select a good coverage

Schedule the selected shifts
Algorithm Details

Generate “good shifts”

Select a good coverage

- Solve linear relaxation
- Use Chvátal [1979] greedy heuristic
- ... embed in a “tree diving” exploration scheme

Feasibility?

- May violate inter shifts constraints
- Use Benders like subproblems
  - Assigns shift to calendar day
    → Benders linear cut
  - Solve the TSPTW
    → Benders combinatorial cut

Schedule the selected shifts
Algorithm

Details

Generate “good shifts”

Select a good coverage

Schedule the selected shifts
  - Greedy heuristic with constraint propagation
  - Use commercial solver for CP
  - ... embed in a “decision pricing” exploration scheme
**Algorithm**

**Local pseudo cut**

- problem: selection of some columns with incompatible time windows
- solution: cuts inspired by Benders feasibility cuts Benders [1962]

Input: Benders cut from tsptw and assignment.
Output: an effective cut in the LP

Input cut pattern:

\[ \sum_{q \in Q | \zeta_q^I^P > 0} z_q \leq m. \]  \tag{12}

Output cut pattern:

\[ \sum_{q \in Q | \zeta_q^I^P > 0} \frac{1}{\zeta_q^L^P} z_q \leq m. \]  \tag{13}
Computational study
Real data 2009 — 3 vehicles — 12 months — 1 second

\( p \) : performance ratio
\( r \) : total completion rate

<table>
<thead>
<tr>
<th></th>
<th>No outages</th>
<th>small outages</th>
<th>full outages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r )</td>
<td>( p )</td>
<td>( r )</td>
</tr>
<tr>
<td><strong>AlgoGreedy</strong></td>
<td>100%</td>
<td>18.82%</td>
<td>27%</td>
</tr>
<tr>
<td><strong>AlgoColGen</strong></td>
<td>100%</td>
<td>30.50%</td>
<td>37%</td>
</tr>
</tbody>
</table>

**Figure:** Task coverage and solution quality

<table>
<thead>
<tr>
<th></th>
<th>No outages</th>
<th>small outages</th>
<th>full outages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t )</td>
<td>( t )</td>
<td>( t )</td>
</tr>
<tr>
<td><strong>AlgoGreedy</strong></td>
<td>47</td>
<td>767</td>
<td>539</td>
</tr>
<tr>
<td><strong>AlgoColGen</strong></td>
<td>3434</td>
<td>180</td>
<td>61</td>
</tr>
</tbody>
</table>

**Figure:** Computation time (in seconds)

4 Intel cores @ 2.6GHz.
Conclusion

- Good to take into account dataset specificities
- Real data are difficult ("lightly infeasible...")
- Column generation is of interest for solving arc routing problem
- An heuristic based on Benders and Dantzig-Wolfe decomposition method


N. Perrier, A. Langevin, and J.F. Campbell. A survey of models and algorithms for winter road maintenance. Part III: Vehicle routing and
