Models for door-to-door multimodal freight transportation

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Summary

- Problem description
- Related works
- Time-Space model
- Implicit Time model
- Results
- Perspectives

Project ANR - Respet

- Aims to develop quantitative approach to door-to-door freight transportation.

- Members:
  - LAAS – CNRS
  - INRIA
  - LIA
  - DHL
  - JASSP

Problem description

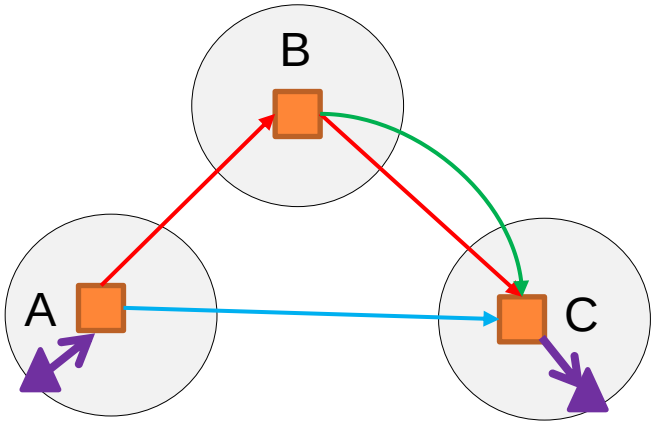
- Given a set L of orders, a set K of containers and a set V of vehicles with known schedule.
  - Assign orders to containers.
  - Schedule transportation over a vehicle network.
  - Containers themselves represent only order groups and do not have themselves source and destination terminals.

Problem description - Data

- Orders:
  - source and destination locations
  - release and due date
  - weight
- Vehicles:
  - set of visited locations (path).
  - load/unload time window at each location.
  - travel cost and distance for each path segment.
- Capacity
  - location storage capacity
  - vehicle transportation capacity
  - container capacity

Problem description - example

Vehicles	Origin		Destination
V1	A – [2 – 3]	B – [5 – 7]	C – [10 – 11]
V2	A – [1 – 2]		C – [4 – 7]
V3	B – [6 – 9]		C – [11 – 13]



Order	Pickup	Source	Dest.	Delivery
▲	2	A	C	15
…	…	…	…	…

➔ Vehicle

Location

Related Works

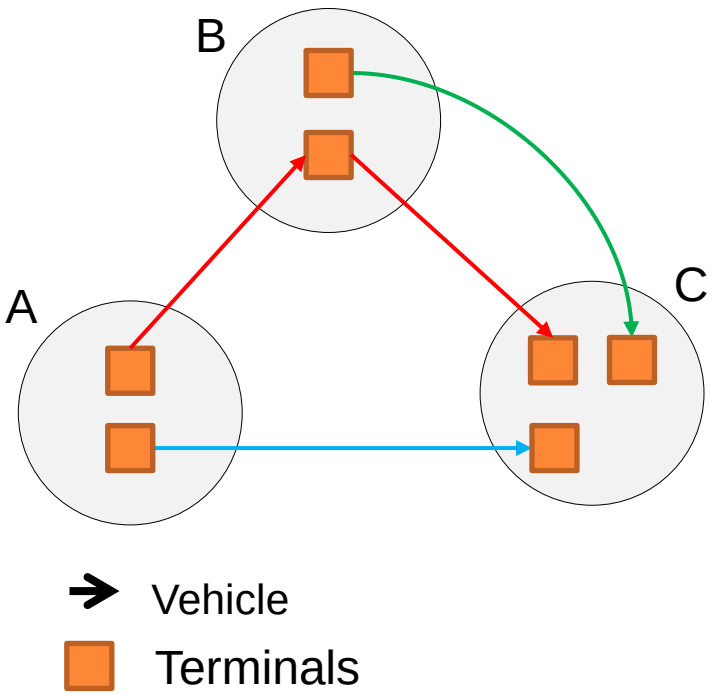
- **Teodor Crainic**. Long-haul freight transportation. *Handbook of Transportation Science, 2003.*
- **SteadieSeifi, M. et al**. Multimodal freight transportation planning: A literature review. *European Journal of Operational Research, 2014.*
- **Burak Ayar, Hande Yaman**. An intermodal multicommodity routing problem with scheduled services. *Computational Optimization and Applications, 2011.*
- **Bauer, Bektas, Crainic**. Minimizing greenhouse gas emissions in intermodal freight transport: an application to rail service. *Journal of the Operational Research Society, 2010.*

Time-space network

- $\bar{N}$ – set of locations
- V – set of vehicles
- T – set of periods
- K – set of containers
- L – set of orders
  - $s_l, d_l \in \bar{N}$ – source and destination
  - $\phi_l, \omega_l \in T$ – release and due date
  - w_l - weight

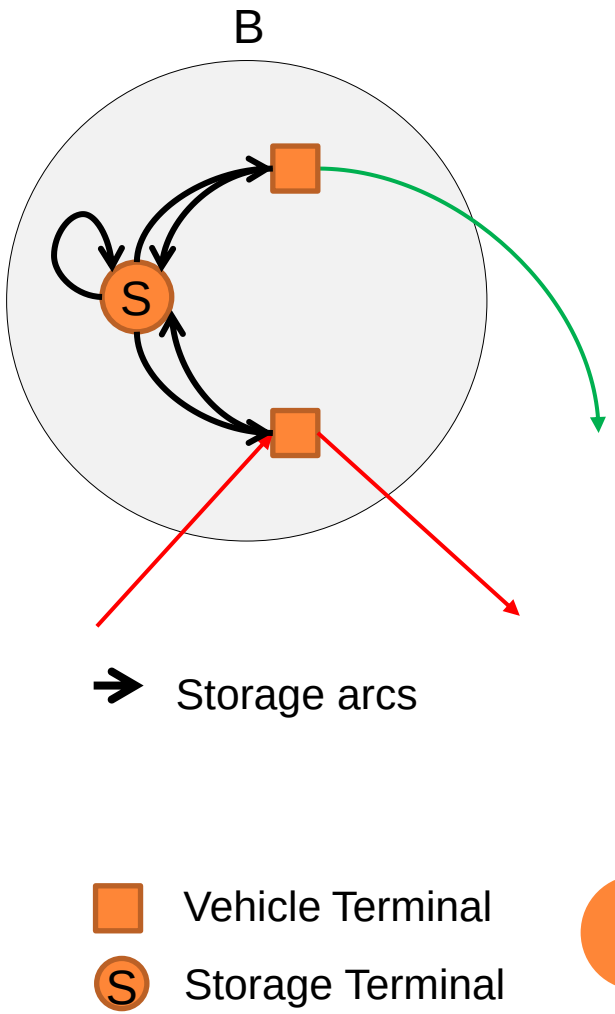
Time-space network - vehicles

- Each vehicle has a graph $G_v = (N_v, A_v)$
- N_v - set of terminals of vehicle v.
- Each $i \in N_v$ corresponds (but not equal) to a location in $\bar{N}$.
  - $\Upsilon_i^v \in T$ – load/unload time window.
- $(i, j) \in A_v$ - path of vehicle v.
  - $c_{ij}, \Delta_{ij}, Q_{ij}$ - cost, travel time and capacity of (i, j).



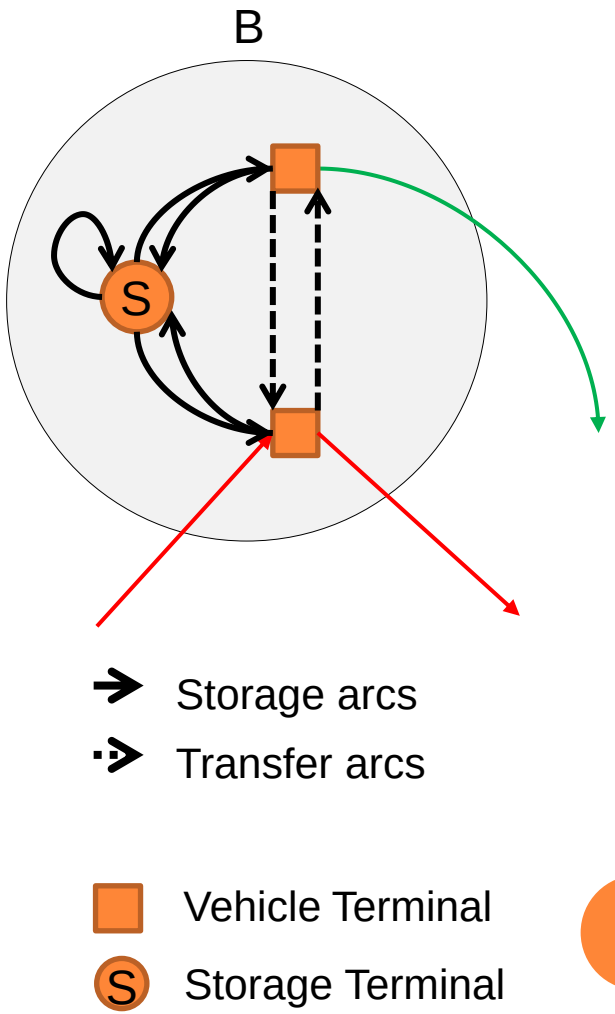
Time-space network - location

- Each location
  - σ_n - Storage terminal
  - A_n^S - set of storage arcs



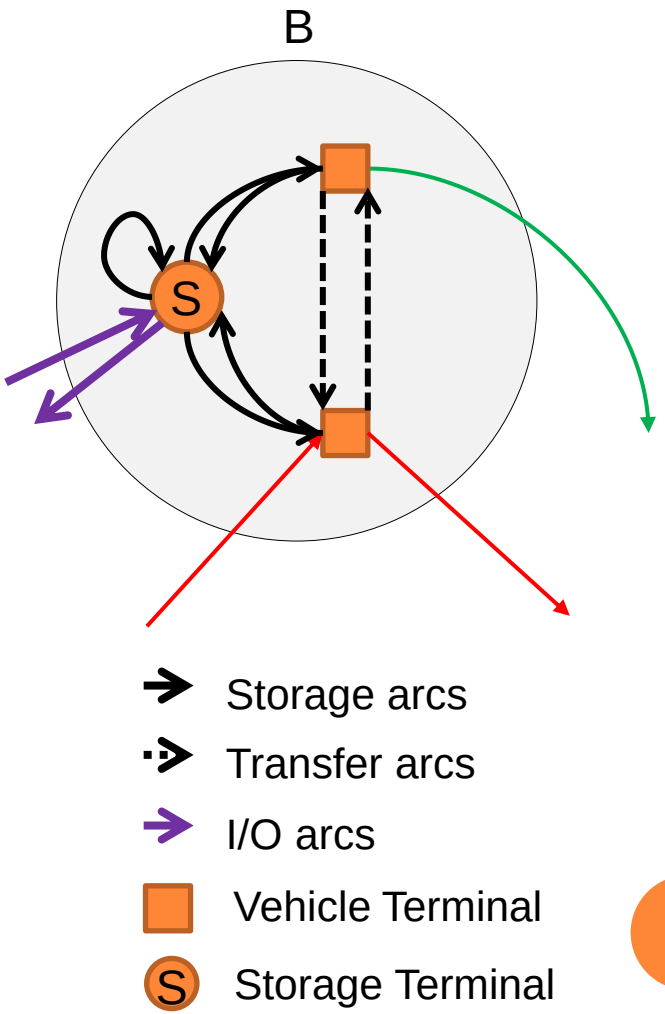
Time-space network - location

- Each location
  - σ_n - Storage terminal
  - A_n^S - set of storage arcs
  - A_n^M - vehicle/mode transfer arcs



Time-space network - location

- Each location
  - σ_n - Storage terminal
  - A_n^S - set of storage arcs
  - A_n^M - vehicle/mode transfer arcs
  - I_n, O_n - incoming and outgoing arcs



Time space model

- Additional parameters:
  - $Q_n, n \in \bar{N} - Q_k, k \in K - Q_{ij}, (i,j) \in A$
  - Location, container and arc capacity, respectively.

- For each period:
  - $G^t = (N^t, A^t)$, where:
  - $N^t = (\cup_{v \in V} N_v) \cup (\cup_{n \in N} \sigma_n)$
  - $A^t = (\cup_{v \in V} A_v) \cup (\cup_{n \in N} A_n^S) \cup (\cup_{n \in N} A_n^M)$

- Complete Time-space network
  - $G = (N, A) = \cup_{t \in T} G^t$

Variables

- $y_{lk} \in \{0,1\}$ – assignment of order l to container k.
- $x_{ijk}^{t} \in \{0,1\}$ - transportation of container k at period t through arc (i, j).

Constraints - Assignment

- every order must be assigned to a container.
- orders assigned to the same container must have the same source and destination

$$\sum_{k \in K} y_{lk} = 1, \qquad \forall l \in L$$

$$y_{lk} + y_{mk} \leq 1, \qquad \forall k \in K, \forall l, m \in L, s_l \neq s_m, d_l \neq d_m$$

Constraints - Capacity

- Vehicle, storage and container capacity constraints

$$\sum_{k \in K} x_{ijk}^{t} \leq Q_{ij}, \qquad \forall (i,j) \in A^{T}, \forall t \in \mathrm{T}$$

$$\sum_{(i,j) \in A_n^S} \sum_{k \in K} x_{ijk}^{t} \leq Q_n, \qquad \forall n \in N, \forall t \in \mathrm{T}$$

$$\sum_{l \in L} w_l y_{lk} \leq Q_k, \qquad \forall k \in K$$

Constraints - Transportation

- Containers must depart and arrive from the source and destination terminals **of its assigned orders**.

$$\sum_{t=\phi_l}^{\omega_l} x_{ijk}^{t} \geq y_{lk}, \qquad \forall l \in L, \forall k \in K, (i,j) = I_n, n = s_l$$

$$\sum_{t=\phi_l}^{\omega_l} x_{ijk}^{t} \geq y_{lk}, \qquad \forall l \in L, \forall k \in K, (i,j) = O_n, n = d_l$$

$$\sum_{k \in K} \sum_{(j,i) \in A} x_{jik}^{t-\Delta_{ji}} = \sum_{k \in K} \sum_{(i,j) \in A} x_{ijk}^{t}, \qquad \forall i \in N, \forall t \in T$$

Time-space Model

Minimize $\sum_{(i,j)\in A}\sum_{k\in K}\sum_{t\in T} c_{ij}x_{ijk}^{t}$

s.t.

$$\sum_{k\in K} y_{lk} = 1, \qquad \forall l \in L$$

$$y_{lk} + y_{mk} \leq 1, \qquad \forall k \in K, \forall l,m \in L, s_l \neq s_m, d_l \neq d_m$$

$$\sum_{k\in K} x_{ijk}^{t} \leq Q_{ij}, \qquad \forall (i,j) \in A^T, \forall t \in \mathrm{T}$$

$$\sum_{(i,j)\in A_n^S}\sum_{k\in K} x_{ijk}^{t} \leq Q_n, \qquad \forall n \in N, \forall t \in \mathrm{T}$$

$$\sum_{l\in L} y_{lk} \leq Q_k, \qquad \forall k \in K$$

$$\sum_{t=\phi_l}^{\omega_l} x_{ijk}^{t} \geq y_{lk}, \qquad \forall l \in L, \forall k \in K, (i,j) = I_n, n = s_l$$

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$$\sum_{k\in K}\sum_{(j,i)\in A} x_{jik}^{t-\Delta_{ji}} = \sum_{k\in K}\sum_{(i,j)\in A} x_{ijk}^{t}, \qquad \forall i \in N, \forall t \in T$$

$$y_{lk}, x_{ijk}^{t} \in \{0,1\}$$

Implicit Time Model

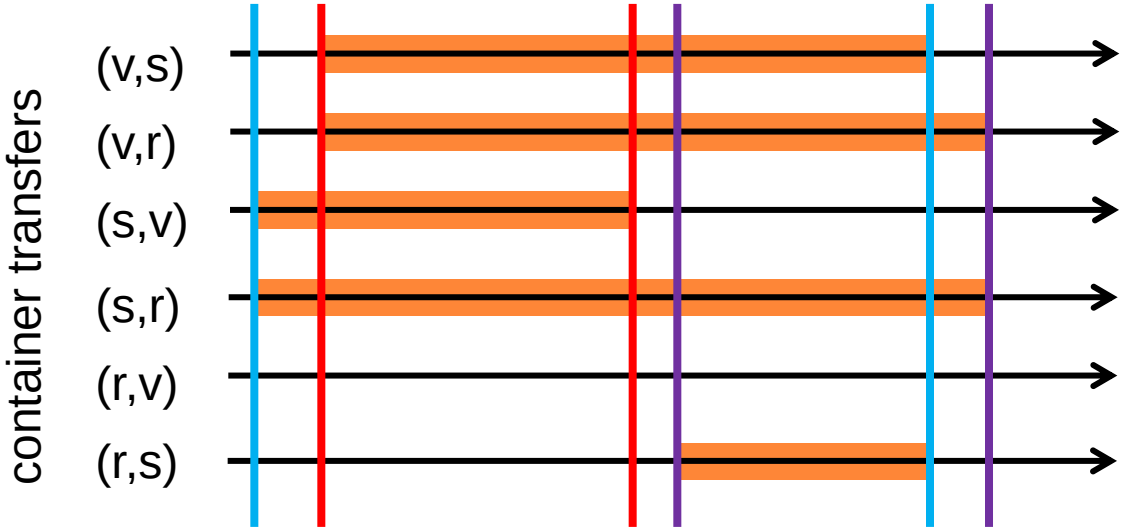
- Since vehicles schedules are known a priori, time periods can be represented implicitly.
- Goal:
  - Reduce model size eliminating atomic decision variables (set of variables for each period).

Location storage capacity

- As periods are not explicitly represented, terminals storage capacity must be modeled differently.
- Known vehicles schedules allow to determine when each container must be stored (in case of vehicle transfer).
- Sets of arcs ($\Upsilon^1, \Upsilon^2, \ldots$) are defined representing when containers must be stored simultaneously at a given interval.

Terminal storage capacity - Example

- Example three vehicles v, s and r
  - Lines represent arrival and departure time of vehicles, box represent interval when containers must be stored



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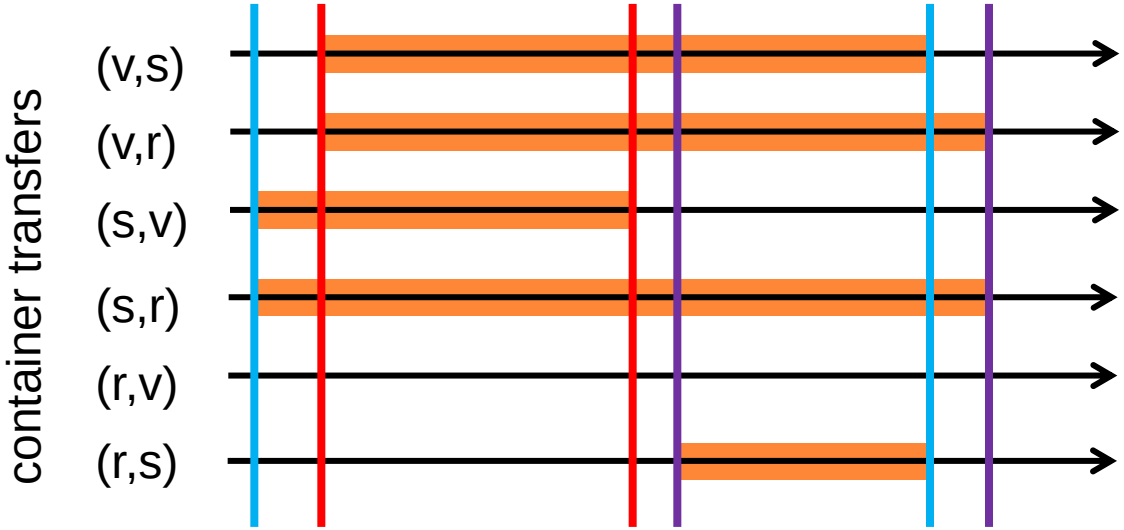

- Set Υ^1 consists of pairs (v,s), (v,r), (s,v) and (s,r) .
- Set Υ^2 consists of pairs (v,s), (v,r), (s,v) and (r,s).

Terminal storage capacity - Constraint

- Let $\overline{\Upsilon_n} = \{\Upsilon_n^1, \Upsilon_n^2, \dots, \Upsilon_n^m\}$.

$$\sum_{(i,j)\in\Upsilon_n^m} \sum_{k\in K} x_{ijk} \leq Q_n, \qquad \forall n \in N, \forall \Upsilon_n^m \in \overline{\Upsilon_n}$$

Implicit Time Model

Minimize $\sum_{(i,j)\in A}\sum_{k\in K} c_{ij}x_{ijk}$

s.t.

$$\sum_{k\in K} y_{lk} = 1, \qquad \forall l \in L$$

$$y_{lk} + y_{mk} \leq 1, \qquad \forall k \in K, \forall l, m \in L, s_l \neq s_m, d_l \neq d_m$$

$$\sum_{k\in K} x_{ijk} \leq Q_{ij}, \qquad \forall (i,j) \in A^T$$

$$\sum_{(i,j)\in \Upsilon_n^m} \sum_{k\in K} x_{ijk} \leq Q_n, \qquad \forall n \in N, \forall \Upsilon_n^m \in \overline{\Upsilon_n}$$

$$\sum_{l\in L} y_{lk} \leq Q_k, \qquad \forall k \in K$$

$$x_{ijk} \geq y_{lk}, \qquad \forall l \in L, \forall k \in K, (i,j) = I_n, n = s_l$$

$$x_{ijk} \geq y_{lk}, \qquad \forall l \in L, \forall k \in K, (i,j) = O_n, n = d_l$$

$$\sum_{k\in K} \sum_{(j,i)\in A} x_{jik} = \sum_{k\in K} \sum_{(i,j)\in A} x_{ijk}, \qquad \forall i \in N$$

$$y_{lk}, x_{ijk} \in \{0,1\}$$

Results

- Quick comparison between models using a network of 6 locations, 20 vehicles and a time horizon of 25 periods.

	Time-space			Implicity time		
	time (s)	#var	#const	time (s)	#var	#const
			10 Orders			
test1	20,98	7739	49774	2,6	1210	10870
test2	9%*	7853	50364	14,4	1230	12280
test3	911,08	6965	46754	6,7	1110	11700
test4	3,51	7439	48414	98,4	1230	11470
test5	45,47	7082	46724	5,4	1230	11470
			12 orders			
test1	11%*	9638	60936	35,6	1616	14136
test2	18%*	9722	60996	124,7	1674	15084
test3	32%*	9650	60960	51,1	1691	14880
test4	11,19	9734	60948	212,1	1736	14916
test5	59,97	9710	60948	5,4	1751	15684
			15 orders			
test1	82%*	12302	76209	69,1	2338	19395
test2	43%*	12332	76299	32,3	2355	20325
test3	65%*	12392	76239	1176,2	2477	19545
test4	44%*	12392	76224	5%*	2429	19680
test5	8%*	12482	76224	3%*	2567	20580

Perspectives

- Investigate instance structure.
- Explore solution methods using a decomposition strategy (column generation).
- Take into account conflicting objectives related to the subject (economical, environmental, QoS, etc) with multiple objectives.

Thank You!