MODELS FOR DOOR-TO-DOOR MULTIMODAL FREIGHT TRANSPORTATION

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SUMMARY

- Problem description
- Related works
- Time-Space model
- Implicit Time model
- Results
- Perspectives
PROJECT ANR - RESPET

Aims to develop quantitative approach to door-to-door freight transportation.

Members:
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- LIA
- DHL
- JASSP
PROBLEM DESCRIPTION

- Given a set $L$ of orders, a set $K$ of containers and a set $V$ of vehicles with known schedule.

- Assign orders to containers.
- Schedule transportation over a vehicle network.

- Containers themselves represent only order groups and do not have themselves source and destination terminals.
**Problem Description - Data**

- **Orders:**
  - source and destination locations
  - release and due date
  - weight

- **Vehicles:**
  - set of visited locations (path).
  - load/unload time window at each location.
  - travel cost and distance for each path segment.

- **Capacity**
  - location storage capacity
  - vehicle transportation capacity
  - container capacity
PROBLEM DESCRIPTION - EXAMPLE

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Origin</th>
<th>Destination</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Order</th>
<th>Pickup</th>
<th>Source</th>
<th>Dest.</th>
<th>Delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>2</td>
<td>A</td>
<td>C</td>
<td>15</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- Vehicle
- Location
**RELATED WORKS**


TIME-SPACE NETWORK

- $\bar{N}$ – set of locations
- $V$ – set of vehicles
- $T$ – set of periods
- $K$ – set of containers
- $L$ – set of orders
  - $s_l, d_l \in \bar{N}$ – source and destination
  - $\phi_l, \omega_l \in T$ – release and due date
  - $w_l$ - weight
Each vehicle has a graph \( G_v = (N_v, A_v) \)

- \( N_v \) - set of terminals of vehicle \( v \).
- Each \( i \in N_v \) corresponds (but not equal) to a location in \( \overline{N} \).
  - \( \gamma^v_i \in T \) – load/unload time window.
- \((i, j) \in A_v\) - path of vehicle \( v \).
  - \( c_{ij}, \Delta_{ij}, Q_{ij} \) - cost, travel time and capacity of \((i, j)\).
TIME-SPACE NETWORK - LOCATION

- Each location
  - $\sigma_n$ - Storage terminal
  - $A^S_n$ - set of storage arcs
TIME-SPACE NETWORK - LOCATION

- Each location
  - $\sigma_n$ - Storage terminal
  - $A^S_n$ - set of storage arcs
  - $A^M_n$ - vehicle/mode transfer arcs
**TIME-SPACE NETWORK - LOCATION**

- Each location
  - $\sigma_n$ - Storage terminal
  - $A^n_S$ - set of storage arcs
  - $A^n_M$ - vehicle/mode transfer arcs
  - $I_n, O_n$ - incoming and outgoing arcs
TIME SPACE MODEL

Additional parameters:
- \( Q_n, n \in \bar{N} - Q_k, k \in K - Q_{ij}, (i,j) \in A \)
- Location, container and arc capacity, respectively.

For each period:
- \( G^t = (N^t, A^t) \), where:
  - \( N^t = (\bigcup_{v \in V} N_v) \cup (\bigcup_{n \in N} \sigma_n) \)
  - \( A^t = (\bigcup_{v \in V} A_v) \cup (\bigcup_{n \in N} A_n^S) \cup (\bigcup_{n \in N} A_n^M) \)

Complete Time-space network
- \( G = (N, A) = \bigcup_{t \in T} G^t \)
**Variables**

- \( y_{lk} \in \{0,1\} \) - assignment of order \( l \) to container \( k \).

- \( x_{ijk}^t \in \{0,1\} \) - transportation of container \( k \) at period \( t \) through arc \( (i,j) \).
Constraints - Assignment

- every order must be assigned to a container.
- orders assigned to the same container must have the same source and destination

\[
\sum_{k \in K} y_{lk} = 1, \quad \forall l \in L
\]

\[
y_{lk} + y_{mk} \leq 1, \quad \forall k \in K, \forall l, m \in L, s_l \neq s_m, d_l \neq d_m
\]
CONTRAIANTS - CAPACITY

Vehicle, storage and container capacity constraints

\[
\sum_{k \in K} x_{ijk}^t \leq Q_{ij}, \quad \forall (i,j) \in A^T, \forall t \in T
\]

\[
\sum_{(i,j) \in A^s} \sum_{k \in K} x_{ijk}^t \leq Q_n, \quad \forall n \in N, \forall t \in T
\]

\[
\sum_{l \in L} w_l y_{lk} \leq Q_k, \quad \forall k \in K
\]
CONSTRAINTS - TRANSPORTATION

- Containers must depart and arrive from the source and destination terminals of its assigned orders.

\[
\sum_{t=\phi_l}^{\omega_l} x_{ijk}^t \geq y_{lk}, \quad \forall l \in L, \forall k \in K, (i,j) = I_n, n = s_l
\]

\[
\sum_{t=\phi_l}^{\omega_l} x_{ijk}^t \geq y_{lk}, \quad \forall l \in L, \forall k \in K, (i,j) = O_n, n = d_l
\]

\[
\sum_{k \in K} \sum_{(j,i) \in A} x_{ijk}^{t-\Delta_ji} = \sum_{k \in K} \sum_{(i,j) \in A} x_{ijk}^t, \quad \forall i \in N, \forall t \in T
\]
**TIME-SPACE MODEL**

Minimize \[ \sum_{(i,j) \in A} \sum_{k \in K} \sum_{t \in T} c_{ij} x_{ijk}^t \]

s.t.

\[ \sum_{k \in K} y_{lk} = 1, \quad \forall l \in L \]
\[ y_{lk} + y_{mk} \leq 1, \quad \forall k \in K, \forall l, m \in L, s_l \neq s_m, d_l \neq d_m \]
\[ \sum_{k \in K} x_{ijk}^t \leq Q_{ij}, \quad \forall (i,j) \in A^T, \forall t \in T \]
\[ \sum_{(i,j) \in A_n^s} \sum_{k \in K} x_{ijk}^t \leq Q_n, \quad \forall n \in N, \forall t \in T \]
\[ \sum_{l \in L} y_{lk} \leq Q_k, \quad \forall k \in K \]
\[ \sum_{t=\phi_l}^{\omega_l} x_{ijk}^t \geq y_{lk}, \quad \forall l \in L, \forall k \in K, (i,j) = I_n, n = s_l \]
\[ \sum_{t=\phi_l}^{\omega_l} x_{ijk}^t \geq y_{lk}, \quad \forall l \in L, \forall k \in K, (i,j) = O_n, n = d_l \]
\[ \sum_{k \in K} \sum_{(j,i) \in A} x_{jik}^{t-\Delta ji} = \sum_{k \in K} \sum_{(i,j) \in A} x_{ijk}^t, \quad \forall i \in N, \forall t \in T \]

\[ y_{lk}, x_{ijk}^t \in \{0,1\} \]
Implicit Time Model

- Since vehicles schedules are known a priori, time periods can be represented implicitly.

Goal:
- Reduce model size eliminating atomic decision variables (set of variables for each period).
LOCATION STORAGE CAPACITY

- As periods are not explicitly represented, terminals storage capacity must be modeled differently.

- Known vehicles schedules allow to determine when each container must be stored (in case of vehicle transfer).

- Sets of arcs \((\gamma^1, \gamma^2, \ldots)\) are defined representing when containers must be stored simultaneously at a given interval.
**Terminal Storage Capacity - Example**

- Example three vehicles v, s and r
  - Lines represent arrival and departure time of vehicles, box represent interval when containers must be stored

![Diagram showing container transfers](image-url)
**Terminal Storage Capacity - Example**

- Example three vehicles $v$, $s$ and $r$
  - Lines represent arrival and departure time of vehicles, box represent interval when containers must be stored.

- Set $\mathcal{Y}^1$ consists of pairs $(v,s)$, $(v,r)$, $(s,v)$ and $(s,r)$.
- Set $\mathcal{Y}^2$ consists of pairs $(v,s)$, $(v,r)$, $(s,v)$ and $(r,s)$. 
Let $\overline{\gamma}_n = \{\gamma^1_n, \gamma^2_n, \ldots, \gamma^m_n\}$. 

$$\sum_{(i,j) \in Y^m_n} \sum_{k \in K} x_{ijk} \leq Q_n, \quad \forall n \in N, \forall \gamma^m_n \in \overline{\gamma}_n$$
**Implicit Time Model**

Minimize \[ \sum_{(i,j) \in A} \sum_{k \in K} c_{ij} x_{ijk} \]

s.t.

\[
\sum_{k \in K} y_{lk} = 1, \quad \forall l \in L
\]

\[
y_{lk} + y_{mk} \leq 1, \quad \forall k \in K, \forall l, m \in L, s_l \neq s_m, d_l \neq d_m
\]

\[
\sum_{k \in K} x_{ijk} \leq Q_{ij}, \quad \forall (i,j) \in A^T
\]

\[
\sum_{(i,j) \in A} \sum_{k \in K} x_{ijk} \leq Q_n, \quad \forall n \in N, \forall Y_n^m \in \overline{Y}_n
\]

\[
\sum_{l \in L} y_{lk} \leq Q_k, \quad \forall k \in K
\]

\[
x_{ijk} \geq y_{lk}, \quad \forall l \in L, (i,j) \in A^T
\]

\[
x_{ijk} \geq y_{lk}, \quad \forall (i,j) \in A^T, (j,i) \in A
\]

\[
\sum_{k \in K} \sum_{(j,i) \in A} x_{jik} = \sum_{k \in K} \sum_{(i,j) \in A} x_{ijk}, \quad \forall i \in N
\]

\[y_{lk}, x_{ijk} \in \{0,1\}\]
RESULTS

- Quick comparison between models using a network of 6 locations, 20 vehicles and a time horizon of 25 periods.

<table>
<thead>
<tr>
<th></th>
<th>Time-space</th>
<th>Implicity time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>time (s)</td>
<td>#var</td>
</tr>
<tr>
<td><strong>10 Orders</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test1</td>
<td>20,98</td>
<td>7739</td>
</tr>
<tr>
<td>test2</td>
<td>9%*</td>
<td>7853</td>
</tr>
<tr>
<td>test3</td>
<td>911,08</td>
<td>6965</td>
</tr>
<tr>
<td>test4</td>
<td>3,51</td>
<td>7439</td>
</tr>
<tr>
<td>test5</td>
<td>45,47</td>
<td>7082</td>
</tr>
<tr>
<td><strong>12 orders</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test1</td>
<td>11%*</td>
<td>9638</td>
</tr>
<tr>
<td>test2</td>
<td>18%*</td>
<td>9722</td>
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<tr>
<td>test3</td>
<td>32%*</td>
<td>9650</td>
</tr>
<tr>
<td>test4</td>
<td>11,19</td>
<td>9734</td>
</tr>
<tr>
<td>test5</td>
<td>59,97</td>
<td>9710</td>
</tr>
<tr>
<td><strong>15 orders</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test1</td>
<td>82%*</td>
<td>12302</td>
</tr>
<tr>
<td>test2</td>
<td>43%*</td>
<td>12332</td>
</tr>
<tr>
<td>test3</td>
<td>65%*</td>
<td>12392</td>
</tr>
<tr>
<td>test4</td>
<td>44%*</td>
<td>12392</td>
</tr>
<tr>
<td>test5</td>
<td>8%*</td>
<td>12482</td>
</tr>
</tbody>
</table>
**Perspectives**

- Investigate instance structure.

- Explore solution methods using a decomposition strategy (column generation).

- Take into account conflicting objectives related to the subject (economical, environmental, QoS, etc) with multiple objectives.
THANK YOU!