Lagrangian relaxation-based lower bound for resource-constrained modulo scheduling.

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Outline

1. Introduction and problem presentation.
2. Time indexed linear programming formulation for the resource constrained modulo scheduling.
3. Lagrangian relaxation
4. Experimental results.
5. Conclusion and future work.

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Introduction and problem presentation

Industrial context : VLIW instruction schedling

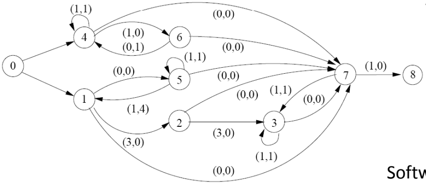
- The instruction scheduling problem is defined by a set of operations to schedule, a set of dependences between these operations, and a target processor micro-architecture. An operation is considered as an instance of an instruction in a program text.
- Modulo scheduling is a software pipelining framework, based only on 1-periodic cyclic schedules with integral period λ.
- The modulo scheduling algorithm must take into account the constraints of the target processor, this is, latencies of operations, resources, and size of the register files.

Introduction and problem presentation

```
int
prod(int n, short a[], short b) {
    int s=0, i;
    for (i=0;i<n;i++) {
        s += a[i]*b;
    }
    return s;
}
```

```
L?__0_8:
LDH_1 g131 = 0, G127
MULL_2 g132 = G126, g131
ADD_3 G129 = G129, g132
ADD_4 G128 = G128, 1
ADD_5 G127 = G127, 2
CMPNE_6 b135 = G118, G128
BRF_7 b135, L?__0_8
```

loops



schedule instructions to end the program in minimum time

↓

Optimize loop performance

↓

Software pipeline
- Cyclic scheduling problem
- **Modulo scheduling**

Introduction and problem presentation

Resource constrained modulo scheduling problem : data

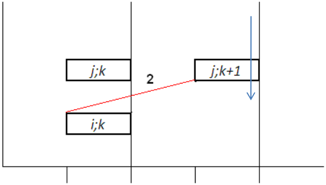
- A set of n generic operations with unit durations and a set of m resources.
- Each resource has a limited availability B_s.
- Each generic operation requires b_i^s units of each resource s.
- A set E of dependences is defined where for each $(i, j) \in E$, a latency θ_i^j and a distance ω_i^j are given.

Introduction and problem presentation

Dependence constraints

Exemple: Let (*i, j*) such that

$\theta_i^j = 2, \omega_i^j = 1$



Start time of instance (i,k)

$$\sigma_i^k = \sigma_i^0 + k\lambda$$

Let $\sigma_i \equiv \sigma_i^0$

Dependence constraint

$$\sigma_j^{k+\omega_i^j} \geq \sigma_i^k + \theta_i^j$$

$$\sigma_i + \theta_i^j - \omega_i^j \lambda \leq \sigma_j \quad \forall (i, j) \in E$$

Resource constraints

Each task uses each resource such that $\exists s, b_i^s > 0$ at time $\sigma_i \bmod \lambda$

Introduction and problem presentation

Resource constrained modulo scheduling problem

- Decision variables : start time $\sigma_i \in \{0, ..., T\}$, where T is the time horizon, for each generic operation $i = 1, \dots, n$

- Minimize λ, subject to
  - *modulo resource constraints*
    $\sum_{\sigma_i \bmod \lambda = \tau} b_i^s \leq B_s$, $\forall \tau \in \{0, \dots, \lambda - 1\}$ and $\forall s \in \{1, \dots, m\}$
  - *dependence constraints*
    $\sigma_j \geq \sigma_i + \theta_i^j - \lambda \omega_i^j$, for all $(i, j) \in E$

Remark 1 : strongly NP-hard

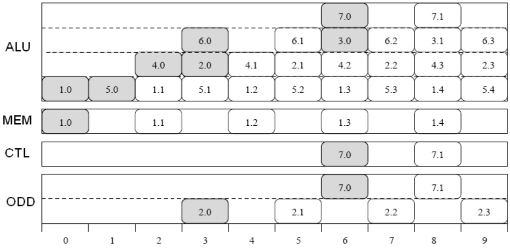
Remark 2 : For a fixed λ, find a feasible solution for a resource-constrained project scheduling problem with minimum and maximum time lags.

Introduction and problem presentation

Exemple.

Resource	Capacity
ALU	4
MEM	1
CTL	1
ODD	2

RESERVATION	ALU	MEM	CTL	ODD
ALU	1	0	0	0
ALUX	2	0	0	1
MUL	1	0	0	1
MULX	2	0	0	1
MEM	1	1	0	0
MEMX	2	1	0	1
CTL	1	0	1	1



$\lambda = 2$

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Time indexed linear programming formulation for the resource constrained modulo scheduling.

Proposed by Dupont de Dinechin (2004). A fixed λ. Binary variables x_i^t such that $x_i^t = 1$ if only if $\sigma_i = t$, $\sigma_i = \sum_{t=0}^{T} t x_i^t$. The feasible set $\mathcal{X}(\lambda)$ is described by (1-4).

$$\sum_{t=0}^{T} x_i^t = 1, \forall i = 1, ..., n. \tag{1}$$

$$\sum_{i=1}^{n} \sum_{k=0}^{\lfloor \frac{T}{\lambda} \rfloor} x_i^{\tau+k\lambda} b_i^s \leq B_s, \forall \tau \in \{0, \ldots, \lambda-1\}, s \in \{1, \ldots, m\} \tag{2}$$

$$\sum_{h=t}^{T} x_i^h + \sum_{h=0}^{t+\theta_i^j-\lambda\omega_i^j-1} x_j^h \leq 1, \forall t \in \{0, \ldots, T\}, (i,j) \in E \tag{3}$$

$$x_i^t \in \{0,1\}, \forall i = 1, ..., n, \forall t \in \{0, \ldots, T\} \tag{4}$$

Time indexed linear programming formulation for the resource constrained modulo scheduling.

Let $\tilde{\mathcal{X}}(\lambda)$, the polyhedron defined by the linear relaxation of $\mathcal{X}(\lambda)$, i.e. constraints (1-3) and (5).

$$0 \leq x_i^t \leq 1, \forall i = 1, ..., n, \forall t \in \{0, \ldots, T\} \quad (5)$$

A method to obtain a lower bound λ_{LP} is to solve repetitively linear programs via linear or binary search to find the smallest λ for which $\tilde{\mathcal{X}}(\lambda)$ is non-empty.

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Lagrangian relaxation.

Let now the linear expression

$$L_{\delta,\lambda}(x) = \sum_{s=1}^{m}\sum_{\tau=0}^{\lambda-1}\delta_{\tau,s}(\sum_{i=1}^{n-1}\sum_{t \bmod \lambda=\tau} b_i^s x_i^t) - \sum_{s=1}^{m}\sum_{\tau=0}^{\lambda-1}\delta_{\tau,s}B_s$$

and the LP

$$\min\{L_{\delta,\lambda}(x) : x \in \hat{\mathcal{X}}(\lambda)\} \tag{6}$$

- $\delta_{\tau,s} \geq 0$, for $\tau \in \{0,\ldots,\lambda-1\}$, $s \in \{1, ldots, m\}$
- $\hat{\mathcal{X}}(\lambda)$ is the polyhedron defined by constraints (1),(3) and (5).
- $L^*_{\delta,\lambda}$ denote its optimal solution value.

Lagrangian relaxation.

Proposition 1.
$\tilde{\mathcal{X}}(\lambda)$ is empty if for some $\delta \geq 0$, $\hat{\mathcal{X}}(\lambda)$ is empty or $L^*_{\delta,\lambda} > 0$.
A lower bound λ_{LR} for the minimal period can be obtained by finding the smallest λ value (through linear of binary search) for which the so-called lagrangian dual problem (LR) defined by :

$$\max_{\delta \geq 0} \min\{L_{\delta,\lambda}(x) : x \in \hat{\mathcal{X}}(\lambda)\} \quad (7)$$

has a non positive optimal solution ($L^*_\lambda \leq 0$). Furthermore, (LR) has the integrity property since $Co\{x \in \{0,1\} \cap \hat{\mathcal{X}}(\lambda)\} = \hat{\mathcal{X}}(\lambda)$, where $Co(X)$ denotes the convex hull of set X. It follows that $\lambda_{LR} = \lambda_{LP}$.

Lagrangian subproblem.

- Lagrangian subproblem is defined by

$$\min\{L_{\delta,\lambda}(x) : x \in \hat{\mathcal{X}}(\lambda)\} \tag{8}$$

if we define weights

$$f_{i,t} = \sum_{s=1}^{m} b_i^s \delta_{t \bmod \lambda, s}. \tag{9}$$

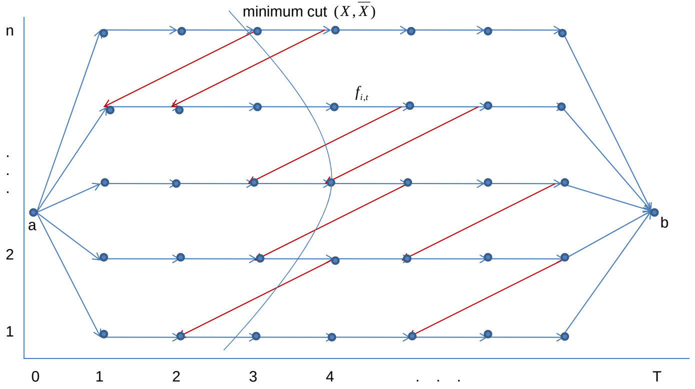
The Lagrangian subproblem can be written as

$$\min\{\sum_{i=1}^{n-1} \sum_{t=0}^{T-1} f_{i,t} x_i^t : x \in \hat{\mathcal{X}}(\lambda)\} \tag{10}$$

A special case of the scheduling problem with start-time-dependent costs described in Möhring et al (2003).

Lagrangian subproblem

- vertex $v_{i,t}$ $\forall i \in \{1 \ldots n\},\ t \in \{0 \ldots T\}$.
- Assignment arcs $(v_{i,t}, v_{i,t+1})$ $\forall i \in \{1 \ldots n\},\ t \in \{0 \ldots T\}$.
- Temporals arcs $(v_{i,t}, v_{i,t+(\theta_i^j - \lambda \omega_i^j)})$ $\forall (i,j) \in E,\ t \in \{0 \ldots T\}$.
- $f_{i,t}$ Arc capacities.



Lagrangian subproblem

Theorem 1 (Möhring et al 2003) describes the transformation of (8) to a minimum cut problem.

Theorem

There is a one to one correspondance between minimum $a - b$ cuts $(X, \bar{X})$ of G wth exactly n forward arcs and the optimal solutions x of (8) by virtue of
$x_{i,t} = 1$ if $(v_{i,t}, v_{i,t+1})$ is a forward arc of the cut $(X, \bar{X})$ and $x_{i,t} = 0$ otherwise. The value $w(x)$ of an optimal solution of (8) equals the capacity $c(X, \bar{X})$ of any minimum cut $(X, \bar{X})ofG$

Solving the lagrangian dual poblem.

Subgradient algorithm for min $\sum_{i=1}^{n-1} \sum_{t=0}^{Tmax-1} f_{i,t} x_i^t : x \in \hat{\mathcal{X}}(\lambda)$

1: $\delta = (\delta_{\tau,s})_{\tau \in \{0,\dots,\lambda-1\}}^{s \in \{1,\dots,m\}} = 0.$
2: Solve max flow and get min cut $(X, \bar{X}) \rightarrow L_{\delta,\lambda}(x)$.
3: **if** $L_{\delta,\lambda}(x) > 0$ **then**
   STOP.
4: **end if**
5: Compute subgradient $g_{s,\tau}^k = \sum_{i=i}^{n-1} b_i^s (\sum_{t \bmod \lambda = \tau} x_{i,t}^k) - B_s$.
6: Compute scalar step size $\gamma^k = \gamma(L^* - L_\delta(x^k))/\|g^k\|^2$.
7: Update the vector of Lagrangian multipliers of the $k+1$th iteration $\rightarrow \delta^{k+1} = [\delta^k + \gamma^k g^k]^+$.
8: If no substantial improvement could be achieved within 15 iterations, STOP.
9: Jump to step 2.

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Experimental results.

We implemented the Lagrangian relaxation for the RCMSP to compute a lower bound for the period λ for instruction scheduling instances of the production compiler for the STMicroelectronics ST200 VLIW processor family.

Experimental results.

TABLE: Lower bound for λ

Instances	λ_0	$\lambda_{LP,LR}$	CPU LP (s)	CPU LR (s)	CPU LR^* (s)
adpcm-st231.1	11	21	14.1	20.22	9.002
gsm-st231.1	24	24	35.85	22.03	18.37
gsm-st231.2	12	26	29.31	24.03	17.589
gsm-st231.5	4	11	13.27	11.25	4.02
gsm-st231.6	3	7	4.12	2.21	0.010
gsm-st231.7	4	11	5.09	4.03	1.006
gsm-st231.10	1	4	1.03	0.98	0.052
gsm-st231.20	3	6	1.29	1.12	0.044
gsm-st231.29	4	11	4.32	3.27	1.005
gsm-st231.40	2	10	13.25	12.27	4.10

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Conclusion and future work

- The results show that the proposed lagrangian relaxation method allows to obtain the same lower bound as the direct solving of the LP formulation with substantially lower computational requirements.
- Heuristic based on lagrangian relaxation
- Direct minimization of λ