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Techniques in Constraint Programming for Combinatorial Optimization Problems

Climbing Depth-Bounded Adjacent Discrepancy Search for Solving Hybrid Flow Shop Scheduling Problems with Multiprocessor Tasks

A. LAHIMER¹, P. LOPEZ¹, M. HAOUARI²

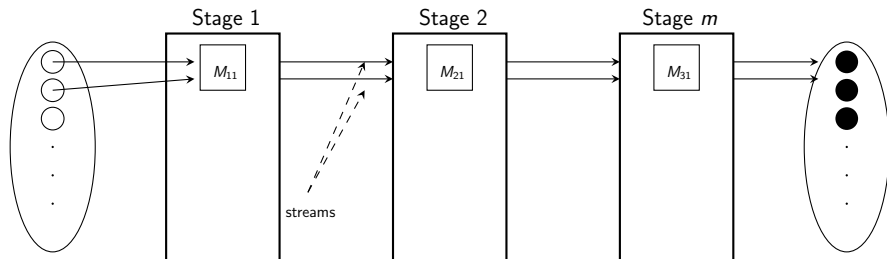
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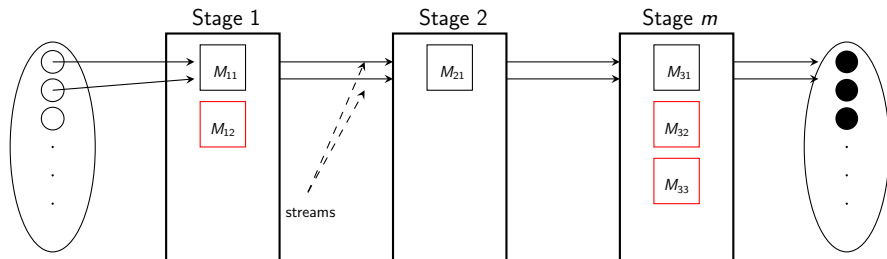
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- 1 Problem Definition
- 2 Discrepancy Methods
- 3 Proposal : Climbing Depth-Bounded Adjacent Discrepancy Search
- 4 Computational Study
- 5 Conclusion

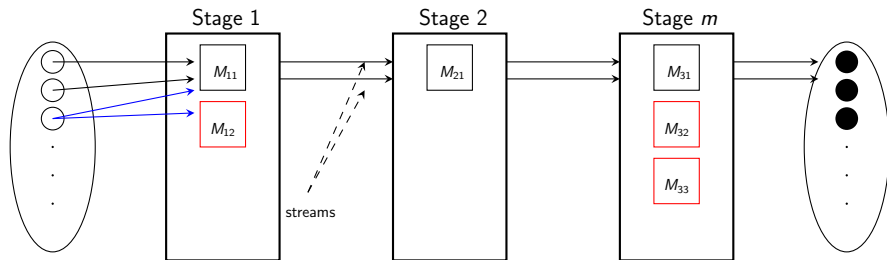
Multiprocessor Hybrid Flow shop



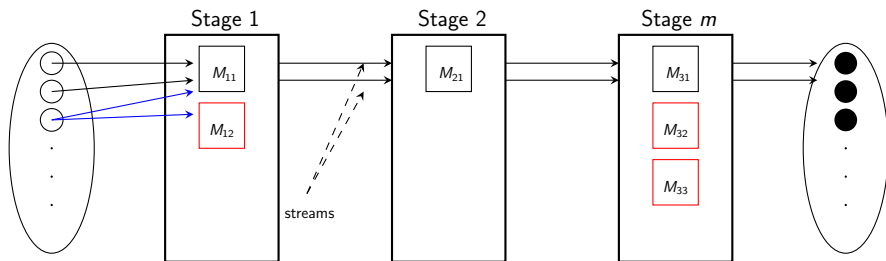
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$$F_m(P_{m_1}, \dots, P_{m_m}) | \text{size}_{ij} | C_{max}$$

Some Applications

- Manufacturing: work-force assignment, transportation problem with recirculation...
- Operating Systems
- Real-time machine vision

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Complexity: NP-hard in the strong sense [J.A. Hoogeveen,1996]

Literature Review

Approaches

- Genetic Algorithm [C. Oğuz *et al.*, 2003]
- Tabu Search [C. Oğuz *et al.*, 2004]
- Ant Colony System [F.S. Şerifoğlu *et al.*, 2006]
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Lower Bounds

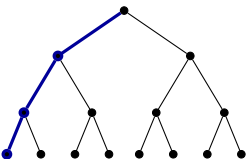
- Specific to F2 [C. Oğuz *et al.*, 2003]
- Adapted to Fm [C. Oğuz *et al.*, 2004]

General Statement

- Genesis: LDS (Limited Discrepancy Search) [Harvey & Ginsberg, 1995]

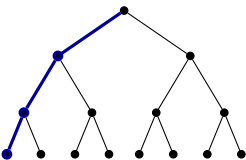
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- A discrepancy = any decision point in the search tree where the choice goes against the heuristic

ILDS: Improved LDS [R. Korf, 1996]

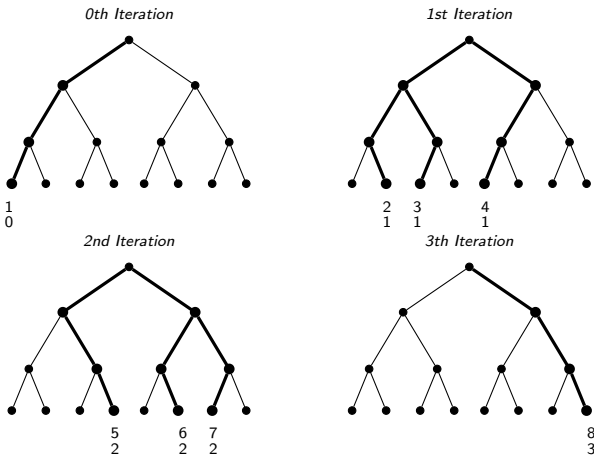


FIGURE: Improved Limited Discrepancy Search

DDS: Depth-bounded Discrepancy Search [T. Walsh, 1997]

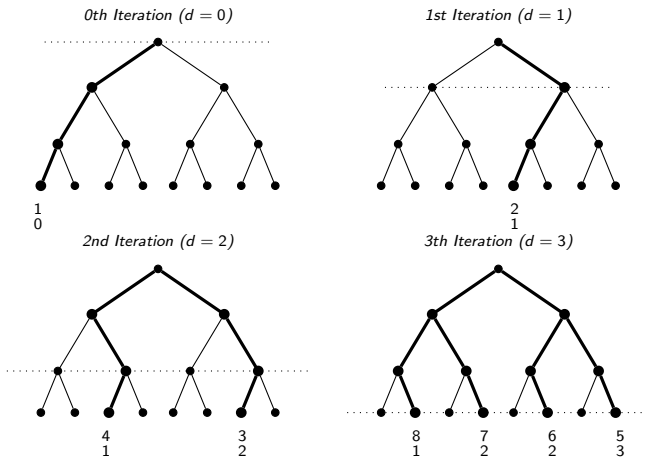


FIGURE: DDS

CDS: Climbing Discrepancy Search [Milano & Roli, 2002]

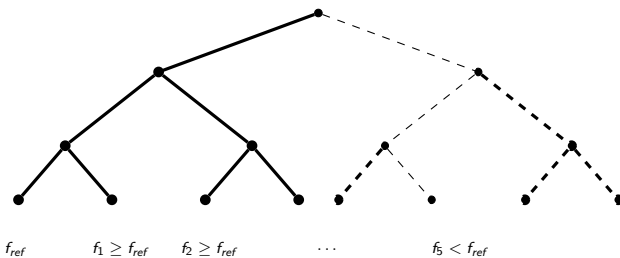


FIGURE: A CDS scenario

DADS: Depth-bounded Adjacent Discrepancy Search

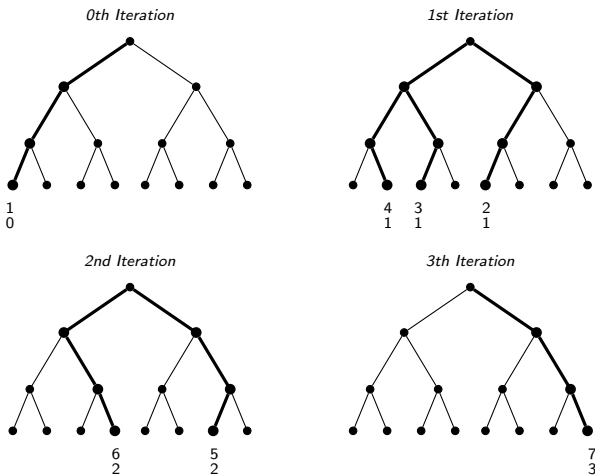


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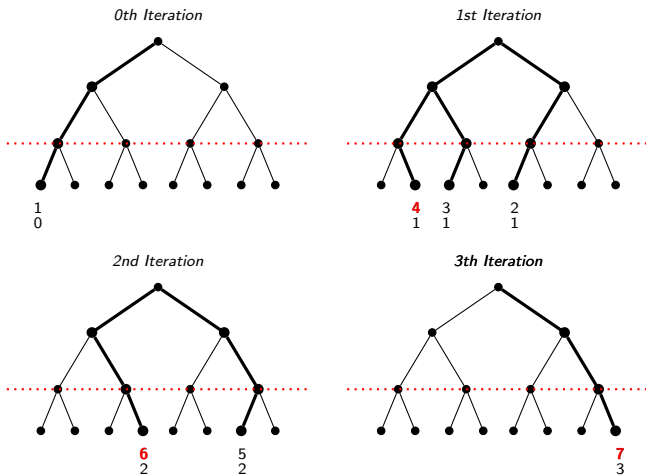
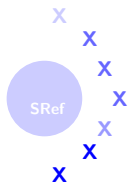


FIGURE: DADS

Climbing DADS



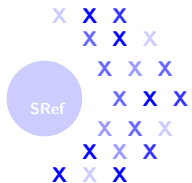
Climbing DADS



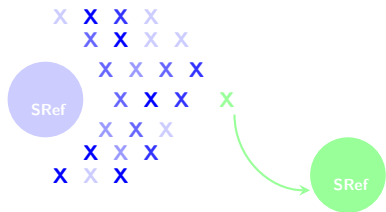
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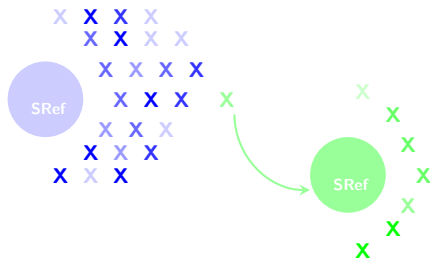
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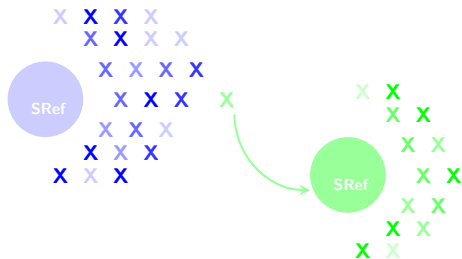
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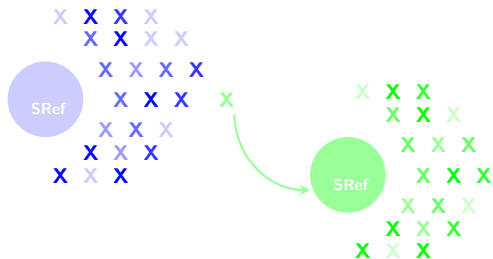
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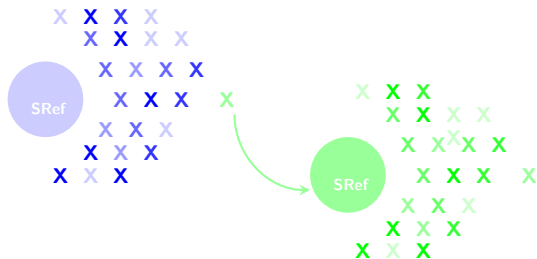
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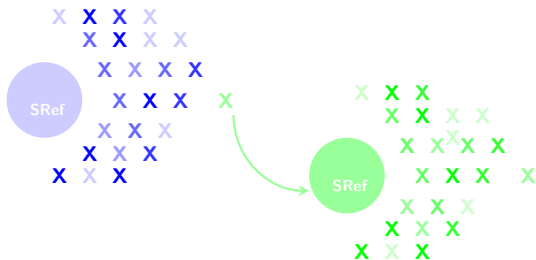
Climbing DADS



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Climbing DADS



Stopping Conditions

- CPU time (60 sec)
- $\text{Cost}(\text{Sol}) = \text{LB}$

Heuristics Selection

- CDADS is strongly based on the quality of the initial solution
- An experimental comparison between various priority rules presented in the literature to consider the most effective

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| Priority Rule | Performance (%) |
|----------------|-----------------|
| NSPT_LastStage | 27 |
| Energy | 25 |
| SPT | 17 |
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Shortest Processing Requirement: $size_{ij}$ increasing order

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$$\mathit{energy}_{ij} = \mathit{size}_{ij} \times p_{ij}$$

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NSPT: Normalized SPT

Schedule Generation Scheme

- Two Types of SGSs
 - Serial SGS [Kelley *et al.*, 1963]
 - Parallel SGS [Brooks *et al.*, 1965]
- Generated Schedules
 - Serial SGSs generate active schedules.
 - Parallel SGSs generate non-delay schedules.
- According to our experimental studies, a parallel SGS is more adapted to our problem.

Lower Bounds

$$LB = \max(LB_s, LB_j)$$

- $LB_s = \max_{i=1..m} LB(i)$

- $LB(i) = \min_{j \in J} \left(\sum_{l=1}^{i-1} p_{lj} \right) + \max(M_1(i), M_2(i), \max_{j \in J} (p_{ij})) + \min_{j \in J} \left(\sum_{l=i+1}^m p_{lj} \right)$

- $M_1(i) = \left\lceil \frac{1}{m_i} \sum_{j \in J} p_{ij} \text{size}_{ij} \right\rceil$

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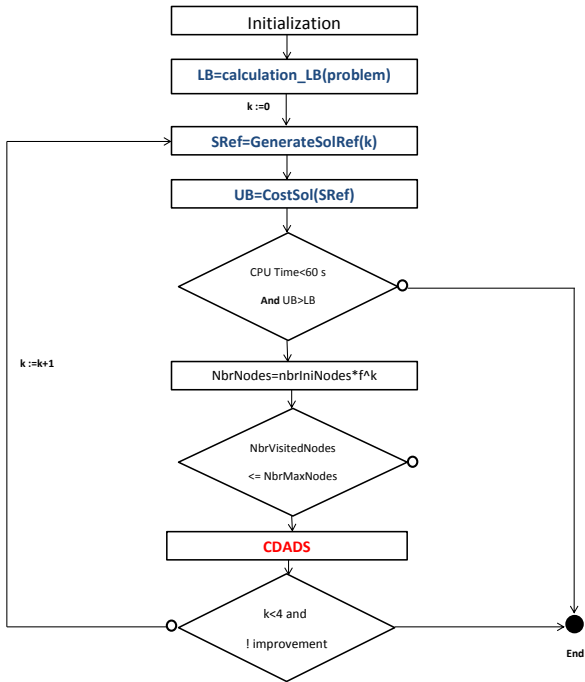
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Test beds

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PC Intel Centrino 2 Duo 2 GHz

OS: Ubuntu

language: C++

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Oğuz *et al.*'s Benchmark, 2004

Size: 300 instances

number of jobs: {5, 10, 20, 50, 100}

number of stages: {2, 5, 8}

2 Categories: 'Type_1' and 'Type_2'

'Type_1': $m_i = 1, \dots, 5$

'Type_2': $m_i = 5$

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Indicators

Deviation (%):

- $100 \times \frac{C_{\max} - LB}{LB}$
- $100 \times \frac{C_{\max} - C_{\max}^*}{C_{\max}^*}$

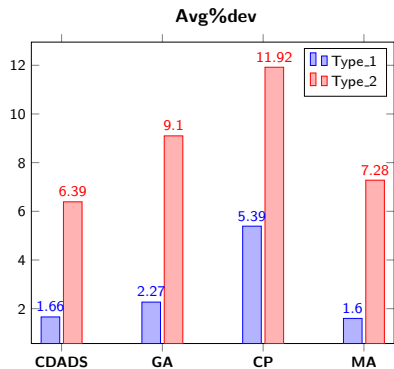
CPU time (sec)

CDADS Performance

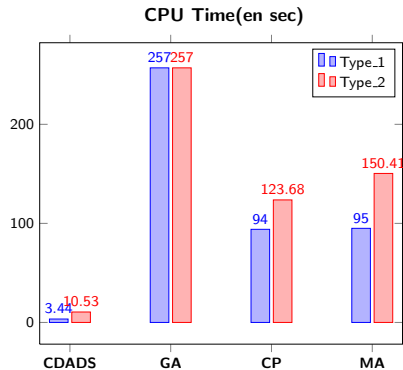
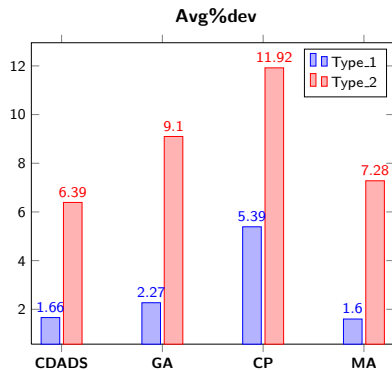
TABLE: CDADS Performance

| n | m | 'Type.1' Problems | | 'Type.2' Problems | |
|-----------------|---|-------------------|-------------|-------------------|--------------|
| | | Avg %dev | CPU (s) | Avg %dev | CPU (s) |
| 5 | 2 | 0.00 | < 0.1 | 0.00 | < 0.1 |
| | 5 | 0.21 | < 0.1 | 0.46 | < 0.1 |
| | 8 | 1.71 | < 0.1 | 0.50 | < 0.1 |
| 10 | 2 | 0.00 | < 0.1 | 1.72 | < 0.1 |
| | 5 | 0.66 | 0.40 | 6.44 | < 0.1 |
| | 8 | 8.47 | < 0.1 | 9.61 | < 0.1 |
| 20 | 2 | 0.05 | 0.10 | 3.34 | 3.10 |
| | 5 | 2.57 | 1.10 | 7.97 | 1.30 |
| | 8 | 5.11 | 0.20 | 15.00 | 1.30 |
| 50 | 2 | 0.49 | 2.30 | 1.74 | 4.20 |
| | 5 | 0.54 | 5.00 | 8.20 | 13.50 |
| | 8 | 1.62 | 6.80 | 12.42 | 33.40 |
| 100 | 2 | 0.08 | 11.10 | 3.32 | 22.80 |
| | 5 | 1.50 | 13.60 | 10.75 | 40.90 |
| | 8 | 1.86 | 11.00 | 14.33 | 47.30 |
| Avg %dev | | 1.66 | | 6.39 | |
| CPU (s) | | | 3.44 | | 10.53 |

CDADS Vs literature



CDADS Vs literature



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The rate of improvement reaches 25

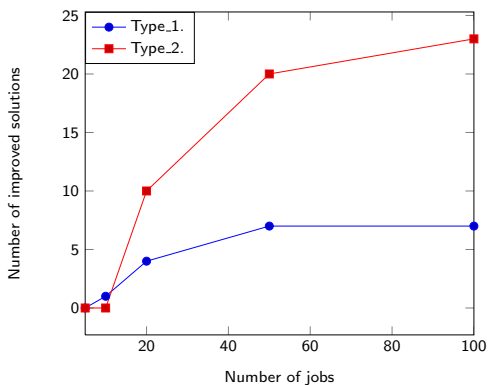


FIGURE: Variation of the number of improved solutions with the number of jobs

Contributions

- CDADS provides better solutions in little CPU time;
- CDADS excels on large instances;
- The proposed LB is efficient [Oğuz & Ercan, 2005];
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Prospects

- Explore the impact of adjacent discrepancies vs. other strategies for limiting the search space ;
- Consider the application of CDADS to simpler problems like classical hybrid flow shop ($size_{ij} = 1, \forall i, j$) ;
- Adapt the proposed implementation of discrepancy search to more general scheduling problems, in particular the Resource-Constrained Project Scheduling Problem ;
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