Ordonnancement de projets avec échec des activités

Groupe MOGISA

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This talk is largely based on

De Reyck, B. and Leus, R. (2008). R&D-project scheduling when activities may fail. *IIE Transactions* 40(4), 367-384.

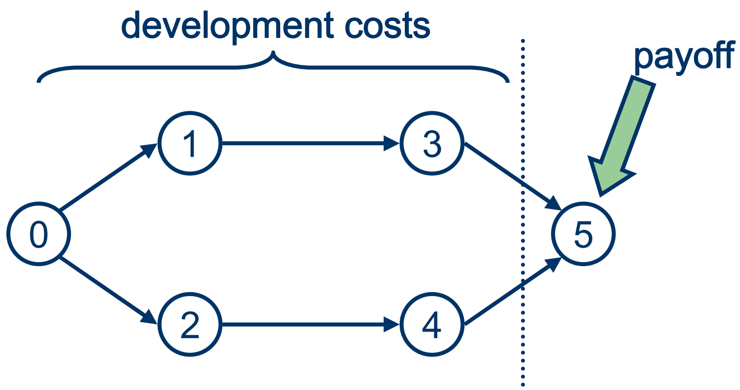
De Reyck, B., Grushka-Cockayne, Y. and Leus, R. (2007). A new challenge in project scheduling: the incorporation of activity failures. *Tijdschrift voor Economie en Management* LII(3), 411-434.

Introduction

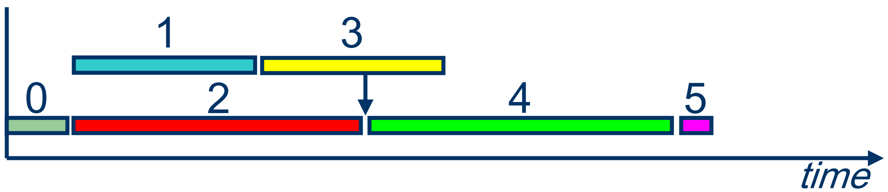
- Project: unique undertaking, aimed at accomplishing a specific non-routine or low-volume task
  $\rightarrow$ Building, software development, infrastructure, …
- Research & Development (R&D) projects, especially NPD: pharmaceuticals, hightech, innovation, ...
- Each project activity has a cost [negative cash flow] and a probability of success – e.g. request for loan, marketing study, apply for building permit, toxicology tests, FDA review, absence of undesirable side-effects, …
- Project pay-off (launch) [positive cash flow] only occurs if *all* activities are successful – especially pharma & agricultural chemicals sector (not modular)
- Time value of money – discounting
- Development of a project schedule with objective: expected NPV (eNPV)

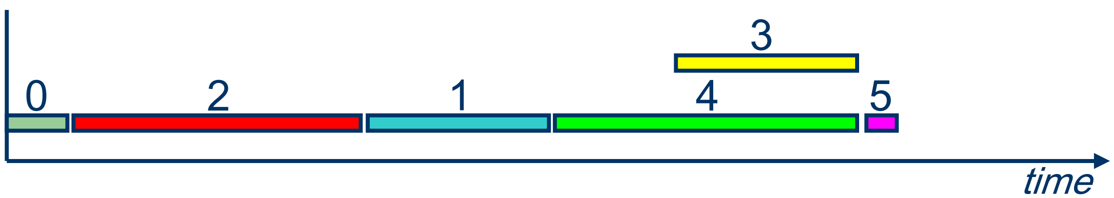
Introduction (2)

precedence network of activities :

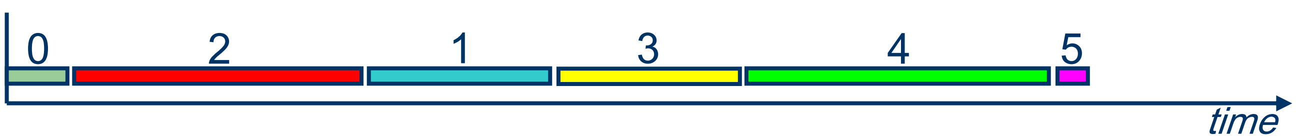


- No (renewable) resource constraints, no duration uncertainty, uncorrelated activity success





"serial" schedule



- Trade-off early project completion *if successful* vs. reduction of costs *if failure*

Problem formulation

N	$= \{0, 1, \ldots, n\}$, the set of project activities
c_i	cash flow of activity $i \in N \backslash \{n\}$, non-positive integer; incurred at the start of the activity
C	integer end-of-project payoff, ≥ 0; received at the start of activity n
d_i	duration of activity $i \in N$
p_i	probability of technical success (PTS) of activity $i \in N \backslash \{n\}$; outcome known at the end of the activity
r	continuous discount rate: the present value of cash flow c incurred at time t equals ce^{-rt}
A	partial order on N representing precedence constraints
δ	project deadline

- We let activity 0 be a dummy representing project start: $c_0 = 0$, $d_0 = 0$, $p_0 = 1$
- Decision variables:

s_i	starting time of activity i; starting-time vector $\mathbf{s}$ is a schedule

- Constraints:
  - A imposes $s_i + d_i \leq s_j$, $\forall (i,j) \in A$
  - deadline δ

Problem formulation & properties

Additional variables:

$$q_i(\mathbf{s}) = \text{probability that all activities ending no later than } s_i \text{ succeed}$$
$$= \prod_{\substack{k \in N: \\ s_k + d_k \leq s_i}} p_k; \text{ remark that } q_n \text{ is a constant.}$$

Objective: maximize the expected net present value (eNPV) of the schedule:

$$\max \qquad q_n C e^{-r s_n} + \sum_{i=1}^{n-1} q_i(\mathbf{s}) c_i e^{-r s_i}$$

The resulting problem is called *R&D Project Scheduling Problem* (*RDPSP*)

Theorem 1: if $r = 0$ and $\delta \geq \sum_{i \in N} d_i$ then an optimal schedule exists that imposes a complete order on N

$\rightarrow$ Problem LCT = “least-cost testing”: solution space restricted to complete orders

Properties (2)

If $r = 0$ and $\delta \geq \Sigma_{i \in N} d_i$ then

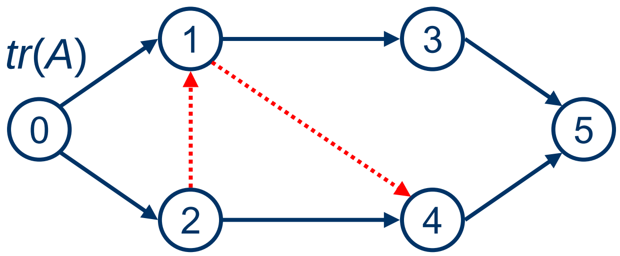
- if $A = \varnothing$ (*no precedence constraints*) then each optimal complete order sequences the activities in non-increasing order of $c_i / (1 - p_i)$ (Mitten, 1960; Butterworth, 1972)
- if A consists of a number of *parallel chains*, a poly-time algorithm exists (Chiu et al., 1999)
- if $G(N,A)$ is *series-parallel*, a poly-time algorithm exists (Monma and Sidney, 1979)

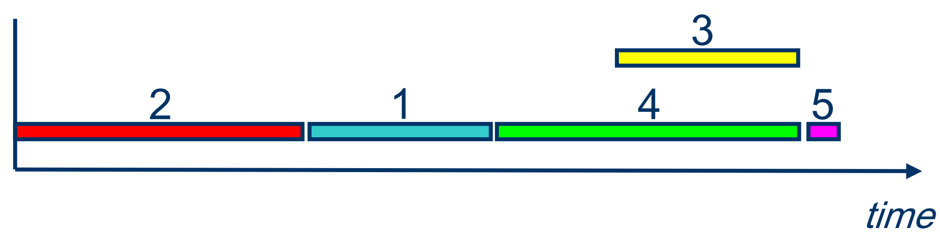
Theorem 2: RDPSP is NP-hard, even if $r = 0$, $C = 0$, all $d_i = 1$ and $\delta \geq \Sigma_{i \in N} d_i$ (reduction from 1 | *prec* | $\Sigma w_j C_j$ (Lenstra and Rinnooy Kan, 1978))

Corollary: LCT is NP-hard under the same conditions as Theorem 2

Order-theoretic approach to scheduling

- Objective function: $\max \quad q_n C e^{-r s_n} + \sum_{i=1}^{n-1} q_i(\mathbf{s}) c_i e^{-r s_i}$
- *E* is an *acyclic extension* of *A* if $E \supseteq A$ and $G(N,E)$ acyclic
- For given extension *E*:
  - values ("information flows") $y_i(E)$ are implicit; we substitute y_i for q_i
  - optimal start times via CPM late-start ($s_0 = 0$ if eNPV ≥ 0; $s_n = \delta$ otherwise)





$A = \{ (0,1),(0,2),(0,3),(0,4),(0,5),(1,3),(1,5),(2,4),(2,5),(3,5),(4,5) \}$

$E = A \cup \{ (1,4),(2,1) \}$

$y_1(E) = p_2$; $y_2(E) = 1$; $y_3(E) = p_1p_2$; $y_4(E) = p_1p_2$; $y_5(E) = p_1p_2p_3p_4$ (with $p_0 = 1$)

- Instead of producing starting times directly, we enumerate all acyclic extensions of *A*. This enumeration is embedded in a B&B procedure.

Computational experiments*

- CPU time is strongly dependent on |*A*| (~ order strength)
- $Imp(\mathbf{s}_1,\mathbf{s}_2) = (g(\mathbf{s}_2) - g(\mathbf{s}_1)) / |g(\mathbf{s}_1)|$
- Truncated B&B. Performance with varying time limits, for $n = 25$:

time limit	opt (/60)	nodes	$Imp(\mathbf{s}_{(0)}, \mathbf{s}_{(i)})$
0	0	0	0.00%
1	20	62,240	+21.67%
5	23	274,900	+23.61%
20	28	996,001	+25.33%
100	35	4,045,108	+27.70%
250	38	8,958,914	+28.60%
1000	41	31,498,214	+31.59%

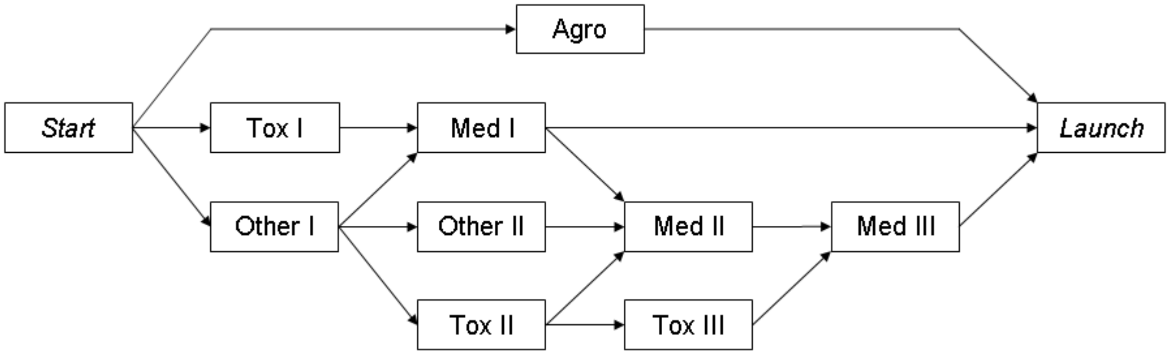
$\mathbf{s}_{(i)}$ = schedule for time limit i ;

$\mathbf{s}_{(0)}$ is a heuristic LB

* Coded in C using MS VC++ 6.0; running on Dell Optiplex GX620, Intel Pentium 4, 2.80 GHz processor, 1 GB RAM

* Instances generated using RanGen (without cash flows and probabilities)

Real-life example



Drug development project, biotech company, Cambridge, England.

task	cash flow (£)	duration (months)	PTS
Agro	−12,000,000	60	100%
Tox I	−300,000	6	75%
Other I	−1,000,000	8	100%
Med I	−200,000	8	80%
Other II	−300,000	8	100%
Tox II	−100,000	6	75%
Med II	−200,000	10	80%
Tox III	−700,000	9	75%
Med III	−400,000	20	60%
Launch	300,000,000	-	-

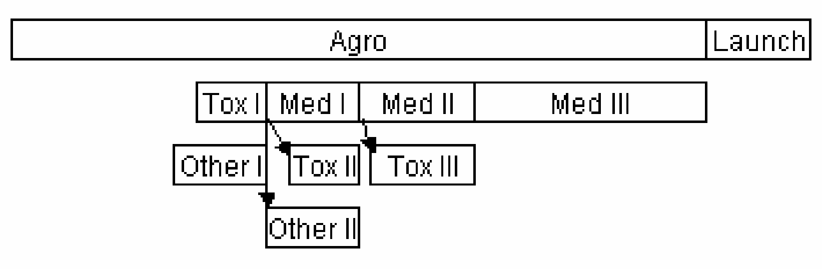
Estimated overall probability of success is 16.2%.

r = 1% per month.

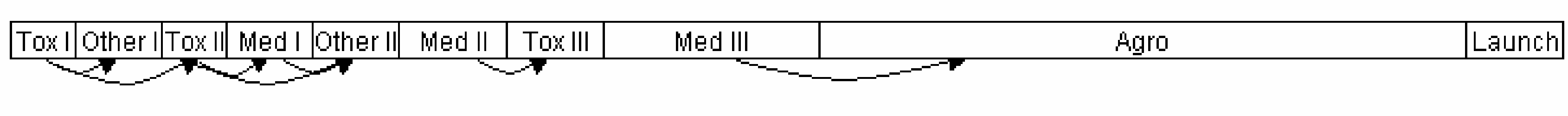
[Crama, P., De Reyck, B., Degraeve, Z. and Chong, W. 2007. R&D project valuation and licensing negotiations at Phytopharm plc. *Interfaces* 37(5), 472–487.]

Real-life example (2)

- CPM Late-start schedule: eNPV of approx. £13 million



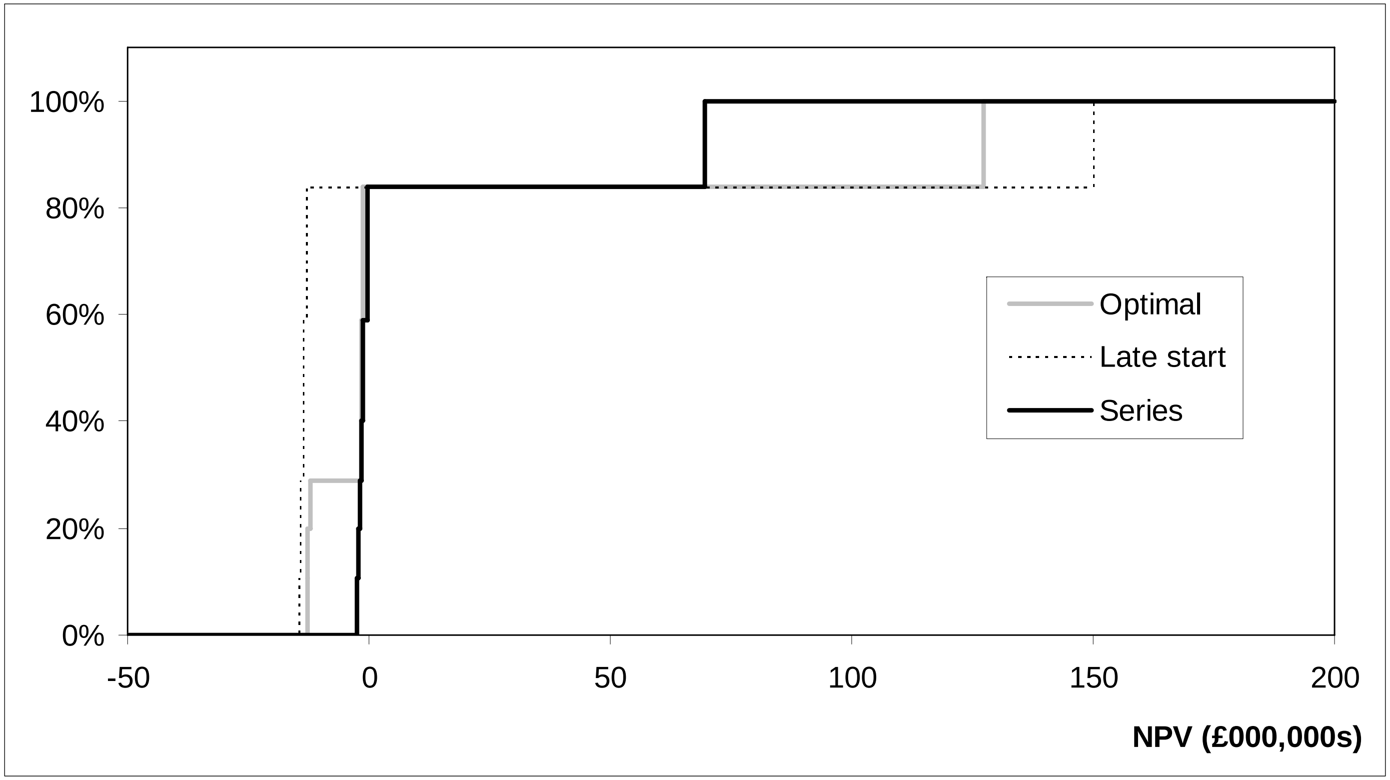
- Serial schedule: eNPV of approx. £10 million



- Optimal schedule: eNPV of approx. £16 million

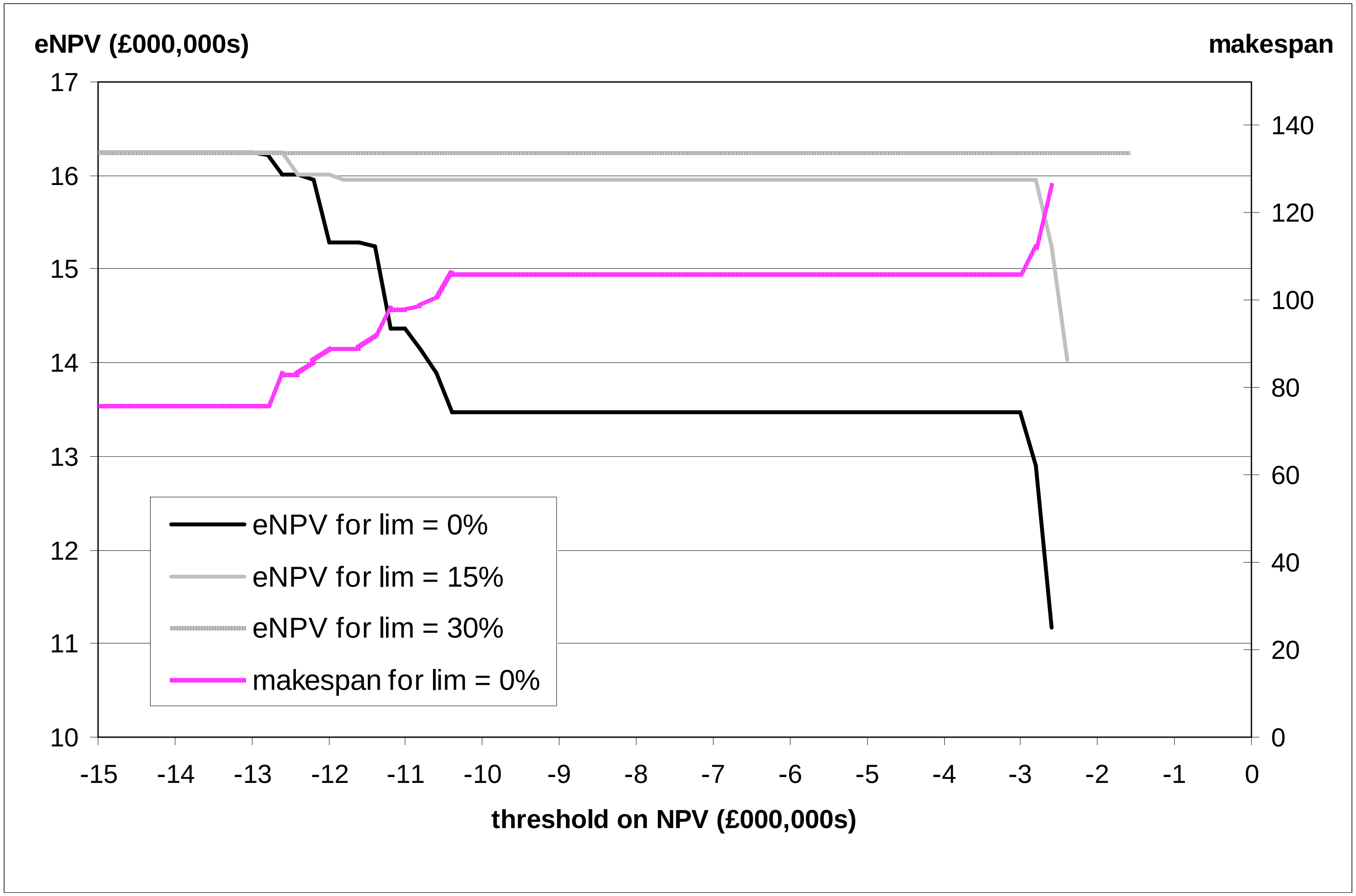
Risk preferences

- *Expected* NPV vs. *actual* project realizations
- cdf of the NPV of a schedule: evaluate entire risk profile
- Determining the cdf of the NPV of an arbitrary schedule in time $O(n \log n)$ (to be compared with stochastic activity durations)



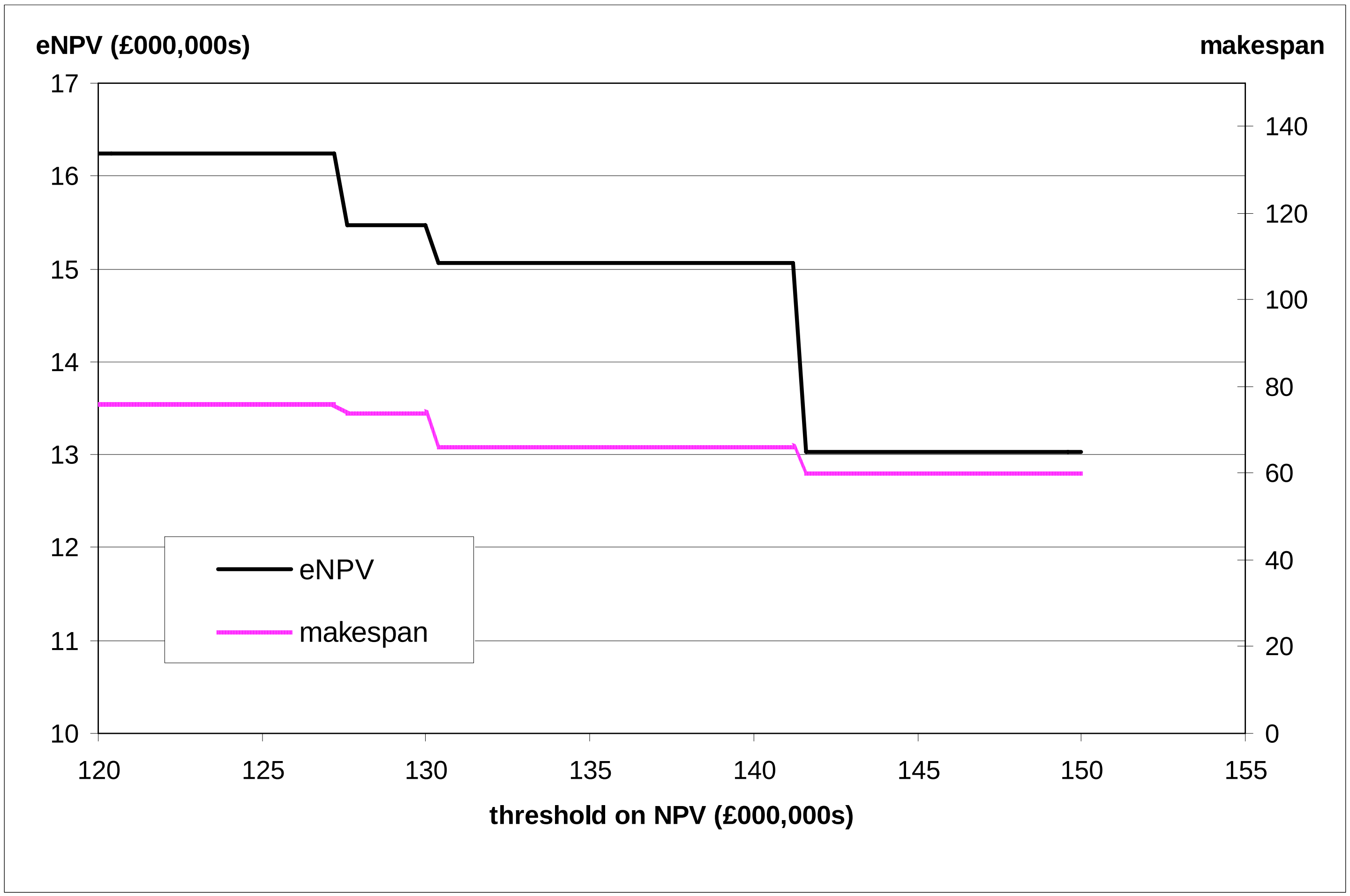
Risk preferences (2)

- « downside risk » = probability that NPV < threshold



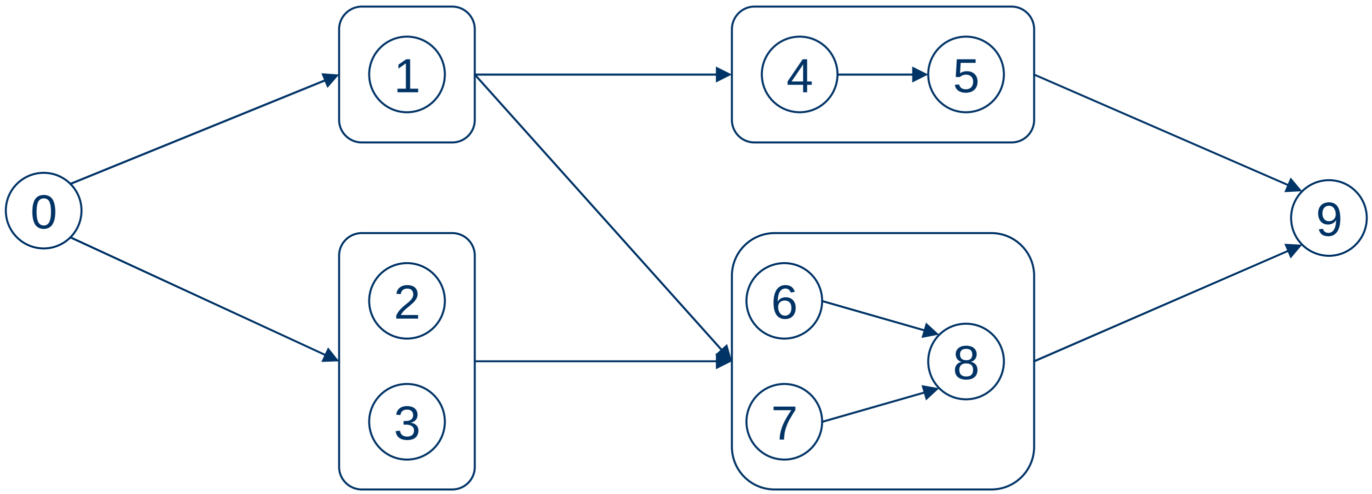
Risk preferences (3)

- « upside potential » = probability that NPV ≥ threshold ?
  Here: NPV *in case of project success* should not be lower than a threshold.



General problem formulation

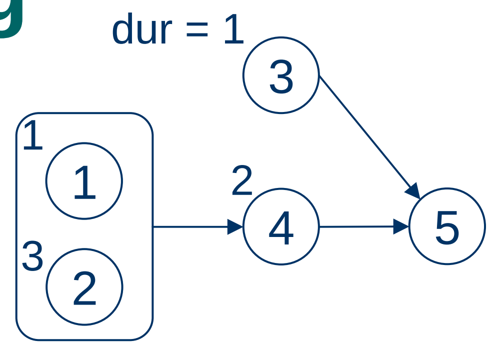
- Literature on sequential testing / scheduling and *k*-out-of-*n* reliability systems:
  1-out-of-*n* = ‘parallel system’; *n*-out-of-*n* = ‘series system’
  $\rightarrow$ only *sequential* testing; no discounting; project success via *no.* of successful act.
- Set of modules $M = \{ 0, 1, ..., m \}$; each module $k \in M$ has a set of activities N_k
- Precedence constraints may apply both between and within modules

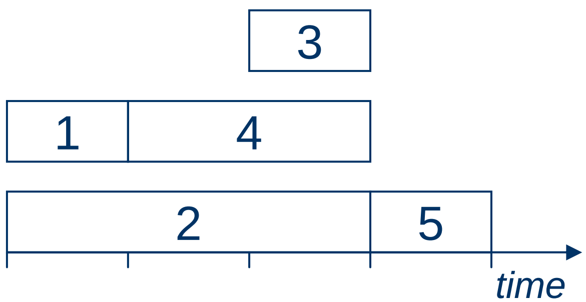


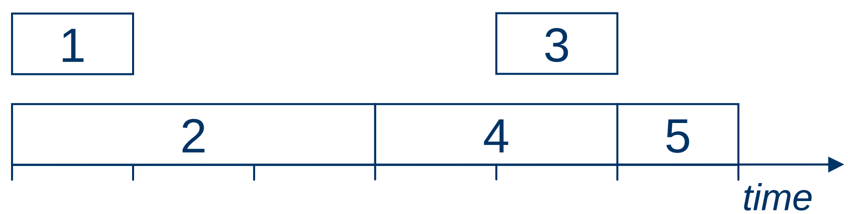
- Each activity has a (fixed) duration, a cost and a probability of success
- A module completes and is successful when one of its activities succeeds
- End-of-project payoff is obtained if all modules are successful

Stochastic scheduling

- What is a solution? A schedule?







- In line with the literature on stochastic programming, especially stochastic scheduling, a solution is a *policy* that defines *actions* at *decision times*
- A *globally optimal* policy is optimal over the class of *all* policies
- In stochastic scheduling, one usually restricts attention to subclasses that have a simple combinatorial representation and where decision points are limited in number
- Activity selection!

Single-module projects

= a 1-out-of-n system: alternative technologies, trials for the same result, or fallback options; *selection* of activities now becomes an issue!

- Define an *elementary policy* as a policy that adheres to a deterministic schedule and imitates it until project completion or the first activity success (*not* selecting a task equates with starting time = + ∞)

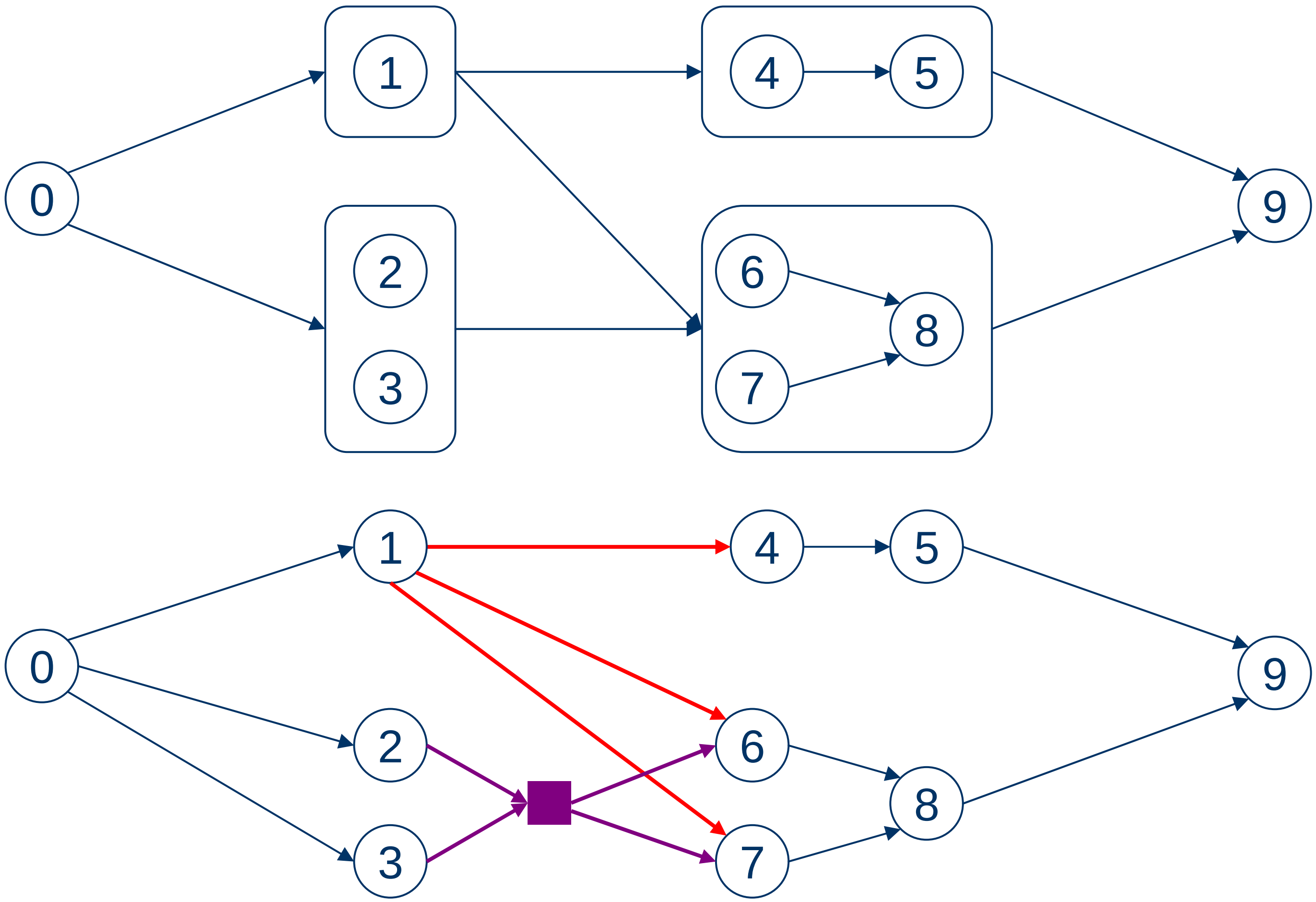
  <u>Theorem 3:</u> an optimal elementary scheduling policy is globally optimal for 1-out-of-n systems

- <u>Theorem 4</u>: if the discount rate is 0 then an optimal schedule exists for 1-out-of-n that imposes a complete order on the set of jobs

- <u>Theorem 5</u>: 1-out-of-n is NP-hard, even if the discount rate is 0

  1-out-of-n is equivalent with n-out-of-n with zero discount rate, for large payoff; Unfortunately, this equivalence no longer holds when the discount rate $\neq 0$ because of the timing of obtaining C

The general problem…



⇒ Use AND/OR-type precedence constraints? Yes, but

– AND/OR precedence constraints combined with non-regular objectives?
Include requirement of “success” in some of the precedence constraints?

Summary & conclusions

- Model for scheduling R&D projects to maximize the expected NPV when the activities have inherent possibility of failure: intermediate step of activity modules
- The problem is NP-hard
- For n-out-of-n systems (RDPSP):
  - Branch-and-bound algorithm
  - Incorporation of risk preferences
- Some characteristics of dominant sets of solutions
  - n-out-of-n: a late-start schedule for an extension of the input graph (determ. NPV)
  - 1-out-of-n: early start may be in order; no determ. NPV anymore ‘
- ‘Stylized’ model: possibly not always of immediate use for decision support