

Jean-Paul Laumond (Editor)

# Robot Motion Planning and Control



Laboratoire d'Analyse et d'Architecture des Systèmes  
Centre National de la Recherche Scientifique  
LAAS report 97438



Previously published as:  
Lectures Notes in Control and Information Sciences 229.  
Springer, ISBN 3-540-76219-1, 1998, 343p.



# Foreword

How can a robot decide what motions to perform in order to achieve tasks in the physical world ?

The existing industrial robot programming systems still have very limited motion planning capabilities. Moreover the field of robotics is growing: space exploration, undersea work, intervention in hazardous environments, servicing robotics . . . Motion planning appears as one of the components for the necessary autonomy of the robots in such real contexts. It is also a fundamental issue in robot simulation software to help work cell designers to determine collision free paths for robots performing specific tasks.

## Robot Motion Planning and Control requires interdisciplinarity

The research in robot motion planning can be traced back to the late 60's, during the early stages of the development of computer-controlled robots. Nevertheless, most of the effort is more recent and has been conducted during the 80's (*Robot Motion Planning*, J.C. Latombe's book constitutes the reference in the domain).

The position (configuration) of a robot is normally described by a number of variables. For mobile robots these typically are the position and orientation of the robot (i.e. 3 variables in the plane). For articulated robots (robot arms) these variables are the positions of the different joints of the robot arm. A motion for a robot can, hence, be considered as a path in the configuration space. Such a path should remain in the subspace of configurations in which there is no collision between the robot and the obstacles, the so-called free space. The motion planning problem asks for determining such a path through the free space in an efficient way.

Motion planning can be split into two classes. When all degrees of freedom can be changed independently (like in a fully actuated arm) we talk about *holonomic motion planning*. In this case, the existence of a collision-free path is characterized by the existence of a connected component in the free configuration space. In this context, motion planning consists in building the free configuration space, and in finding a path in its connected components.

Within the 80's, Roboticians addressed the problem by devising a variety of heuristics and approximate methods. Such methods decompose the configuration space into simple cells lying inside, partially inside or outside the free space. A collision-free path is then searched by exploring the adjacency graph of free cells.

In the early 80's, pioneering works showed how to describe the free configuration space by algebraic equalities and inequalities with integer coefficients (i.e. as being a semi-algebraic set). Due to the properties of the semi-algebraic sets induced by the Tarski-Seidenberg Theorem, the connectivity of the free configuration space can be described in a combinatorial way. From there, the road towards methods based on Real Algebraic Geometry was open. At the same time, Computational Geometry has been concerned with combinatorial bounds and complexity issues. It provided various exact and efficient methods for specific robot systems, taking into account practical constraints (like environment changes).

More recently, with the 90's, a new instance of the motion planning problem has been considered: planning motions in the presence of kinematic constraints (and always amidst obstacles). When the degrees of freedom of a robot system are not independent (like e.g. a car that cannot rotate around its axis without also changing its position) we talk about *nonholonomic motion planning*. In this case, any path in the free configuration space does not necessarily correspond to a feasible one. Nonholonomic motion planning turns out to be much more difficult than holonomic motion planning. This is a fundamental issue for most types of mobile robots. This issue attracted the interest of an increasing number of research groups. The first results have pointed out the necessity of introducing a Differential Geometric Control Theory framework in nonholonomic motion planning.

On the other hand, at the motion execution level, nonholonomy raises another difficulty: the existence of stabilizing smooth feedback is no more guaranteed for nonholonomic systems. Tracking of a given reference trajectory computed at the planning level and reaching a goal with accuracy require non-standard feedback techniques.

Four main disciplines are then involved in motion planning and control. However they have been developed along quite different directions with only little interaction. The coherence and the originality that make motion planning and control a so exciting research area come from its *interdisciplinarity*. It is necessary to take advantage from a common knowledge of the different theoretical issues in order to extend the state of the art in the domain.

## About the book

The purpose of this book is not to present a current state of the art in motion planning and control. We have chosen to emphasize on recent issues which have been developed within the 90's. In this sense, it completes Latombe's book published in 1991. Moreover an objective of this book is to illustrate the necessary interdisciplinarity of the domain: the authors come from Robotics,

Computational Geometry, Control Theory and Mathematics. All of them share a common understanding of the robotic problem.

The chapters cover recent and fruitful results in motion planning and control. Four of them deal with nonholonomic systems; another one is dedicated to probabilistic algorithms; the last one addresses collision detection, a critical operation in algorithmic motion planning.

*Nonholonomic Systems* The research devoted to nonholonomic systems is motivated mainly by mobile robotics. The first chapter of the book is dedicated to nonholonomic path planning. It shows how to combine geometric algorithms and control techniques to account for the nonholonomic constraints of most mobile robots. The second chapter develops the mathematical machinery necessary to the understanding of the nonholonomic system geometry; it puts emphasis on the nonholonomic metrics and their interest in evaluating the combinatorial complexity of nonholonomic motion planning. In the third chapter, optimal control techniques are applied to compute the optimal paths for car-like robots; it shows that a clever combination of the maximum principle and a geometric viewpoint has permitted to solve a very difficult problem. The fourth chapter highlights the interactions between feedback control and motion planning primitives; it presents innovative types of feedback controllers facing nonholonomy.

*Probabilistic Approaches* While complete and deterministic algorithms for motion planning are very time-consuming as the dimension of the configuration space increases, it is now possible to address complicated problems in high dimension thanks to alternative methods that relax the completeness constraint for the benefit of practical efficiency and probabilistic completeness. The fifth chapter of the book is devoted to probabilistic algorithms.

*Collision Detection* Collision checkers constitute the main bottleneck to conceive efficient motion planners. Static interference detection and collision detection can be viewed as instances of the same problem, where objects are tested for interference at a particular position, and along a trajectory. Chapter six presents recent algorithms benefiting from this unified viewpoint.

The chapters are self-contained. Nevertheless, many results just mentioned in some given chapter may be developed in another one. This choice leads to repetitions but facilitates the reading according to the interest or the background of the reader.

## On the origin of the book

All the authors of the book have been involved in PROMotion. PROMotion was a European Project dedicated to robot motion planning and control. It has progressed from September 1992 to August 1995 in the framework of the Basic Research Action of ESPRIT 3, a program of research and development in Information Technologies supported by the European Commission (DG III). The work undertaken under the project has been aimed at solving concrete problems. Theoretical studies have been mainly motivated by a practical efficiency. Research in PROMotion has then provided methods and their prototype implementations which have the potential of becoming key components of recent programs in advanced robotics.

In few numbers, PROMotion is a project whose cost has been 1.9 MEcus<sup>1</sup> (1.1 MEcus supported by European Community), for a total effort of more than 70 men-year, 179 research reports (most them have been published in international conferences and journals), 10 experiments on real robot platforms, an International Spring school and 3 International Workshops. This project has been managed by LAAS-CNRS in Toulouse; it has involved the “Universitat Politecnica de Catalunya” in Barcelona, the “Ecole Normale Supérieure” in Paris, the University “La Sapienza” in Roma, the Institute INRIA in Sophia-Antipolis and the University of Utrecht.

J.D. Boissonnat (INRIA, Sophia-Antipolis), A. De Luca (University “La Sapienza” of Roma), M. Overmars (Utrecht University), J.J. Risler (Ecole Normale Supérieure and Paris 6 University), C. Torras (Universitat Politecnica de Catalunya, Barcelona) and the author make up the steering committee of PROMotion. This book benefits from contributions of all these members and their co-authors and of the work of many people involved in the project.

On behalf of the project committee, I thank J. Wejchert (Project officer of PROMotion for the European Community), A. Blake (Oxford University), H. Chochon (Alcatel) and F. Wahl (Braunschweig University) who acted as reviewers of the project during three years. Finally I thank J. Som for her efficient help in managing the project and M. Herrb for his help in editing this book.

Jean-Paul Laumond  
LAAS-CNRS, Toulouse  
August 1997

---

<sup>1</sup> US \$ 1  $\approx$  1 Ecu

# List of Contributors

A. Bellaïche  
Département de Mathématiques  
Université de Paris 7  
2 Place Jussieu  
75251 Paris Cedex 5  
France  
abellaic@mathp7.jussieu.fr

A. De Luca  
Dipartimento di Informatica  
e Sistemistica  
Università di Roma “La Sapienza”  
Via Eudossiana 18  
00184 Roma  
Italy  
adeluca@giannutri.caspur.it

P. Jiménez  
Institut de Robòtica  
i Informàtica Industrial  
Gran Capità, 2  
08034-Barcelona  
Spain  
jimenez@iri.upc.es

J.P. Laumond  
LAAS-CNRS  
7 Avenue du Colonel Roche  
31077 Toulouse Cedex 4  
France  
jpl@laas.fr

J.D. Boissonnat  
INRIA Centre de Sophia Antipolis  
2004, Route des Lucioles BP 93  
06902 Sophia Antipolis Cedex,  
France  
boissonn@sophia.inria.fr

F. Jean  
Institut de Mathématiques  
Université Pierre et Marie Curie  
Tour 46, 5ème étage, Boite 247  
4 Place Jussieu  
75252 Paris Cedex 5  
France  
jean@math.jussieu.fr

F. Lamiroux  
LAAS-CNRS  
7 Avenue du Colonel Roche  
31077 Toulouse Cedex 4  
France  
lamiroux@laas.fr

G. Oriolo  
Dipartimento di Informatica  
e Sistemistica  
Università di Roma “La Sapienza”  
Via Eudossiana 18  
00184 Roma  
Italy  
oriolo@giannutri.caspur.it

M. H. Overmars  
Department of Computer Science,  
Utrecht University  
P.O.Box 80.089,  
3508 TB Utrecht,  
the Netherlands  
markov@cs.ruu.nl

C. Samson  
INRIA Centre de Sophia Antipolis  
2004, Route des Lucioles BP 93  
06902 Sophia Antipolis Cedex,  
France  
Claude.Samson@sophia.inria.fr

P. Souères  
LAAS-CNRS  
7 Avenue du Colonel Roche  
31077 Toulouse Cedex 4  
France  
soueres@laas.fr

F. Thomas  
Institut de Robòtica  
i Informàtica Industrial  
Gran Capità, 2  
08034-Barcelona  
Spain  
thomas@iri.upc.es

J.J. Risler  
Institut de Mathématiques  
Université Pierre et Marie Curie  
Tour 46, 5ème étage, Boite 247  
4 Place Jussieu  
75252 Paris Cedex 5  
France  
risler@math.jussieu.fr

S. Sekhavat  
LAAS-CNRS  
7 Avenue du Colonel Roche  
31077 Toulouse Cedex 4  
France  
sepanta@laas.fr

P. Švestka  
Department of Computer Science,  
Utrecht University  
P.O.Box 80.089,  
3508 TB Utrecht,  
the Netherlands  
petr@cs.ruu.nl

C. Torras  
Institut de Robòtica  
i Informàtica Industrial  
Gran Capità, 2  
08034-Barcelona  
Spain  
torras@iri.upc.es



# Table of Contents

|   |            |
|---|------------|
| <b>Guidelines in Nonholonomic Motion Planning for Mobile Robots</b> | <b>1</b>   |
| <i>J.P. Laumond, S. Sekhavat, F. Lamiroux</i>                       |            |
| 1 Introduction  | 1          |
| 2 Controllabilities of mobile robots                                | 2          |
| 3 Path planning and small-time controllability                      | 10         |
| 4 Steering methods  | 13         |
| 5 Nonholonomic path planning for small-time controllable systems    | 23         |
| 6 Other approaches, other systems                                   | 42         |
| 7 Conclusions   | 44         |
| <b>Geometry of Nonholonomic Systems</b>                             | <b>55</b>  |
| <i>A. Bellaïche, F. Jean, J.-J. Risler</i>                          |            |
| 1 Symmetric control systems: an introduction                        | 55         |
| 2 The car with $n$ trailers   | 73         |
| 3 Polynomial systems  | 82         |
| <b>Optimal Trajectories for Nonholonomic Mobile Robots</b>          | <b>93</b>  |
| <i>P. Souères, J.-D. Boissonnat</i>                                 |            |
| 1 Introduction  | 93         |
| 2 Models and optimization problems                                  | 94         |
| 3 Some results from Optimal Control Theory                          | 97         |
| 4 Shortest paths for the Reeds-Shepp car                            | 107        |
| 5 Shortest paths for Dubins' Car                                    | 141        |
| 6 Dubins model with inertial control law                            | 153        |
| 7 Time-optimal trajectories for Hilare-like mobile robots           | 161        |
| 8 Conclusions   | 166        |
| <b>Feedback Control of a Nonholonomic Car-Like Robot</b>            | <b>171</b> |
| <i>A. De Luca, G. Oriolo, C. Samson</i>                             |            |
| 1 Introduction  | 171        |
| 2 Modeling and analysis of the car-like robot                       | 179        |
| 3 Trajectory tracking   | 189        |
| 4 Path following and point stabilization                            | 213        |
| 5 Conclusions   | 247        |
| 6 Further reading   | 249        |

|   |            |
|---|------------|
| <b>Probabilistic Path Planning</b> .....                            | <b>255</b> |
| <i>P. Švestka, M. H. Overmars</i>                                   |            |
| 1 Introduction.....   | 255        |
| 2 The Probabilistic Path Planner .....                              | 258        |
| 3 Application to holonomic robots .....                             | 266        |
| 4 Application to nonholonomic robots .....                          | 270        |
| 5 On probabilistic completeness of probabilistic path planning..... | 279        |
| 6 On the expected complexity of probabilistic path planning .....   | 285        |
| 7 A multi-robot extension .....                                     | 291        |
| 8 Conclusions .....   | 300        |
| <b>Collision Detection Algorithms for Motion Planning</b> .....     | <b>305</b> |
| <i>P. Jiménez, F. Thomas, C. Torras</i>                             |            |
| 1 Introduction.....   | 305        |
| 2 Interference detection.....                                       | 306        |
| 3 Collision detection .....   | 317        |
| 4 Collision detection in motion planning .....                      | 335        |
| 5 Conclusions .....   | 338        |