Trace Channel Nets

Jean Fanchon , LAAS du CRNS, 7 av. du Col. Roche, 31077 Toulouse France . fanchon@laas.fr

Abstract

This paper presents a new class of nets which includes and extends both Coloured Nets and Fifo Nets by defining weights on edges and markings of places as traces on a concurrent (trace) alphabet. Considering different independence relations on the alphabet, from the maximal one to the empty one (yielding words), Trace Channel Nets open a hierarchy of semantics on a single net structure. Furthermore a field of investigation results from the relationship between the independence on the alphabet and the behaviours of the net. In particular we show that the boundedness of a TCNet is related to particular independence relations, maximal w.r.t. boundedness, that TCNets can be applied to the study of Communicating Finite State Machines (using communication through a trace channel), and that they define a hierarchy of partial order semantics for Nets.

Keywords: Mazurkiewicz traces, Fifo Nets, Coloured Nets, concurrency, asynchronous communication, concurrent automata, recognizability.

1 Introduction.

Partial orders and "true concurrency" models have been widely generalised in the last two decades for modeling distributed and/or concurrent processes, while their relationships with Petri Nets Theory played a central role ([BC] [BD] [Gi] [Gr] [K1] [NSW] [Pr] [Vo]). Among the many variations on Petri Nets, the ones concerning the structure of arcs labels, tokens and markings led to important developments [Je]. In the present paper, we define a new class of nets, Trace Channel Nets, TCNets in short, where the arcs labels and the markings of the places are traces of a concurrent alphabet [Ma]. They can be viewed as bridging the gap between Simple Coloured Nets, where markings are unordered multisets of colours, and Fifo Nets [FM], where markings are totally ordered multisets.

While Fifo Nets and Communicating Finite State Machines have been used to model processes with asynchronous communication and with total orders on messages, Trace Channel Nets (and Partial Order Channel Nets, see [FB] for the original report) propose in particular a good model for Partial Order Connections and Partial Order Protocols, developed for multimedia applications [ACC][CDL]. The trace or partial order structure of a channel's marking models any precedence or causality relation between alphabetic elementary components, like the partial order on the messages of a POC. Furthermore it acts as a concurrent behaviour "specification" for the output transitions .

Since the late 80's, different formal language theory approaches and results have been successfully extended or adapted to trace languages, [Oc][GPZ], and more recently, to pomsets languages [Dr][LW]. Trace Channel Nets can be comprehensively seen as a 'natural' generalisation of existing classes of Nets, corresponding to the extention from words to traces, and benefit from these advances. In Fifo Nets the firing of a transition truncates a prefix of the words marking its input places (the label of the corresponding input arc), and appends suffixes (the words labelling the output arcs), to the markings of its output places. Trace Channel Nets generalise this paradigm: arcs are labelled by traces on a concurrent alphabet, and the notions of trace prefix, residue and concatenation are used to define the TCNet's firing rules (instead of their 'word' versions).

A concurrent alphabet is a pair (A, I) where I is a binary irreflexive relation on an alphabet A, called independence relation. A trace is an equivalence class of words of A^* obtained by permutations of independent letters. The quotient set denoted $(A/\approx_I)$ is a cancellative quotient monoid of A^* called the trace monoid of (A, I). The intuition of the semantics of TCNets is that when two transitions t_1 and t_2 input the letters a and b into the same channel, the order of their firings will be relevant for the resulting marking only if the letters are dependent in the alphabet.

If t_1 and t_2 output the letters a and b from a channel, the markings of the channel induce an order on the firings of t_1 and t_2 only if the two letters are dependent.

A first application of TCNets concerns Communicating Finite State Machines. CFSMs use fifo channels to exchange messages, and are a typical case of Fifo Nets. Their extention to trace channels is natural: two safe nets share a trace channel, the first one may input into the channel, the second one may output from it. While the loss of messages has been introduced in CFSM by [CFP], inducing easier verification, Trace Channels relax the total order condition. We give a sufficient condition preserving the rationality of the languages of the two communicating nets, when the channel is taken into account. This condition depends on the rational structure of these languages, and on the labels of the arcs adjacent to the channel, taking into account the dependence relation of the alphabet. It enlights the existence of minimal dependences satisfying the condition, i.e. preserving the rationality of the languages; we think that such a property is typical of the interest of TCNets.

We define the concurrent semantics of TCNets by means of deterministic stably concurrent automata [DK]. They are deterministic transition systems having “local” concurrency relations, defined for each marking on the set of transitions firable from it, and satisfying “good” properties (the diamond, the cube and the inverse cube axioms). The advantages of this definition is that we gain for TCNets the properties worked out for this type of automata, in particular the ones relating recognizable and co-rational sub-languages when the automaton is finite [Dr], i.e. when the net is bounded. Besides the concurrent firings of transitions with no common adjacent places, the trace structure of markings gives the possibility to fire concurrently two transitions that are ouput of the same place, provided that they consume two independent prefixes of the place marking. The concurrency relations on transitions depend on the independence between the letters of the trace alphabet. If we relax dependence on letters, we may add new behaviours to the net, but also increase the degree of concurrency in the existing ones.

Independence relations on an alphabet are ordered by inclusion and have a maximal and a minimal element. Simple Coloured Nets and Fifo Nets can be considered as two extreme cases of TCNets, corresponding to these particular relations. Starting from a Fifo Net and considering the different independences on the token alphabet, we define a full partial order of semantics materialized by automata morphisms from the ones with lower independences towards the more concurrent ones. Furthermore, considering the independences from the point of view of the induced behaviours, some of them appear as maximal for the boundedness of the net, as we mentioned above for CFSMs.

The three TCNets of the figure 1 share the same structure and the same alphabet with different independence relations. The transition t_3 may occur in the Fifo Net net only after the firing of the sequence t_1t_2 , while it may occur in the other ones after any firings of t_1 and t_2 . After the firing of t_3, the Fifo Net may only fire the sequence $t_4t_6t_5t_4$. The increasing independence relations from left to right induce new firing sequences for the transitions t_4, t_5 and t_6, say $t_4t_6t_4t_5$ for the middle net ,and $t_4t_4t_5t_6$ for the net on the right . The concurrent semantics of section 6 is based on equivalence classes of firing sequences. The two sequences $t_4t_6t_4t_5$ and $t_4t_6t_5t_4$, both enabled in the middle net after the firing of t_3, are equivalent, and compose a partial word on t_4, t_5 and t_6, in which the firings of t_4 and t_5 after the one of t_6 are concurrent .

In section 2 we present the necessary background on Trace Theory. Section 3 contains the basic definitions of TCNets and interleaving semantics. Section 4 focuses on the behaviour of the transitions adjacent to a single channel (place), and gives a characterization of the constraints induced by a channel on the behaviour of a TCNet. Section 5 applies the results of the previous section to the study of bounded nets communicating through a trace channel. Section 6 defines the concurrent semantics of TCNets in terms of Automata with Concurrency Relations and shows these ones are stably concurrent. Section 7 defines the hierarchy of concurrent semantics on a single net structure , based on the lattice of independence relations on the channels alphabet.

Related works. If we restrict to Net Theory, besides the above mentioned papers, we can point out Coloured Fifo Nets as defined in [BG] where two types of places are used in Nets, i.e. Fifo places holding words, and bag places holding multisets of colours. In [Fa], the two types of places were defined, but using monochrome tokens for the bags places.

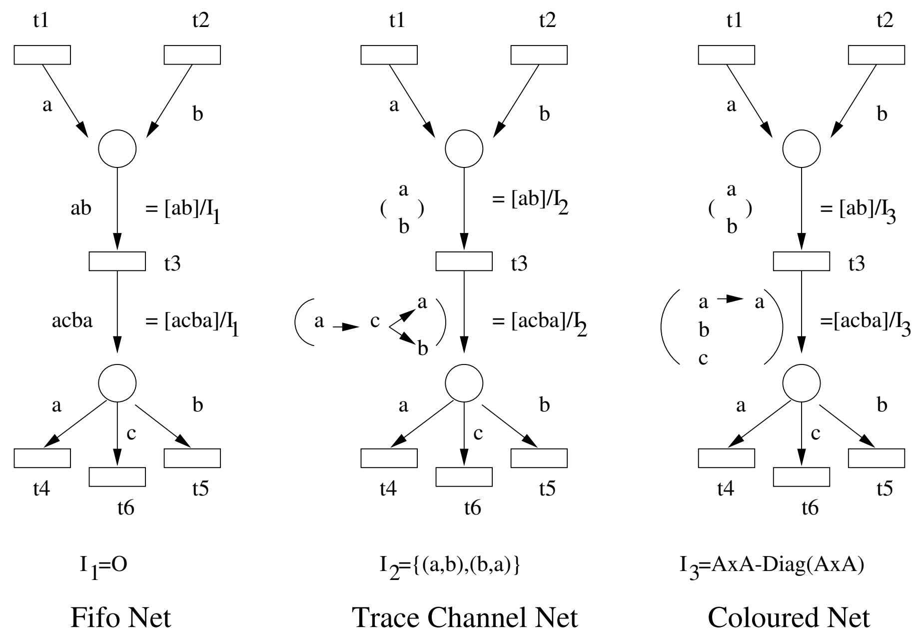

Figure 1.

2 Traces.

A **trace (or concurrent) alphabet** is a pair (A, I) where A is a set, and I a binary symmetric irreflexive relation on A called the **independence** relation. Its complement $D = A \times A - I$, which is symmetric and reflexive, is called the **dependence** relation.

The **Trace monoid** $(A/\approx_I)$(denoted usually $M(A, I)$, but the letter M is already used for markings) is the quotient of the free monoid A^* by the congruence $\approx_I$, the congruence w.r.t. concatenation generated by the set of pairs $\{(ab, ba) \in A^* \times A^* | (a, b) \in I\}$. Let u be a word in A^*, we denote by $[u]_I$ the class of u in $(A/\approx_I)$. Elements of $(A/\approx_I)$ are called traces. The **monoidal operation** (concatenation) $\circ$ on traces $[u]_I \circ [v]_I$ is defined by $[u]_I \circ [v]_I = [uv]_I$. The monoid $(A/\approx_I)$ is **cancellative,** i.e. $[u]_I \circ [v]_I \circ [u']_I = [u]_I \circ [w]_I \circ [u']_I \Longrightarrow [v]_I = [w]_I$ for any $u, u', v, w \in A^*$, or equivalently $u_1 \approx_I u'_1 \wedge u_2 \approx_I u'_2 \wedge u_1 v u_2 \approx_I u'_1 w u'_2 \Longrightarrow v \approx_I w$ for any $u_1, u'_1, u_2, u'_2, v, w \in A^*$.

The empty trace $[\varepsilon]_I$ is also denoted ε. The number of occurrences of a letter $a \in A$ in a trace $s = [u]_I$ is $|s|_a = |u|_a$, the alphabet of a trace s is $\alpha(s) = \{a \in A,\ |s|_a \neq 0\}$, and the size of a trace s is $|s| = \sum_{a \in A} |s|_a$, and $|\varepsilon| = 0$. The multiset $[s]$ of a trace s is $[s] \in \aleph^A$such that $\forall a \in A,\ [s](a) = |s|_a$ ($\aleph$denotes the set of natural numbers).

A trace $[u]_I$ is a **prefix** of a trace $[v]_I$, denoted $[u]_I \preceq [v]_I$, by the definition $[u]_I \preceq [v]_I \Longleftrightarrow \exists w \in A^*, [v]_I = [uw]_I$. The **set of prefixes** of a (trace) language L is denoted $Pref(L)$. If $[u]_I \preceq [v]_I$, we define the **residue** $[v]_I - [u]_I$ as follows: $[v]_I - [u]_I = [w]_I \Longleftrightarrow [v]_I = [uw]_I$. The residue is uniquely defined due to the cancellativity of $\circ$. If $\{s_i\}_{i \in J}$ is a finite indexed set of traces such that $i \neq j \Longrightarrow \alpha(s_i) I \alpha(s_j)$, i.e. any two traces have independent alphabets, then we denote by $\bigcirc_{i \in J} s_i$ the trace concatenation of the elements of $\{s_i\}_{i \in J}$, which is independent of the order of concatenations.

A trace $[u]_I$ is **connected** iff it cannot be decomposed into independent factors, i.e. it cannot be written $[u]_I = [vw]_I$ such that $[v]_I \neq \varepsilon \neq [w]_I$ and $\alpha([v]_I) \times \alpha([w]_I) \cap D = \oslash$. A connected trace $[v]_I$ is a **connected component** of a trace $[u]_I$ iff $[u]_I = [vw]_I$ for some w such that $\alpha([v]_I) \times \alpha([w]_I) \cap D = \oslash$. The co-star iteration of a trace language L is defined by $L^{co-*} = Comp(L)^*$where $Comp(L)$ is the set of connected components of elements of L. A subset of a monoid (resp. trace monoid) is **rational (resp. co-rational)** iff it is generated from finite subsets by finite applications of concatenation, union and star (resp. co-star) operations. An **iterative factor** of a language (trace language) L is a word (a trace) t such that $ut^*v \subseteq L$ for some words u, v. A language $L \subseteq (A/\approx_I)$ is **star-connected** iff every iterative factor is connected for the dependence relation D. A subset of a monoid is **recognizable** iff it is saturated by a finite index monoid congruence. Ochmanski's theorem states in particular that a language $L \subseteq (A/\approx_I)$ is star-connected iff it is recognizable iff it is co-rational [Oc].

3 TCNets structure and interleaving semantics.

This section presents the basic definitions of TCNets, i.e elementary firing rules and related objects. The definitions are very similar to those of Fifo Nets, except the change from the free monoid A^* to the trace one $(A/\approx_I)$.

3.1 TCNets structure.

A **TCNet** is a tuple $N = (A, I, C, T, W)$ where :

1. (A, I) is a trace alphabet ,
2. C, T, are disjoints finite sets . C is the set of channels or places, T the set of transitions,
3. W is the flow relation: $W : C \times T \cup T \times C \longrightarrow (A/\approx_I)$.

A **marking** M is a map from channels to traces : $M : C \longrightarrow (A/\approx_I)$.
The **pre-set** (input set) and the **post-set** (output set) of $x \in C \cup T$ are
${}^\bullet x = \{y \in C \cup T, W(y, x) \neq \varepsilon\}$ and $x^\bullet = \{y \in C \cup T, W(x, y) \neq \varepsilon\}$.

Channel alphabets. The input (resp.output) alphabet of a channel c denoted $\alpha_i(c)$ (resp.$\alpha_o(c)$), is the set of letters in the labels of its input (resp.output) arcs: $\alpha_i(c) = \bigcup\{\alpha(W(t, c)), t \in T\}$ (resp. $\alpha_o(c) = \bigcup\{\alpha(W(c, t)), t \in T\}$). The alphabet of c is $\alpha(c) = \alpha_i(c) \cup \alpha_o(c)$.

Normalised Nets. If the input and output alphabets of c coincide, i.e. $\forall c \in C$, $\alpha_i(c) = \alpha_o(c) = \alpha(c)$ the net is said to be normalised .

Local alphabets. A particular form of TCNets arises where the alphabets of channels are disjoint, i.e. $c \neq c' \Longrightarrow \alpha(c) \cap \alpha(c') = \oslash$, with "local" independences. Each channel has its own concurrent alphabet $(\alpha(c), I(c))$, and this case is easy to achieve by mappings $\alpha(c) \longrightarrow \alpha(c) \times \{c\}$. Except in the section 7, where we consider explicitly local alphabets, the rest of the paper does not depend on this "localisation" property.

We also distinguish semi-alphabetic and alphabetic channels: a channel c is **in-alphabetic**, (resp **out-alphabetic**) iff its input arcs (resp. its ouput arcs) are labelled by at most single letters, i.e $\forall t \in T, |W(t, c)| \leq 1$ (resp. $\forall t \in T, |W(c, t)| \leq 1$). It is alphabetic iff it has both properties.

3.1.1 Two particular cases.

The special cases correspond to the minimal (empty) and maximal independence relations.

Fifo Nets coincide with the TCNets where the independence relation is empty. If $I = \oslash$, the flow relation is then valued on words : $W : C \times T \cup T \times C \longrightarrow A^*$. All the usual definitions and properties of Fifo Nets are those of TCNets with empty independence .

Simple Coloured Nets are tuples $N = (A, C, T, W, l)$, where the flow function maps arcs on multisets of colours, i.e. $W : C \times T \cup T \times C \longrightarrow \aleph^A$. The **maximal independence** relation for a trace alphabet is $I_m = A \times A - \triangle(A)$, due to its irreflexivity ($\triangle(A)$ is the diagonal of the product). A TCNet with the maximal independence I_m behaves like a Simple Coloured Net for the interleaving semantics: it is equivalent in a marking to have a bag containing 2 a's or a trace with a connected component $a.a$. For the concurrent semantics this is not the case because in the first case (2 a's) a transition taking one a may fire twice concurrently, which cannot occur in the second. The section 7.4 aggregates Simple Coloured Nets to TCNets w.r.t. concurrency.

3.2 Interleaving semantics.

The interleaving semantics is based on individual transition firing rules, using prefix, residue and concatenation on the trace monoid $(A/\approx_I)$. The left and right cancellativity of $(A/\approx_I)$ are used in the section below.

3.2.1 Transition firing rule.

Let $N = (A, I, C, T, W, l)$ be a TCNet. We say that a transition t **is enabled** at a marking M, denoted $M[t>$, iff it satisfies:

$$M[t> \Longleftrightarrow \forall c \in C, W(c,t) \preceq M(c)$$

If t is enabled at M, the firing of t **leads to a new marking M'**, denoted $M[t > M'$, defined as follows:

$$M[t > M' \iff M[t> \bigwedge \forall c, M'(c) = (M(c) - W(c,t)) \circ W(t,c)$$

The definition of the new marking can be expressed in the following way, due to properties of traces (see section 2):

$$M[t > M' \iff M[t> \bigwedge \forall c, W(c,t) \circ M'(c) = M(c) \circ W(t,c)$$

3.2.2 Firing sequences, reachable markings.

A **sequence of transitions** $\theta = t_1 t_2 ... t_n$ is **enabled** at marking M , and leads to marking M', denoted $M[\theta > M'$, if and only if there is a sequence of markings $M = M_0, M_1, M_2 ... M_n = M'$ such that $M_{i-1}[t_i > M_i \; for \; i = 1, .., n$.

The set of **firing sequences** of a marked net (N, M) where M is a marking of N, is

$$FS(N, M) = \{\theta \in T^*, M[\theta >\}$$

The set of **reachable markings** is

$$R(N, M_0) = \{M : C \rightarrow (A/\approx_I) | \exists \theta \in FS(N, M_0), M_0[\theta > M\}$$

A channel $c \in C$ is **bounded** in (N, M_0) iff $\exists n \in \aleph, \forall M \in R(N, M_0)$, $|M(c)| \leq n$. The marked net (N, M_0) is bounded iff all its channels are bounded .

4 Channel languages.

In this section, we focus on a single channel in the net. The labels of its input and output arcs induce specific structure on its markings, and relations between the sequences of input and output transitions. In particular the balanced language of a net w.r.t. a channel models exactly the constraints induced by the channel on the behaviour of the net.

4.1 Input and output traces of channels.

We extend the flow function W to sequences of transitions. For a channel c, W yields two monoid morphisms $W(\_, c) : T^* \longrightarrow (A/\approx_I)$ and $W(c, \_) : T^* \longrightarrow (A/\approx_I)$, defining the input and output traces of the channel c for a given word $\theta = t_1 t_2 ... t_n \in T^*$:

$$W(\theta, c) = W(t_1, c) \circ W(t_2, c) \circ .... \circ W(t_n, c)$$

$$W(c, \theta) = W(c, t_1) \circ W(c, t_2) \circ ..... \circ W(c, t_n)$$

We use the notation $W(\_, c)(L) = \{W(\theta, c), \theta \in L\}$.

In the examples of the figure 1, if we let $\{c_1\} = t_1^\bullet$, we have $W(t_1t_2t_1, c_1) = [aba]_I$, with $I = I_1, I_2$ or I_3 depending on the chosen net.

By induction on the length of a firing sequence, we prove the following properties.

4.1.1 Lemma (firing sequence property).

Let be (N, M) is a marked TCNet ,then

1. $\forall \theta \in FS(N, M), \forall c \in C,\ W(c, \theta) \preceq M(c) \circ W(\theta, c)$.
2. $M[\theta > M' \Longrightarrow \forall c \in C,\ M'(c) = (M(c) \circ W(\theta, c)) - W(c, \theta)$.

Proof.

Let be $\theta = t_1t_2...t_n \in FS(N, M)$, and $\theta' = t_1t_2...t_{n-1}$, with $W(c, \theta') \preceq M(c) \circ W(\theta', c)$ and $M[\theta' > M', M'(c) = (M(c) \circ W(\theta', c)) - W(c, \theta')$. We have $M'[t_n > M'' \Longrightarrow W(c, t_n) \preceq M'(c)$, and then $W(c, \theta) = (W(c, \theta') \circ W(c, t_n)) \preceq (W(c, \theta') \circ M'(c)) \preceq (M(c) \circ W(\theta', c))$, which implies $W(c, \theta) \preceq (M(c) \circ W(\theta, c))$

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4.2 Balanced languages w.r.t. channels.

We characterize the constraints induced by a single channel on the net behaviours. For that purpose, we consider a net N, a channel c, and we define the hiding of c in N, $N \setminus c$, as the net where the channel c and its adjacent arcs have been suppressed. We show that the firing sequences of N are the ones of $N \setminus c$ which we call balanced w.r.t. c, i.e. such that for any of its prefixes, the corresponding output from c is a trace prefix of its inputs in c.

4.2.1 Definition (balanced language).

Let c be a channel, M a marking, the **balanced language of** N **w.r.t.** c **and** M is

$$BL(c, M) = \{\theta \in T^* | \forall \nu \in T^*,\ \nu \preceq \theta \Longrightarrow W(c, \nu) \preceq M(c) \circ W(\nu, c)\}$$

We then define the "hiding" of a channel c in a net N, denoted $N \setminus c$ and then compare its firing sequences with the ones of the complete net.

4.2.2 Definition (hiding of a channel).

Let be a net $N = (A, I, C, T, W, l)$, the **hiding of** c **in** N is the net

$N \setminus c = (A, I, C - \{c\}, T, W \setminus c, l)$

where $W \setminus c$ is the restriction of the flow mapping W to the set $((C - \{c\}) \times T) \cup (T \times (C - \{c\}))$.

We denote by $M \setminus c$ the restriction of a marking M to $C - \{c\}$. The firing language of N is the intersection of the firing language of $N \setminus c$ and the balanced language of N w.r.t. c , with the hypothesis that ${}^\bullet c \cap c^\bullet = \oslash$, i.e. no transition is both an input and an output for c.

4.2.3 Proposition.

If c is a channel such that ${}^\bullet c \cap c^\bullet = \oslash$, then:

$$FS(N, M) = FS(N \setminus c, M \setminus c) \cap BL(c, M)$$

Proof.

The inclusion $FS(N, M) \subseteq FS(N \setminus c, M \setminus c) \cap BL(c, M)$ is obvious, due to the previous lemma 4.1.1. The inverse inclusion is proved by induction on the length of a sequence. Let be $\theta.t \in FS(N \setminus c, M \setminus c) \cap BL(c, M)$, and such that $\theta \in FS(N, M)$, with $M[\theta > M'$.

- If $t \notin c^\bullet$ then we have $(M' \setminus c)[t >\Longrightarrow M'[t >$ and $\theta.t \in FS(N, M)$.
- Otherwise $t \notin^\bullet c$, $M'(c) = (M(c) \circ W(\theta, c)) - W(c, \theta)$, and $\theta.t \in BL(c, M) \Longrightarrow W(c, \theta.t) = W(c, \theta) \circ W(c, t) \preceq M(c) \circ W(\theta, c) \circ W(t, c) = M(c) \circ W(\theta, c)$, which by left cancellativity, cancelling the prefix $W(c, \theta)$ in both sides, implies $W(c, t) \preceq M'(c)$. Using $(M' \setminus c)[t >$in $N \setminus c$ we conclude that $M'[t >$, and $\theta.t \in FS(N, M)$. This proves the induction step. The proof for a single transition sequence is obvious.

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4.3 Relations between inputs and outputs in balanced languages.

We find interesting for understanding TCNets to characterize algebraically the relationships between the projections of a balanced language on the sets ${}^\bullet c$ and $c^\bullet$ of input and output transitions, i.e. $BL(c, M)/{}^\bullet c$ and $BL(c, M)/c^\bullet$. The following lemma may seem verbose, but facilitates the next sections.

4.3.1 Lemma.

If ${}^\bullet c \cap c^\bullet = \oslash$, then

1. If $\theta \in ({}^\bullet c)^*$and $\theta' \in (c^\bullet)^*$then $\theta\theta' \in BL(c, M) \Longleftrightarrow W(c, \theta') \preceq M(c) \circ W(\theta, c)$
2. If $\theta \in ({}^\bullet c)^*$ then $\{\theta' \in (c^\bullet)^*, \theta\theta' \in BL(c, M)\} = W(c, \_\,)^{-1}(Pref(M(c) \circ W(\theta, c)))$
3. If $\theta \in (c^\bullet)^*$ then $\{\theta' \in ({}^\bullet c)^*, \theta'\theta \in BL(c, M)\} = W(\_\,, c)^{-1}((W(c, \theta) \circ (A/_{\approx I})) - M(c))$
4. $\theta \in BL(c, M) \Longrightarrow (\theta/{}^\bullet c)(\theta/c^\bullet) \in BL(c, M)$

Proof.

1)The implication from left to right is direct from definitions. Now if $\nu \preceq \theta\theta'$, then either $\nu/{}^\bullet c = \theta \bigwedge \nu/c^\bullet \preceq \theta'$ or $\nu/c^\bullet = \varepsilon$, and thus $W(c, \nu) \preceq M(c) \circ W(\nu, c)$.

2,3,4) Direct consequences of 1), with the use of the following equivalences:

$W(c, \theta') \preceq M(c) \circ W(\theta, c) \Longleftrightarrow \theta' \in W(c, \_\,)^{-1}(Pref(M(c) \circ W(\theta, c)))$

$W(c, \theta) \preceq M(c) \circ W(\theta', c) \Longleftrightarrow \theta' \in W(\_\,, c)^{-1}((W(c, \theta) \circ (A/_{\approx I})) - M(c))$

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4.3.2 Local enabling relation.

We associate to any sequence of input transitions θ, the set $En(c, \theta, M)$ of sequences of output transitions θ' such that $\theta\theta' \in BL(c, M)$, and which we call **enabled** after θ:

$En(c, \_\,, M) : ({}^\bullet c)^* \longrightarrow \wp((c^\bullet)^*)$ with $En(c, \theta, M) = \{\theta' \in (c^\bullet)^*, \theta\theta' \in BL(c, M)\}$

In the same way we can define a **requirement** mapping :

$Rq(c, \_\,, M) : (c^\bullet)^* \longrightarrow \wp(({}^\bullet c)^*)$ with $Rq(c, \theta, M) = \{\theta' \in ({}^\bullet c)^*, \theta'\theta \in BL(c, M)\}$.

A natural question that arises is: if $L \subseteq ({}^\bullet c)^*$ is a recognizable language , at which conditions is $En(c, L, M) \subseteq (c^\bullet)^*$ a recognizable language ? The proposition below gives a sufficient condition, based on the image by $W(\_\,, c)$ of the iterative factors of L. The proposition relies on the Ochmanski's theorem , which identifies recognizable , co-rational and star-connected subsets of $(A/\approx_I)$, and the following fact:

If L is a rational language, $L \subseteq T^*$, and $\varphi : T^* \longrightarrow (A/\approx_I)$ a monoid morphism, then $\varphi(L)$ is co-rational if and only if the image of any iterative factor of L by φ is connected in $(A/\approx_I)$.

4.3.3 Proposition (rationality of $En(c, L, M)$).

If $L \subseteq ({}^\bullet c)^*$ is a rational language such that for each iterative factor F of L, $W(\_, c)(F) \subseteq (A/\approx_I)$ is connected, then $En(c, L, M) \subseteq (c^\bullet)^*$ is rational.

Proof.

We have $En(c, L, M) = W(c,\_)^{-1}(Pref(M(c) \circ W(\_, c)(L)))$. If $W(\_, c)(F)$ is connected for each iterative factor F of L, then $L' = W(\_, c)(L)$ is star-connected. $M(c).L'$ and $Pref(M(c).L')$ are also star-connected, because $M(c)$ is finite, and because the iterative factors of the prefix language $Pref(K)$ of a language K , defined by a co-rational expression are the iterative factors of K. This is due to the following :

$Pref(K^{co-*}) = Pref(Comp(K)^*) = Comp(K)^*.Pref(Comp(K))$ (where $Comp(K)$ is the set of connected components of K).

$Pref(M(c).L')$ is then recognizable, and the inverse homomorphic image of a recognizable language being recognizable too, $En(c, L, M) = W(c,\_)^{-1}(Pref(M(c).L'))$ is recognizable (and rational) in $(c^\bullet)^*$.

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Note that this condition is not necessary as shown in the example in figure 2.d of section 5.

5 Bounded Petri Nets communicating through Trace Channels.

Communicating Finite State Machines use Fifo channels to exchange messages, and are a typical case study for Fifo nets. Their extention to trace channels is natural. We only study communication through a single channel, and throughout this section we consider a TCNet $N = (A, I, C, T, W, l)$ and a channel $c \in C$ such that $N \setminus c$ has two disjoint connected components $N_i = (A, I, C_i, T_i, W_i, l_i), i = 1, 2$, such that ${}^\bullet c \subseteq T_1$ and $c^\bullet \subseteq T_2$. If M is a marking of N, we denote by M_1 and M_2 its restrictions to C_1 and C_2.

5.1 Components languages.

We suppose that the nets (N_i, M_i) are bounded, and so, $FS(N_i, M_i), i = 1, 2$, are rational languages in T_i^*. The problem we adress first is to find sufficient conditions on the net such as to ensure the rationality of the projections $FS(N, M)/T_i, i = 1, 2$. As far as we have $FS(N, M)/T_1 = FS(N_1, M_1)$, the question is : when is $FS(N, M)/T_2$ rational? The proposition below relies on proposition 4.3.3, and considers the image by the input flow function $W(\_, c)$ of the iterative factors of $FS(N_1, M_1)$.

5.1.1 Proposition .

If the sets $FS(N_i, M_i), i = 1, 2$, are rational languages, and if for every iterative factor F of $FS(N_1, M_1)$, $W(\_, c)(F)$ is connected, then $FS(N, M)/T_i, i = 1, 2.$ are rational languages.

Proof.

The two nets $N_i, i = 1, 2$, being disjoint, we have $FS(N\setminus c, M\setminus c) = FS(N_1, M_1)|||FS(N_2, M_2)$, where $|||$ is the shuffle operator on languages.

From the proposition 4.2.3 , we then have $FS(N, M) = (FS(N_1, M_1)|||FS(N_2, M_2)) \cap BL(c, M))$ and using the lemma 4.3.1 (4), $\theta \in FS(N, M) \implies (\theta/T_1)(\theta/T_2) \in FS(N.M)$. We infer that $FS(N, M)/T_2 = \{u \in FS(N_2, M_2),\ u/{}^\bullet c \in En(FS(N_1, M_1)/{}^\bullet c)\}$, using the lemma 4.3.1 (2).

Thus $FS(N, M)/T_2$ is the intersection of two languages, $FS(N_2, M_2)$, which is rational, and $\{u \in T_2^*,\ u/{}^\bullet c \in En(FS(N_1, M_1)/{}^\bullet c)\}$, which is rational iff $En(FS(N_1, M_1)/{}^\bullet c)\}$ is. We can then use the proposition 4.3.3 to conclude the proof.

○

Note that the condition of the proposition is valid for some initial marking of c iff it is valid for any initial marking of c.

5.2 Empty channel languages.

Emptiness of the communication channel is an important property, used in particular in the definition of half-duplex systems. We focus now on the firing sequences which preserve the emptyness. For this purpose, we define the **empty channel language** of (N, M) as $L_{c,\varepsilon} = \{\theta \in FS(N, M), W(\theta, c) = W(c, \theta)\}$. If we suppose that $M(c) = \varepsilon$, we have

$L_{c,\varepsilon} = \{\theta \in FS(N, M), M[\theta > M' \Longrightarrow M'(c) = \varepsilon\}$.

With the hypothesis that $FS(N, M)/T_i, i = 1, 2$ are rational languages , i.e. a property ensured by the previous proposition, and using the same type of arguments, we give a sufficient condition to ensure the rationality of the projections $L_{c,\varepsilon}/T_1$ and $L_{c,\varepsilon}/T_2$. The same type of argument as in the previous proposition are used, but in this case we consider also the iterative factors of the receiving net language.

5.2.1 Proposition.

Let $L_i = FS(N, M)/T_i, i = 1, 2$ be rational, then if for every iterative factor F of L_1, $W(\_, c)(F)$ is connected, and if for every iterative factor F of L_2, $W(c, \_)(F)$ is connected, then $L_{c,\varepsilon}/T_i, i = 1, 2$, are rational languages.

Proof.

Let be $B = W(\_, c)(L_1) \cap W(c, \_)(L_2)$, then B is star-connected if the condition of the proposition is satisfied : an iterative factor of B is an iterative factor of $W(\_, c)(L_1)$ and $W(c, \_)(L_2)$, and if both of these latter are star-connected, then B is also. In that case, $L_{c,\varepsilon}/T_1 = L_1 \cap W(\_, c)^{-1}(B)$ and $L_{c,\varepsilon}/T_2 = L_2 \cap W(c, \_)^{-1}(B)$ are recognizable, as intersections of recognizable sets, the inverse homomorphic images of recognizable sets being so.

○

The connectivity of $W(\_, c)(F)$ and $W(c, \_)(F)$ used in the two propositions above depends on the relation I. If I is too large, the conditions for the rationality of the languages of the component nets are not verified. More generally in section 6, we will show the existence of maximal independences on channels alphabets w.r.t. the rationality of the firing language of a net.

5.3 Examples.

The figure 2 shows four TCNets, each one composed of a pair of disjoint component nets, denoted N_1 and N_2, which communicate through a trace channel. The nets in 2.a and 2.b have bounded components, and the proposition 5.1.1. applies. We can ensure the rationality of the projected behaviours.

In the net 2.a , we have $FS(N_1) = Pref((t_1t_2)^*)$, then t_1t_2 is the only iterative factor of the input language, and we need to ensure that $W(\_, c)(t_1t_2) = [abcd]_I$ is connected while $W(t_i, c), i = 1, 2$ do not need to be connected. This is the case for $I = \{(a, b), (b, a), (a, c), (c, a), (b, c), (c, b)\}$, and the projected behaviours on T_2 are $FS(N)/T_2 = Pref((Perm(t'_1.t'_2.t'_3).t'_4)^*) \cap FS(N_2)$ (the iterative factor is any permutation of the sequence $t'_1.t'_2.t'_3$ followed by t'_4).

In the net 2.b, $FS(N_1) = Pref(t_1^*t_2^*)$ has two iterative factors, t_1 and t_2, and the proposition applies only if $W(t_i, c), i = 1, 2$ are connected , i.e $\{(a, b), (c, d)\} \subset D$. If $I = (\{a, b\} \times \{c, d\}) \cup (\{c, d\} \times \{a, b\})$, then $FS(N)/T_2 = Pref((t'_1.t'_2.)^*|||(t'_3.t'_4)^*) \cap FS(N_2)$ which can be expressed as a rational expression (the shuffle of rational languages is rational).

The net 2.c is a counter-example to the rationality of $FS(N_1)/{}^\bullet c$, $FS(N_1)/{}^\bullet c = \{t_1^n t_2^n, n \in \aleph\}$, and $\{W(\theta, c) | \theta \in FS(N)\} = \{[a^n b^n]_I, n \in \aleph\}$, which cannot be rational, independently from I.If aIb, $FS(N)/T_2 = \{\nu \in T_2^*, |\nu|_{t'_1} = |\nu|_{t'_2}\} \cap FS(N_2)$.

The net 2.d. shows that the condition of the proposition 5.1.1 is not necessary: If a and b are independent from c and d, with aDb and cDd, then $W(\_, c)(t_1t_2) = [abcd]_I$ is not connected but the induced behaviour on T_2 is rational : $FS(N)/T_2 = Pref((t'_1t'_2)^*) \cap FS(N_2)$.

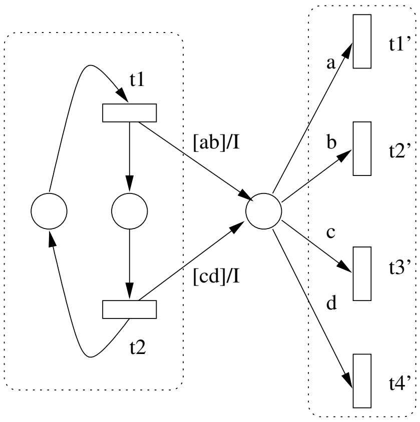


fig 2.a

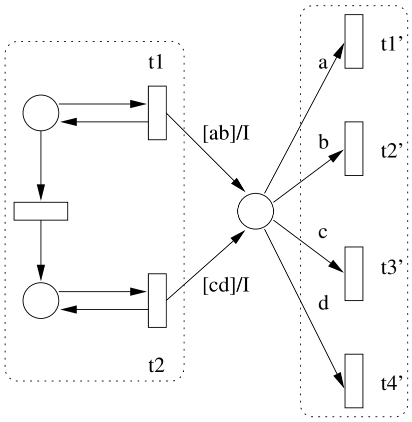


fig 2.b

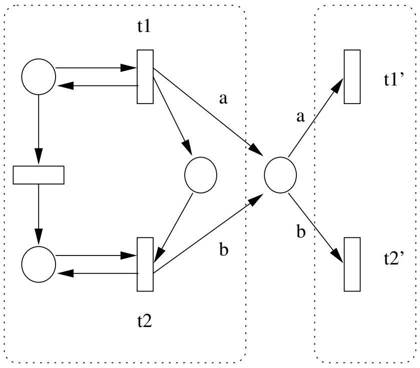


fig 2.c

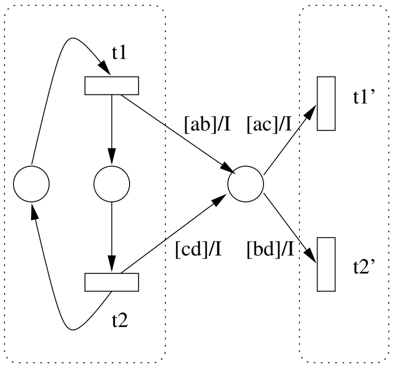


fig 2.d

Figure 2.

6 Concurrent semantics.

6.1 Modeling by automata with concurrency relations.

We define the concurrent semantics for TCNets, by **deterministic automata with concurrency relations** [DK]. For each TCNet N, we define an automaton $Aut(N)$, and we show that $Aut(N)$ is **stably concurrent**. This type of automata has many advantages:

- Equivalence classes of sequential behaviours correspond to maximally concurrent firable partial orders of transitions [BDK][Vo]. We do not need to define explicitly firing rules for steps of transitions or partial words of transitions.
- If the net is bounded, the automaton is finite, and co-rationality and recognizability are related for the sub-languages of the automaton [Dr].
- A simulation relation between TCNets can be defined by means of automata morphisms, which take concurrency of behaviours into account. These morphisms relate nets with the same structure but different dependence relations on the tokens alphabet, and particular dependences can be underscored in this framework.

In this section we need to distinguish the transitions of the net which coincides with the *events* of the automaton, from the *transitions* of the automaton, i.e. triples composed by a source marking, an event and a target marking. The **concurrency relations on the events** of the automaton, denoted $\|_M$, depend on the marking M where the events are enabled and on the independence relation I. More precisely two events are concurrent at a marking M iff they are both enabled at M, and they neither both output dependent letters from the same channel, nor both input dependent letters in the same channel. This is formalised in the item 4 of the definition below.

6.1.1 Definition.

The automaton of a TCNet $N = (A, I, C, T, W, l)$, denoted $Aut(N)$, is the tuple
$Aut(N) = (Q, E, \Theta, \{\|_M\}_{M \in Q})$ where

1. $Q = \{M : C \longrightarrow (A/\approx I)\}$ is the **set of states**, coinciding with the markings of N.
2. $E = T$, the **set of events** coincides with the set of transitions of N .
3. $\Theta \longrightarrow\subseteq Q \times E \times Q$ is the **transition set** , defined by $(M, t, M') \in \Theta \Longleftrightarrow M[t > M'$.
4. $\|_M \subseteq E \times E$, defined by $t \|_M t' \Longleftrightarrow M[t > \bigwedge M[t' > \bigwedge t I_E t'$, is the **concurrency relation**. The auxiliary relation on events I_E does not rely on M :

$$tI_Et' \Longleftrightarrow \forall c \in C,\ \alpha(W(c,t)) \times \alpha(W(c,t')) \cap D = \oslash \bigwedge \alpha(W(t,c)) \times \alpha(W(t',c) \cap D = \oslash$$

We recall that D is the complement of I in $A \times A$.
Note that if the pre and post sets of two transitions are disjoint, they are related by I_E:
$t^\bullet \cap t'^\bullet = \oslash \bigwedge {}^\bullet t \cap {}^\bullet t' = \oslash \Longrightarrow tI_Et'$.
Note also that for Fifo Nets, the two properties are equivalent, and in that case two transitions can be concurrent only if they do not share some input or output channel:
$I = \oslash \Longrightarrow (t^\bullet \cap t'^\bullet = \oslash \bigwedge {}^\bullet t \cap {}^\bullet t' = \oslash \Longleftrightarrow tI_Et')$
To satisfy the definition of automata with concurrency relations, we have to verify the so-called (forward) **diamond** property, which ensures that the order in which we fire concurrent events is irrelevant w.r.t. the obtained marking.
Note that the proofs of the three lemmas below rely on the fact that the dependence relation D is reflexive, and for that reason this construction cannot be used to give a concurrent semantics to Coloured Nets. We shall overcome this restriction in the section 7.4.

6.1.2 Lemma (diamond property).

$$t \|_M t' \Longrightarrow \exists M',\ M[t.t' > M' \bigwedge M[t'.t > M'$$

Proof .
If $W(c,t') \preceq M(c)$, $W(c,t) \preceq M(c)$ and $\alpha(W(c,t)) \times \alpha(W(c,t')) \cap D = \oslash$, then $W(c,t') \preceq M(c) - W(c,t)$, and $M[t.t' >$. Furthermore $W(c,t'.t) = W(c,t.t')$ and $W(t.t',c) = W(t'.t,c)$. Then if $M[t.t' > M'$, we have
$M'(c) = ((M(c) - W(c,t)) \circ W(t,c)) - W(c,t')) \circ W(t',c) = (M(c) - W(c,t.t')) \circ W(t.t',c) = (M(c) - W(c,t'.t)) \circ W(t'.t,c)$
and we conclude that $M[t'.t > M'$.
○
Note that the reverse implication $(\exists M',\ M[t.t' > M' \bigwedge M[t'.t > M') \Longrightarrow t \|_M t'$ does not hold; the left hand side may be due to $W(c,t) = W(c,t')$ or $W(t,c) = W(t',c)$ for some $c \in C$, and we do not consider in that case the transitions to be concurrent.

6.2 Stability property.

The automaton $Aut(N)$ is in fact a deterministic **stably concurrent automaton** because it is deterministic (obvious from definitions) and it satisfies the following cube and inverse cube properties [DK][Dr].

6.2.1 Lemma (cube property).

Let be $M \in Q$, and three events t_1, t_2, t_3 such that $t_1 \parallel_M t_2$, $t_2 \parallel_M t_3$, and $t_1 \parallel_{M_2} t_3$, then $t_1 \parallel_M t_3$, $t_2 \parallel_{M_1} t_3$, and $t_1 \parallel_{M_3} t_2$, where $M[t_i > M_i, i = 1, 2, 3$.

Proof .

We have from hypothesis $i \neq j \Longrightarrow t_i I_E t_j$ for $\{i, j\} \subset \{1, 2, 3\}$, and then $t_1 \parallel_M t_3$. From hypothesis, $W(t_i, c) \preceq M(c), i = 1, 2, 3$, and then $\bigcirc_{i=1,2,3} W(t_i, c) \preceq M(c)$, due to $i \neq j \Longrightarrow \alpha(W(t_i, c)) \times \alpha(W(t_j, c)) \cap D = \oslash$. Thus for any $k \in \{1, 2, 3\}$ we have $\bigcirc_{i \neq k} W(t_i, c) \preceq M(c) - W(t_k, c)$. We can conclude that $(i \neq k \Longrightarrow M_k[t_i >, k = 1, 2, 3)$.

$\bigcirc$

6.2.2 Lemma (inverse cube property).

Let be $M \in Q$, and three events t_1, t_2, t_3 such that $t_1 \parallel_M t_3$, $t_2 \parallel_{M_1} t_3$, and $t_1 \parallel_{M_3} t_2$, then $t_1 \parallel_M t_2$, $t_2 \parallel_M t_3$, and $t_1 \parallel_{M_2} t_3$ where $(M[t_i > M_i, i = 1, 2, 3)$.

Proof.

We have to prove that $M[t_2 > M_2$ for some M_2, and $(M_2[t_i >, i = 1, 3)$.

$(M_i[t_2 >, i = 1, 3) \Longleftrightarrow \forall c \in C, (W(c, t_2) \preceq (M(c) - W(c, t_i)) \circ W(t_i, c), i = 1, 3)$. But due to $t_2 \parallel_{M_1} t_3$ and $t_1 \parallel_{M_3} t_2$, we have $(\alpha(W(c, t_i)) \times \alpha(W(c, t_2)) \cap D = \oslash, i = 1, 3)$, and then $(W(c, t_2) \preceq M(c) \circ W(t_i, c), i = 1, 3)$. With $\alpha(W(t_1, c)) \times \alpha(W(t_3, c)) \cap D = \oslash$ we conclude that $W(c, t_2) \preceq M(c)$, and thus $M[t_2 >$.

The property $(M_2[t_i >, i = 1, 3)$ is proved similarly to the previous lemma.

$\bigcirc$.

A stably concurrent automaton being characterized by the cube and inverse cube properties , the following proposition resumes the three lemmas above:

6.2.3 Proposition.

The automaton $Aut(N)$ of a TCNet N is a **deterministic stably concurrent automaton**.

6.3 Computations sequences and concurrent behaviours.

The difference between firing sequences and computation sequences is that the latter record the intermediate markings. A **computation sequence** is either the empty one (noted ε) or a finite sequence $u = u_1, u_2, ..u_n$, where $u_i = (M_{i-1}, t_i, M_i), i = 1, n$. M_0 and M_n are the domain $dom(u)$ and codomain $cod(u)$ of u. The sequence $evseq(u) = t_1 t_2 ..t_n$ is a firing sequence of $FS(N, M_0)$. The **set of computation sequences** of $Aut(N)$ is denoted $CS(N)$, and is a partial monoid, where the concatenation of u and v is defined iff $dom(v) = cod(u)$. We define on the monoid of computation sequences a congruence denoted $\sim$, generated by the **permutation of concurrent transitions**:

$$(M, t_1, M_1)(M_1, t_2, M') \sim (M, t_2, M_2)(M_2, t_1, M') \Longleftrightarrow t_1 \parallel_M t_2$$

extended to a congruence w.r.t. the partial concatenation. Note that two equivalent sequences have the same domain , codomain and event sets.

The set of **concurrent behaviours** is the quotient $Beh(N) = CS(N)/\sim \cup \{0\}$, where 0 stands for the undefined concatenation and is absorbing. As traces in standard trace monoids, the concurrent behaviours in $Beh(N)$ can be represented by partial words of transitions, using the same construction. Note that we do not consider infinite sequences or computations in the present work.

In the original work [FB], we defined a concurrent semantics of TCNets based on steps and partial words firing rules, following [Vo]. The one presented here yields maximally concurrent behaviours, while the previous one was a “weak” semantics, where any increment of the partial order between events in a behaviour yields another admissible behaviour, allowing the firing of the partial word $t \rightarrow t'$ at a marking M where the concurrent step $t \, co \, t'$ is enabled . The two semantics coincide on maximal concurrency.

6.3.1 Marked nets and automata with initial state.

If we consider a marked TCNet (N, M_0), then the **automaton with initial state** $Aut(N, M_0)$is the tuple

$$Aut(N, M_0) = (Q, E, \Theta, \{\|_M\}_{M \in Q}, M_0)$$

where the set of states is $Q = R(N, M_0)$, and the other components are the same as in $Aut(N)$.

Note that this automaton has the same cube and inverse cube properties as $Aut(N)$. We denote by $CS(N, M_0)$ and $Beh(N, M_0)$ the sets of computation sequences and concurrent behaviours of $Aut(N, M_0)$, i.e. the computation sequences (and concurrent behaviours) of $Aut(N)$ having as domain the initial state M_0.

7 From Fifo Nets to Simple Coloured Nets.

We explicitate in this section the **partial order of semantics** induced by the different independence relations on a single net structure. We relate in this section TCNets with the same alphabets, sets of channels and transitions that differ only by the independence relations and flow functions. For simplicity, we **do not work up to nets isomorphisms**, in particular transitions or channels renamings (they do not constitute a drawback here, but should be introduced to define operators and observations on TCNets).

The figure 3 below illustrates the main aspects of the section. It represents a trace net N where the independence relation is left undefined, and the spectrum of semantics on the net structure of N, induced by the inclusion order on independence relations.

For the relation $I = \{(a,c),(c,a),(b,c),(c,b),(b,d),(d,b)\}$, the net is bounded, while for $I' = \{(a,c),(c,a),(a,d),(d,a),(b,c),(c,b),(b,d),(d,b)\}$ it is unbounded. In this case for all independences I for which the trace $W(t_1.t_2, c) = [abcd]_I$ is connected, the net is bounded, while the boundedness is lost when $W(t_1.t_2, c)$ is disconnected: in the former case, all transitions t'_1, t'_2, and t'_3 have to fire once between two occurrences of t'_4, inducing a bound on the markings of the channel output of t_1 and t_2. Note the similarity between this particular situation and the properties worked out in the proposition 4.3.3 and section 5 about bounded nets communicating through trace channels.

In the following, we first define the quotient net N/I' obtained from the net N by replacing its independence relation I by a relation I' such that $I \subseteq I'$. We show that there is a concurrent automata morphism between $Aut(N)$ and $Aut(N/I')$. The relation between N and $N' = N/I'$ is a partial order denoted $N \sqsubseteq N'$, defined on sets of nets with the same underlying structure, having a set of Fifo Nets as minimal elements , and with a unique maximal element, a quotient N/I_m, where I_m is the maximal independence relation.

In this section, we consider TCNets with disjoint channels alphabets. We could define formally a set of concurrent alphabets $\{(A_c, I_c), c \in C\}$ with $A_c = \alpha(c)$, yielding monoids $(A_c/\approx_{I_c})$. For simplicity and readability , we use (A, I) and $(A/\approx_I)$, and thus we take $I \subseteq I'$ as a shortcut for $I_c \subseteq I'_c$, $\forall c \in C$. Proofs do not suffer from this simplification, all traces being "local" to a channel, and we gain in readability.

7.1 Incrementing independence.

Starting from the definition of a Fifo Net $N = (A, \oslash, C, T, W, l)$, we can define on N a TCNet semantics associated to an independence relation I on A , considering the arcs labels up to the monoid congruence $\approx_I$, and using the firing rules in $(A/\approx_I)$. More generally, let be I and I' two independence relations on A, with $I \subseteq I'$, there is a cannonical monoid morphism

$$\varphi_{I,I'} : (A/\approx_I) \longrightarrow (A/\approx_{I'})$$

which maps $(A/\approx_I)$ on $(A/\approx_{I'})$ defined by $\varphi_{I,I'}([u]_I) = [u]_{I'}$. We denote by $\circ_I$ and $-_I$ the trace concatenation and residue in $(A/\approx_I)$. A net $N = (A, I, C, T, W, l)$ can be quotiented by the relation I' as follows:

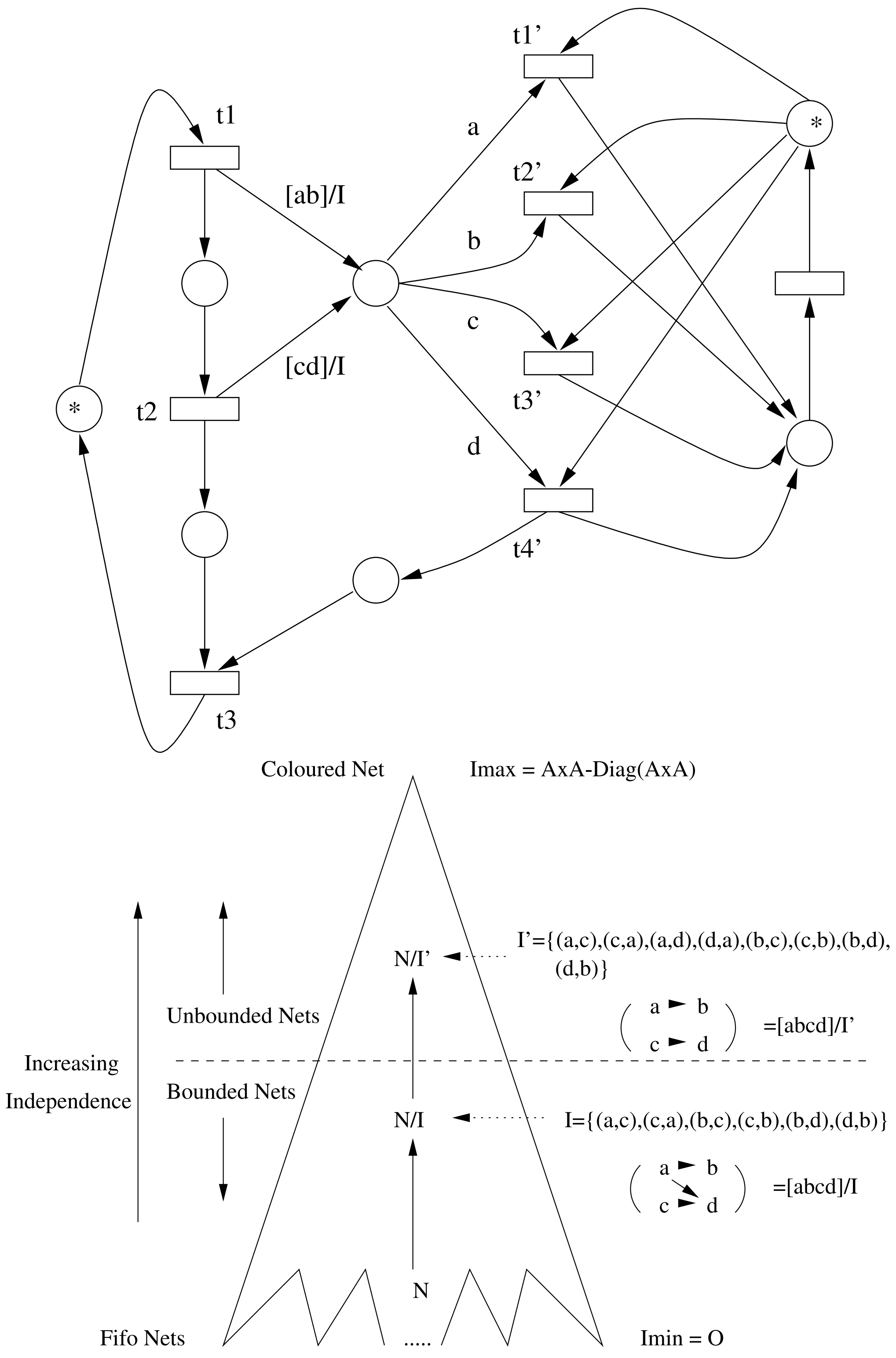

t1'
t1
a
[ab]/I
t2'
b
c
[cd]/I
t2
t3'
d
t4'
t3
*
*
Coloured Net
Imax = AxA-Diag(AxA)
I'={(a,c),(c,a),(a,d),(d,a),(b,c),(c,b),(b,d),
(d,b)}
N/I'
Unbounded Nets
a ► b
c ► d
=[abcd]/I'
Increasing
Independence
Bounded Nets
N/I
I={(a,c),(c,a),(b,c),(c,b),(b,d),(d,b)}
a ► b
c ► d
=[abcd]/I
N
Fifo Nets
.....
Imin = O

Figure 3.

7.1.1 Definition (quotient net).

Let be $N = (A, I, C, T, W, l)$a TCNet, and a independence relation I' on A such that $I \subseteq I'$, the **quotient** N/I' is the trace net $N/I' = (A, I', C, T, W', l)$ where $W' = \varphi_{I,I'} \circ W$.

For convenience we denote by $s_{I'}$ the image $\varphi_{I,I'}(s) \in (A/\approx_{I'})$ of a trace $s \in (A/\approx_I)$.

The main property is that the firing sequences of a net N are preserved in the quotient, N/I' and the images of its reachable markings by $\varphi_{I,I'}$ are reachable markings of N/I'.

7.1.2 Lemma .

Let be $N = (A, I, C, T, W, l)$ a TCNet, and $I \subseteq I'$, then

1. $FS(N, M) \subseteq FS(N/I', M_{I'})$
2. $\varphi_{I,I'}(R(N, M)) \subseteq R(N/I', M_{I'})$.

Proof.

Based on the fact that if $I \subseteq I'$, then $[v]_I = [uw]_I \Longrightarrow [v]_{I'} = [uw]_{I'}$ and $[u]_I \preceq [v]_I \Longrightarrow [u]_{I'} \preceq [v]_{I'}$. $[v]_{I'} - [u]_{I'} = ([v]_I - [u]_I)_{I'}$. Thus we infer:

$W(c, t) \preceq M(c) \Longrightarrow W(c, t)_{I'} \preceq M(c)_{I'}$.

$M'(c) = (M(c) -_I W(c, t)) \circ_I W(t, c) \Longrightarrow M'(c)_{I'} = (M(c)_{I'} -_{I'} W(c, t)_{I'}) \circ_{I'} W(t, c)_{I'}$.

We have proved that

$M[t > M'$ in $N \Longrightarrow M_{I'}[t > M^{'}_{I'}$ in N/I. The proposition follows directly.

○

The relation between $Aut(N)$ and $Aut(N/I')$ is a morphism of concurrent automata, following the definition of [DK]. Such morphisms ensure that the concurrent behaviours of the source net can be executed in the target system in a more (or at least equal) concurrent way .

7.1.3 Definition (automata morphism).

A morphism between the automata $Aut = (Q, E, \Theta, \{\|_M\}_Q, M_0)$ and $Aut' = (Q', E', \Theta', \{\|_M\}_{Q'}, M'_0)$ is a pair $(\pi, \eta) : Aut \longrightarrow Aut'$ where

$\pi : Q \longrightarrow Q'$,$\eta : E \longrightarrow E'$, such that:

1. $(M, t, M') \in \Theta \Longrightarrow (\pi(M), \eta(t), \pi(M')) \in \Theta'$
2. $\pi(M_0) = M'_0$
3. $t \|_M t' \Longrightarrow \eta(t) \|_{\pi(M)} \eta(t')$.

7.1.4 Proposition.

Let (N, M_0) be a marked TCNet, with $N = (A, I, C, T, W, l)$, and $I \subseteq I'$, the pair $(\varphi_{I,I'}, id_E)$ is an automata morphism

$$(\varphi_{I,I'}, id_E) : Aut(N, M_0) \longrightarrow Aut(N/I', M_{0I'})$$

Proof.

The item 1 of the definition 7.1.3 is proved in the previous lemma.

For the item 3), we have $M[t > M'$ in $N \Longrightarrow M_{I'}[t > M^{'}_{I'}$ in N/I, (see lemma 7.1.2). Furthermore $I \subseteq I' \Longrightarrow D' \subseteq D$, then

$\alpha(W(c, t)) \times \alpha(W(c, t')) \cap D = \oslash \Longrightarrow \alpha(W(c, t)_{I'}) \times \alpha(W(c, t)_{I'}) \cap D' = \oslash$. The same holds for arcs in $T \times C$, and $tI_E t' \Longrightarrow tI^{'}_E t'$.

○

Increasing the independence on letters preserves the firing sequences and may enlarge their equivalence classes w.r.t. the congruence $\sim$, incrementing concurrency. Viewing behaviours as partial words of transitions, we have the relation $Beh(N) \subseteq Aug(Beh(N/I'))$, where Aug operates on its argument by incrementing the order relations of the included partial words.

The morphism may also introduce new transitions, i.e. transitions which are not the image $(M_{I'}, t, M_{I'}^{'}) \in \Theta'$ of a transition $(M, t, M') \in \Theta$. The situation is specified as follows:

$\exists M \in R(N, M_0), t \in T \; : \; M_{I'}[t > \bigwedge (\forall M^{'} \in R(N, M_0), M_{I'} = M_{I'}^{'} \Longrightarrow \neg M^{'}[t >)$.

It seems that the ongoing study of the different situations induced by an increment of independence should yield interesting properties and a new insight on net structures.

7.2 The independence partial order .

As mentioned previously, the maximal independence for a trace alphabet is $I_m = \{A \times A - \triangle(A)\}$, more precisely $I_m = \cup_{c \in C}(\alpha(c) \times \alpha(c) - \triangle(\alpha(c)))$. We say that two TCNets N and N' are **structure equivalent** , denoted $N \leftrightarrow N'$, iff they have the same quotient by the relation I_m, i.e. $N/I_m = N'/I_m$. The relation $\leftrightarrow$ is obviously transitive and symmetric. We denote by $[[N]]$ the class of TCNets equivalent to N.

The **independence partial order** $([[N]], \sqsubseteq)$ is composed of pairs $N \sqsubseteq N'$ such that N' is a quotient N/I' for some relation I' . It is a transitive relation on TCNets, due to the following property : if $N = (A, I, C, T, W, l)$ and $I \subseteq I' \subseteq I''$, then $N/I'' = (N/I')/I''$. Antisymmetry follows from the one of $\subseteq$.

7.2.1 Definitions.

Let be two TCNets $N = (A, I, C, T, W, l)$and $N' = (A, I', C, T, W', l)$then

1. $N \sqsubseteq N' \Longleftrightarrow I \subseteq I' \bigwedge N' = N/I'$.
2. $N \leftrightarrow N' \Longleftrightarrow N/I_m = N'/I_m$.
3. $[[N]] = \{N', N \leftrightarrow N'\}$

7.2.2 Lemma.

The equivalence $\leftrightarrow$ is the the symmetric transitive closure of $\sqsubseteq$.

Proof.

1)$\sqsubseteq \subset \leftrightarrow$, due to $N/I_m = (N/I')/I_m$.

2)$\leftrightarrow \subseteq (\sqsubseteq \cup \sqsubseteq^{-1})^*$. If $N/I_m = N'/I_m$, then both nets N, N' are related to the same net by $\sqsubseteq$. ○

For any Fifo Net N, $[[N]]$ contains all the nets which can deduced from N by permutations on the arcs labels $W(x, y) \in A^*$, and these are the minimal elements of $([[N]], \sqsubseteq)$. The first result of the definition of $([[N]], \sqsubseteq)$ is the existence of particular independences w.r.t. different properties, minimal independences for liveness or reachability properties, maximal ones for boundedness as shown in the next section.

7.3 Critical independences w.r.t. boundedness.

We consider boundedness property w.r.t. the independence order, and the lemma below states that if N' is bounded and $N \sqsubseteq N'$, then N is bounded too .

7.3.1 Lemma.

Let $N = (A, I, C, T, W, l)$be a TCNet, and $I \subseteq I'$, if $(N/I', M_{0I'})$ is bounded, then (N, M_0) is bounded .

Proof.

If $(N/I', M_{0I'})$ is bounded, the images of the reachable markings of (N, M_0) by $\varphi_{I,I'}$ belong to $R(N/I', M_{0I'})$, and are bounded by the same bound, i.e. $\exists n \in \aleph, \forall c \in C, \forall M \in R(N, M_0)$, $|M_{I'}(c)| \leq n$. Due to the equality $|M(c)| = |M_{I'}(c)|$, the net (N, M_0) is bounded.

○

A immediate consequence is that if N is unbounded and $N \sqsubseteq N'$, then N' is unbounded. The lemma above induces the existence of **critical** independences, i.e. maximal w.r.t. the boundedness of a net.

7.3.2 Proposition.

If $N = (A, I, C, T, W, l)$is bounded , and N/I_m is not bounded, then we can find a (non unique) relation I', with $I \subseteq I'$ such that

1. N/I' is bounded.
2. if $I' \subset I''$, then N/I'' is unbounded.

Proof.

The set of independence relations on the finite alphabet is finite, say indexed by $[n]$, ordered by inclusion. The subset $\{I_j,\ N/I_j \textit{ is bounded} \bigwedge I \subseteq I_j\}$ is not empty and has maximal elements. By the previous lemma, any of these maximal elements satisfy the proposition.

○

We can relate this result to the propositions of the section 5, in which the sufficient conditions for the rationality of the components languages yield also maximal independence relations, those ensuring the connectedness of specific traces of $(A/\approx_I)$.

7.4 Simple Coloured Nets.

The TCNet N/I_m has a concurrent semantics in terms of stably concurrent automata as defined previously, but this definition forbids concurrent firing of transitions which output a common letter in the same channel. In particular it forbids autoconcurrency of the net transitions. For that reason we have to define specific firing rules and concurrent semantics for a coloured net, and then introduce it in the framework of the present section.

A Simple Coloured Net $N = (A, C, T, W, l)$, has the weight function and the markings valued as multisets on A : $W : C \times T \cup T \times C \longrightarrow \aleph^A$, $M : C \longrightarrow \aleph^A$. The firing rules are defined as follows:

1. $M[t > \Longleftrightarrow \forall c \in C, W(c,t)) \leq M(c)$
2. $M[t > M' \iff M[t > \bigwedge \forall c, M'(c) = (M(c) - W(c,t)) + W(t,c)$

where the relation $\leq$ and operators $-$ and $+$ are the standard ones on multisets, i.e. defined componentwise.

The main difference between the automaton with concurrency relations $Aut_{co}(N)$ defined for a Coloured Nets and the ones of TCNets lies in the concurrency relation. Due to this difference, autoconcurrent firings of a transition are now allowed, and the new definition invalidates both the cube and inverse cube properties.

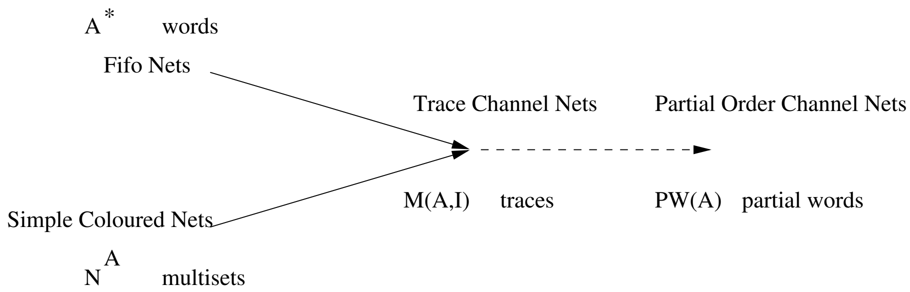
$Aut_{co}(N) = (Q, E, \Theta, \{\|_M\}_{M \in Q})$ where

1. $Q = \{M : C \longrightarrow \aleph^A\}$ is the set of states,
2. $E = T$, the set of events coincides with the set of transitions of N .
3. $\Theta \longrightarrow \subseteq Q \times E \times Q$ is the transition set, defined by $(M, t, M') \in \Theta \Longleftrightarrow M[t > M'$.
4. $\|_M \subseteq E \times E$, defined by $t \|_M t' \iff \forall c \in C, W(c,t) + W(c,t') \leq M(c)$.

The diamond property of $Aut_{co}(N)$ is obvious.

Let be $u \in (A/\approx_I)$, we denote by $[u] \in \aleph^A$ the multiset such that $\forall a \in A$, $[u](a) = |u|_a$.

To any TCNet , we associate a unique Coloured Net by replacing the flow function by its multisets value .



7.4.1 Definition.

Let be $N = (A, I, C, T, W, l)$ a TCNet, the **coloured net associated** to N, is $N_{co} = (A, C, T, [W], l)$ where $[W](x, y) = [W(x, y)]$ for $(x, y) \in C \times T \cup T \times C$.

7.4.2 Lemma.

Let be $N = (A, I, C, T, W, l)$ a TCNet then the pair $([\_], id_E)$ is an automata morphism
$([\_], id_E) : Aut(N, M) \longrightarrow Aut_{co}(N_{co}, [M])$

Proof.

It is based on the fact that $\forall u, v \in (A/\approx_I)$, $u \preceq v \Longrightarrow ([u] \leq [v] \bigwedge [v - u] = [v] - [u])$. The two consequences are:

1. $M[t > M'$ in $N \implies [M][t > [M']$ in N_{co}.
2. If $t \parallel_M t'$ in N, then $M[t >$, $M[t' >$, and $\forall c \in C$, $\alpha(W(c, t)) \times \alpha(W(c, t')) \cap D = \oslash$. This implies that $W(c, t) \circ W(c, t') \preceq M(c)$ and due to $[W(c, t) \circ W(c, t')] = [W(c, t)] + [W(c, t')]$ $t \parallel_M t' \Longrightarrow \forall c \in C, [W(c, t)] + [W(c, t')] \leq M(c)$.

   ○

   This last section has completed the independence partial order $([[N]], \sqsubseteq)$ by the Simple Coloured Net N_{co}.

8 Conclusion.

The figure 4 above shows the inclusions between four classes of Nets, related to the inclusions of the underlying markings and edge labels sets. In the present paper, we have presented a framework introducing the two arrows on the left. Although traces can be viewed as particular partial words, an embedding morphism w.r.t. concatenation has not been defined yet, and the arrow on the right is ongoing work.

Trace Channel Nets seem to be a 'sensible' generalisation of two existing and until now unrelated classes of Petri Nets. They induce Net Theory to follow the development of Formal Language Theory from words languages to traces and to partial words languages. Even if they inheritate their Turing power from Fifo Nets, different tractable classes of interesting problems arise from their definition, and hopefully they will result in specification and optimisation techniques for systems of communicating processes. In particular the use of trace channels in CFSMs should result useful in protocols specifications and proofs, to be applied for Partial Order Connections and Protocols. They seem well adapted to represent concurrent systems where data are structured by order or dependence relations, and that is the case when data are themselves behaviours specifications. Notions of observability, equivalence, refinement and implementation, compositionality or modularity should be developed for a practical use of the model, and are the topic of ongoing work.

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